Calculate The Derivative Of The Function. Then Find The Value Of The Derivative As Specified. F(x)= 8/x+2

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Understanding how to differentiate functions is fundamental in calculus, as derivatives measure how a function changes at any given point. In this article, we will focus on a specific function: $F(x) = \frac{8}{x} + 2$. Our goal is twofold: first, to find the derivative $F'(x)$, which tells us the rate of change of the function at any value of x, and second, to determine the value of this derivative for specific x-values. We will explore the steps involved in differentiation, apply the relevant rules, and interpret the results to deepen our understanding of the function’s behavior.

Understanding the Function $F(x) = \frac{8}{x} + 2$

Breaking Down the Function

The function $F(x) = \frac{8}{x} + 2$ is composed of two parts:

- $\frac{8}{x}$ — a rational function that involves division by x
- $+ 2$ — a constant term

When differentiating such a function, we will need to apply specific rules suitable for each component, particularly the power rule and the constant rule.

Step-by-Step Differentiation of $F(x)$

Rewriting the Function for Ease of Differentiation

To differentiate $F(x) = \frac{8}{x} + 2$, it’s often easier to express the rational term as a power of x:

$$F(x) = 8x^{-1} + 2$$

This transformation simplifies the application of differentiation rules.

Applying Differentiation Rules

We will use the following rules:

- **Power Rule:** If $f(x) = ax^{n}$, then $f'(x) = a n x^{n-1}$
- **Constant Rule:** The derivative of a constant is zero.

Calculating the Derivative $F'(x)$

1. Differentiate $8x^{-1}$:
    - Apply the power rule: $\frac{d}{dx} [8x^{-1}] = 8 \times (-1) x^{-1 - 1} = -8 x^{-2}$
2. Differentiate 2:
    - Derivative of a constant is zero: $\frac{d}{dx} [2] = 0$

Putting it all together, we obtain:

$$F'(x) = -8 x^{-2} + 0 = -8 x^{-2}$$

Rewriting the derivative back into a more familiar form:

$$F'(x) = -\frac{8}{x^{2}}$$

Interpreting The Derivative $F'(x)$

The derivative $F'(x) = -\frac{8}{x^{2}}$ provides insights into the behavior of the original function:

- Since x^{2} is always positive for $x \neq 0$, the sign of $F'(x)$ is negative for all $x \neq 0$.
- This indicates that the function $F(x)$ is decreasing everywhere in its domain (excluding $x=0$).

- The magnitude of the derivative decreases as $|x|$ increases, implying the rate of change becomes less steep for larger $|x|$.

Finding The Value Of The Derivative at Specific Points

Selecting Points for Evaluation

To fully understand the behavior at particular points, evaluate $F'(x)$ at specific x-values:

1. $x = 1$
2. $x = -1$
3. $x = 2$
4. $x = -3$

Calculations for Each Point

At $x = 1$

```
F'(1) = -8 / (1)^2 = -8 / 1 = -8
```

The derivative is -8, indicating the function decreases at a rate of 8 units per unit increase in x at $x=1$.

At $x = -1$

```
F'(-1) = -8 / (-1)^2 = -8 / 1 = -8
```

The derivative is again -8; the function decreases at the same rate at $x=-1$ as at $x=1$.

At $x = 2$

```
F'(2) = -8 / (2)^2 = -8 / 4 = -2
```

The rate of decrease is less steep compared to at $x=1$, with the derivative value -2.

At $x = -3$

```
F'(-3) = -8 / (-3)^2 = -8 / 9 \approx -0.8889
```

The derivative is approximately -0.8889, indicating a gentle decline at this point.

Summary of Results and Their Significance

The derivative $F'(x) = -\frac{8}{x^{2}}$ reveals several key features of the function:

- **Always Negative (except at $x=0$):** The function is decreasing for all $x \neq 0$.
- **The rate diminishes as $|x|$ increases:** The magnitude of the derivative gets smaller with larger $|x|$, implying the function flattens out at distant points.
- **Vertical asymptote at $x=0$:** Since the derivative involves x^{2} in the denominator, the derivative is undefined at $x=0$, corresponding to the original function's vertical asymptote.

Conclusion

In this comprehensive exploration, we differentiated the function $F(x) = \frac{8}{x} + 2$, resulting in the derivative $F'(x) = -\frac{8}{x^{2}}$. This derivative provides essential insights into the function's behavior, showing that it is decreasing everywhere except at $x=0$ where it is undefined. Evaluating the derivative at specific points reveals how the rate of change varies across the domain, emphasizing the importance of derivatives in understanding the dynamics of mathematical functions. Mastery of these differentiation techniques equips students and professionals alike to analyze a wide array of functions and their applications in science, engineering, and beyond.

Frequently Asked Questions

What is the derivative of the function f(x) = 8/x + 2?

The derivative of $f(x) = 8/x + 2$ is $f'(x) = -8/x^2$.

How do you find the derivative of the function f(x) = 8/x + 2?

Rewrite $8/x$ as $8x^{-1}$, then differentiate term-by-term: $d/dx\, [8x^{-1}] = -8x^{-2}$, and $d/dx\, [2] = 0$. So, $f'(x) = -8/x^2$.

What is the value of the derivative f'(x) at x = 4 for the function f(x) = 8/x + 2?

At x = 4, f'(4) = -8 / (4)^2 = -8 / 16 = -0.5.

Calculate the derivative of f(x) = 8/x + 2 and evaluate it at x = 1.

Derivative: f'(x) = -8/x^2. At x = 1, f'(1) = -8 / 1^2 = -8.

If f(x) = 8/x + 2, what is the slope of the tangent line at x = 2?

First find f'(2): f'(2) = -8 / (2)^2 = -8 / 4 = -2. So, the slope at x=2 is -2.

How do you interpret the derivative of f(x) = 8/x + 2 at a specific point?

The derivative at a point gives the slope of the tangent line to the graph of f(x) at that point, indicating the rate of change of the function there.

Additional Resources

Calculate The Derivative Of The Function. Then Find The Value Of The Derivative As Specified. F(x) = 8/x + 2

Understanding how to differentiate functions is a fundamental skill in calculus, essential for analyzing rates of change, slopes of curves, and optimization problems. The process of calculating derivatives allows mathematicians, scientists, and engineers to interpret the behavior of functions more deeply. In this article, we will explore the derivative of the function $F(x) = \frac{8}{x} + 2$, step-by-step, and then determine the value of this derivative at specific points. This exercise not only reinforces the mechanics of differentiation but also clarifies how to interpret the derivative's meaning in context.

Understanding the Function: $F(x) = \frac{8}{x} + 2$

The given function $F(x) = \frac{8}{x} + 2$ is a rational function combined with a constant. It can be re-expressed to facilitate differentiation:

$$
F(x) = 8x^{-1} + 2
$$

Rewriting the function in this form makes it easier to apply differentiation rules, specifically the power rule, which is central to calculus.

Basics of Differentiation: Key Rules

Before diving into the specific derivative, it’s important to recall some fundamental differentiation rules:

- Power Rule: $\frac{d}{dx} [x^n] = n x^{n-1}$

- Constant Multiple Rule: $\frac{d}{dx} [k \cdot f(x)] = k \cdot f'(x)$

- Constant Rule: $\frac{d}{dx} [c] = 0$

- Sum Rule: $\frac{d}{dx} [f(x) + g(x)] = f'(x) + g'(x)$

Applying these rules systematically is essential for accurately calculating derivatives.

Calculating the Derivative of $F(x) = 8x^{-1} + 2$

Let's proceed with the differentiation:

Step 1: Differentiate $8x^{-1}$

Using the power rule:

$$
\frac{d}{dx} [8x^{-1}] = 8 \times \frac{d}{dx} [x^{-1}] = 8 \times (-1) x^{-2} = -8 x^{-2}
$$

Step 2: Differentiate the constant 2

Since the derivative of any constant is zero:

$$
\frac{d}{dx} [2] = 0
$$

Step 3: Combine the derivatives

Using the sum rule:

\[

$$
F'(x) = -8 x^{-2} + 0 = -8 x^{-2}
$$

Step 4: Rewrite the derivative in a more familiar form

Expressing the derivative with positive exponents:

$$
F'(x) = -\frac{8}{x^{2}}
$$

This is the derivative of the function $F(x) = \frac{8}{x} + 2$.

Interpreting the Derivative $F'(x)$

The derivative $F'(x) = -\frac{8}{x^{2}}$ tells us about the rate of change of the original function at any point x. Because x^{2} is always positive for $x \neq 0$, and the numerator is negative, the derivative is negative for all $x \neq 0$. This indicates that $F(x)$ is decreasing everywhere in its domain (excluding zero, where the function is undefined).

Finding the Derivative's Value at Specific Points

Now, let's find the derivative's value at particular points, say $x = 1$ and $x = 2$. This practical step helps visualize how the function’s rate of change varies with x.

At $x = 1$:

$$
F'(1) = -\frac{8}{(1)^2} = -\frac{8}{1} = -8
$$

Interpretation: At $x = 1$, the function decreases at a rate of 8 units per unit increase in x.

At $x = 2$:

$$
F'(2) = -\frac{8}{(2)^2} = -\frac{8}{4} = -2
$$

Interpretation: At $x = 2$, the function decreases at half the rate compared to $x=1$. This trend signifies that as x increases, the magnitude of the rate of change diminishes, consistent with the reciprocal nature of the original function.

Applications and Significance of the Derivative Calculation

Calculating derivatives like $F'(x)$ is crucial in various fields:

- Physics: Derivatives represent velocities and accelerations.

- Economics: Marginal costs and revenues are derivatives of cost and revenue functions.

- Biology: Rate of change in populations over time.

- Engineering: Analyzing how systems respond to changing inputs.

Understanding the derivative of $F(x) = \frac{8}{x} + 2$ enables one to predict how the function behaves as x varies, such as identifying where the function decreases rapidly versus slowly.

Features and Pros/Cons of the Differentiation Approach

Features:

- Systematic: Uses well-established rules for clarity and accuracy.

- Flexible: Can handle more complex functions by combining rules like chain rule, product rule, etc.

- Educational: Reinforces fundamental calculus principles.

Pros:

- Simplifies complex functions into manageable steps.

- Provides precise quantitative measures of change.

- Facilitates understanding of the function's behavior.

Cons:

- Can be error-prone if rules are misapplied.

- Less straightforward for functions involving multiple variables or more complex compositions.

- Requires a solid grasp of calculus rules and algebraic manipulation.

Conclusion

Calculating the derivative of $F(x) = \frac{8}{x} + 2$ demonstrates the power and elegance of calculus. By converting the function into a power form and applying the power rule, we arrive at a simple yet insightful expression for the rate of change:

$$
F'(x) = -\frac{8}{x^{2}}
$$

This derivative reveals that the function is decreasing everywhere in its domain, with the rate diminishing as x increases. Evaluating the derivative at specific points like $x=1$ and $x=2$ provides tangible insights into how quickly the function changes at different values.

Understanding how to compute derivatives accurately is a cornerstone of mathematical analysis, and mastering these techniques opens doors to solving real-world problems across science, engineering, and economics. Whether analyzing the slope of a curve or predicting the behavior of complex systems, the principles demonstrated here are foundational tools for any calculus enthusiast.

Note: Always be cautious with the domain of the original function, especially with rational functions involving division by zero. In this case, $x \neq 0$ is necessary for the function and its derivative to be defined.

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