Compute The Surface Integral Over The Given Oriented Surface: =9 4, Portion Of The Plane =1 In The Octant

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Understanding how to compute surface integrals is fundamental in multivariable calculus, especially when dealing with vector fields and different geometrical surfaces. In this comprehensive guide, we will walk through the process of calculating the surface integral over a specific oriented surface, which is a portion of a plane within the first octant. The problem involves the surface defined by the equation z = 4 - 9x/4, restricted to the octant where x, y, and z are all non-negative, and the orientation of the surface is specified. By the end of this article, you'll have a clear understanding of the steps involved, the underlying concepts, and how to apply these techniques to similar problems.

Understanding the Problem Setup

Before diving into calculations, it’s crucial to interpret the problem properly.

Given Data and Context

- The surface is a portion of the plane: z = 4 - (9/4) x.
- The surface lies in the first octant, meaning x □ 0, y □ 0, z □ 0.
- The surface is oriented in a specific manner, which influences the direction of the surface normal vector.

- The goal is to compute the surface integral of a vector field over this surface, which involves integrating the dot product of the vector field with the surface's differential area element.

Note: While the problem statement mentions "=9 4" and "portion of the plane =1," it appears to be shorthand for defining the plane and the specific portion within the octant. Clarification of the limits and the vector field involved will be addressed in the upcoming sections.

Step 1: Visualizing and Parametrizing the Surface

To compute a surface integral, the first step involves parametrizing the surface.

Parametrization of the Plane

Since the surface is a portion of the plane $z = 4 - (9/4) x$, and bounded within the first octant, we can describe the surface using parameters x and y:

- Let’s choose parameters: $x = x$, $y = y$
- Then, $z = 4 - (9/4) x$

However, note that z depends only on x, so for a fixed x, y varies freely within the bounds determined by the octant and the plane's intersection.

Parametric equations:

$$
\vec{r}(x, y) = \langle x, y, 4 - \frac{9}{4} x \rangle
$$

where:

- $x \geq 0$
- $y \geq 0$
- z □ 0 implies $4 - \frac{9}{4} x \geq 0$ □ $x \leq \frac{16}{9}$

Limits of integration:

- x ranges from 0 to $\frac{16}{9}$
- For each fixed x, y varies from 0 to the boundary of the surface in the y-direction, which, since the plane extends infinitely unless bounded, but given the "portion" of the plane in the octant, perhaps bounded by the axes or other constraints, typically y ranges from 0 to some maximum. If no other boundary is specified, the surface may be considered unbounded, but in typical problems, the limits are finite. For demonstration, assume y ranges from 0 to some y_max(x), which might be arbitrary or specified.

In many problems, the boundary in y is determined by the intersection with another plane or boundary. If none is specified, for simplicity, assume the surface is bounded by y from 0 to some maximum y, or that the problem involves a specific vector field and boundary conditions.

Simplified Assumption for Calculation

Suppose the problem considers the vector field $\vec{F} = \langle P, Q, R \rangle$, and the surface is bounded at x = 0 and x = 16/9, y from 0 to some y_max, or perhaps y is bounded by a particular curve.

For the purpose of this explanation, assume the surface is limited by y from 0 to y = 3, just as an example.

Step 2: Computing the Surface Element d$\vec{S}$

The differential surface element depends on the parametrization.

Calculating the Surface Normal Vector

Given the parametrization:

$$
\vec{r}(x, y) = \langle x, y, 4 - \frac{9}{4} x \rangle
$$

the partial derivatives are:

$$
\frac{\partial \vec{r}}{\partial x} = \langle 1, 0, -\frac{9}{4} \rangle
$$

$$
\frac{\partial \vec{r}}{\partial y} = \langle 0, 1, 0 \rangle
$$

The surface element vector is given by the cross product:

$$
\vec{r}_x \times \vec{r}_y
$$

Calculating this:

$$
\vec{r}_x \times \vec{r}_y =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 0 & -\frac{9}{4} \\
0 & 1 & 0
\end{vmatrix}
= \mathbf{i} \left(0 \times 0 - (-\frac{9}{4}) \times 1 \right) - \mathbf{j} \left(1 \times 0 - (-\frac{9}{4}) \times 0 \right) + \mathbf{k} \left(1 \times 1 - 0 \times 0 \right)
$$

Simplifying:

$$
= \mathbf{i} \left(0 + \frac{9}{4} \right) - \mathbf{j} (0 - 0) + \mathbf{k} (1 - 0)
$$

$$
= \left\langle \frac{9}{4}, 0, 1 \right\rangle
$$

Orientation: The surface's orientation is specified, which determines whether we take this vector or its negative. For now, assume the orientation is such that the normal vector points outward in the positive z-direction.

Step 3: Setting Up the Surface Integral

The general form of the surface integral over a vector field $\vec{F} = \langle P, Q, R \rangle$ is:

$$
\iint_{S} \vec{F} \cdot d\vec{S}
$$

where:

$$
d\vec{S} = \vec{r}_x \times \vec{r}_y \, dx \, dy
$$

Thus, the integral becomes:

$$
\iint_{D} \vec{F}(\vec{r}(x, y)) \cdot (\vec{r}_x \times \vec{r}_y) \, dx \, dy
$$

where D is the projection of the surface onto the xy-plane, i.e., the domain for x and y.

Example: Computing the Surface Integral for a Specific Vector Field

Suppose the vector field is:

$$
\vec{F} = \langle y, x, z \rangle
$$

Then, the dot product becomes:

\[

$$
\vec{F}(\vec{r}(x, y)) \cdot \left\langle \frac{9}{4}, 0, 1 \right\rangle
$$

where:

$$
\vec{F}(\vec{r}(x, y)) = \langle y, x, 4 - \frac{9}{4} x \rangle
$$

The dot product:

$$
= y \times \frac{9}{4} + x \times 0 + \left(4 - \frac{9}{4} x \right) \times 1
$$

Simplifies to:

$$
\frac{9}{4} y + 4 - \frac{9}{4} x
$$

The surface integral becomes:

$$
\iint_{D} \left(\frac{9}{4} y + 4 - \frac{9}{4} x \right) dx dy
$$

with the domain D defined by:

- $x \in [0, 16/9]$
- $y \in [0, y_{max}(x)]$

Assuming y ranges from 0 to some boundary, for simplicity, say y from 0 to 3.

Step 4: Computing the Double Integral

Assuming y from 0 to 3:

\[
\int_{x=0}^{16/9} \int_{y=0}^{3} \left(\frac{9}{4} y + 4 - \frac{9}{4} x \right) dy dx
\]

First, integrate with respect to y:

\[
\int_{0}^{3} \left(

Frequently Asked Questions

What is the main goal when computing the surface integral over the given oriented surface?

The main goal is to evaluate the surface integral of a vector or scalar field over the specified surface, taking into account its orientation and boundaries.

How do you determine the surface element dS for the portion of the

plane in the octant?

The surface element dS can be found by parameterizing the surface and computing the magnitude of the cross product of the partial derivatives, or by using the projection method, considering the plane's equation and the bounds in the octant.

What is the significance of the orientation in computing the surface integral?

The orientation determines the direction of the surface normal vector, which affects the sign and value of the surface integral, ensuring consistency with the problem's specified direction.

How do you set up the bounds for the octant portion of the plane?

Since the surface is in the octant (positive x, y, z), the bounds are typically from 0 to the maximum extent of the plane in each coordinate, with z expressed in terms of x and y based on the plane equation.

What is the equation of the plane given in the problem, and how does it relate to the surface integral?

The plane's equation is typically of the form ax + by + cz = d; in this case, with the given parameters, it defines the surface over which the integral is computed, and the equation guides the parameterization.

What steps are involved in converting the surface integral into a double integral over the projection domain?

First, parameterize the surface or express z in terms of x and y, then determine the Jacobian or the surface element dS, and finally integrate over the projected region in the xy-plane within the octant boundaries.

What are common challenges faced when computing surface integrals over portions of planes in the octant?

Challenges include correctly setting the bounds in the positive octant, accurately computing the surface element, maintaining the proper orientation, and simplifying the resulting double integral.

Are there any specific formulas or techniques recommended for this particular problem?

Yes, using the parameterization z = (d - ax - by)/c, calculating dS using the cross product of partial derivatives, and converting the surface integral into a double integral over the xy-plane are standard approaches for such problems.

Additional Resources

Compute The Surface Integral Over The Given Oriented Surface: =9 4, Portion Of The Plane =1 In The Octant

In the realm of vector calculus, surface integrals serve as a powerful tool to evaluate flux, flow, and other physical phenomena across surfaces. Today, we delve into a specific problem: calculating the surface integral over an oriented surface defined by a portion of a plane within the octant, characterized by the parameters "=9 4" and "portion of the plane =1." While these parameters may seem cryptic at first glance, they unfold into a fascinating mathematical exploration that combines geometric intuition with analytical rigor. This article aims to clarify the concepts, approach, and step-by-step process involved in solving such a problem, making the complex accessible and understandable.

Understanding the Problem: Surface Integral and Its Context

Before diving into calculations, it's essential to understand what a surface integral entails and how it applies to the problem at hand.

What is a Surface Integral?

A surface integral extends the idea of integrating functions over a line or an area, allowing us to compute quantities like flux across a surface. Specifically, for a vector field F and a surface S, the flux integral:

$$
\iint_S \mathbf{F} \cdot \mathbf{n}\, dS
$$

measures how much of the field F passes through S. Here, n is the unit normal vector to the surface, and dS is an infinitesimal element of surface area.

The Given Data:

- The surface is a portion of a plane.
- The plane is defined or characterized by the parameters "=9 4" and "portion of the plane =1."
- The surface resides in the octant, meaning all coordinates are positive ($x, y, z \geq 0$).

Deciphering the Parameters:

While the notation "=9 4" and "portion of the plane =1" might seem abstract, in typical calculus problems, these could represent:

- The plane equation coefficients, e.g., $9x + 4y + z = c$.
- The specific part of the plane (bounded region) lying within the first octant, possibly between certain

limits.

Assuming standard interpretations, the problem likely involves:

- A plane expressed as $9x + 4y + z = 1$.
- The portion of this plane located within the octant ($x, y, z \geq 0$).

This assumption aligns with common textbook problems where the plane intersects the coordinate axes, forming a bounded region.

Defining the Surface: The Geometric Setup

Step 1: Establish the Plane Equation

Given the parameters, we interpret the plane as:

$$
9x + 4y + z = 1
$$

This is a linear plane with intercepts on the axes:

- When $y = 0, z = 0$:

$$
x = \frac{1}{9}
$$

- When $x = 0, z = 0$:

$$
y = \frac{1}{4}
$$

- When $x = 0, y = 0$:

$$
z = 1
$$

Step 2: Identify the Portion of the Plane in the Octant

Within the octant where all variables are positive, the bounded region is the triangle formed by the intercepts:

- $(x, 0, 0)$ with $x \in [0, 1/9]$,
- $(0, y, 0)$ with $y \in [0, 1/4]$,
- $(0, 0, z)$ with $z \in [0, 1]$.

The part of the plane between these intercepts forms a triangular surface bounded in the positive octant.

Parameterization of the Surface

To evaluate the surface integral, we need a parameterization of the surface. Since the surface is part of a plane, a convenient approach is to express it using two parameters, say x and y, and solve for z:

$$
z = 1 - 9x - 4y
$$

Parameterization:

$$
\mathbf{r}(x, y) = \langle x, y, 1 - 9x - 4y \rangle
$$

where:

- $x \geq 0$,
- $y \geq 0$,
- $z \geq 0 \Rightarrow 1 - 9x - 4y \geq 0$.

Region of Integration in the xy-plane:

$$
1 - 9x - 4y \geq 0 \Rightarrow 4y \leq 1 - 9x \Rightarrow y \leq \frac{1 - 9x}{4}
$$

with:

$$
x \geq 0, \quad y \geq 0, \quad y \leq \frac{1 - 9x}{4}
$$

and

\[
x \leq \frac{1}{9}
\]

The feasible region \(R \) in the \(xy \)-plane is:

\[
0 \leq x \leq \frac{1}{9}
\]
\[
0 \leq y \leq \frac{1 - 9x}{4}
\]

Calculating the Surface Element \(dS \)

Given the parameterization \(\mathbf{r}(x, y) \), the surface element \(dS \) can be computed using the magnitude of the cross product of the partial derivatives:

\[
\mathbf{r}_x = \frac{\partial \mathbf{r}}{\partial x} = \langle 1, 0, -9 \rangle
\]
\[
\mathbf{r}_y = \frac{\partial \mathbf{r}}{\partial y} = \langle 0, 1, -4 \rangle
\]

The surface element:

\[

$$
dS = |\mathbf{r}_x \times \mathbf{r}_y| \, dx\,dy
$$

Cross product:

$$
\mathbf{r}_x \times \mathbf{r}_y =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 0 & -9 \\
0 & 1 & -4
\end{vmatrix}
$$

Calculating the determinant:

$$
= \mathbf{i} (0 \times -4 - (-9) \times 1) - \mathbf{j} (1 \times -4 - (-9) \times 0) + \mathbf{k} (1 \times 1 - 0 \times 0)
$$

$$
= \mathbf{i} (0 + 9) - \mathbf{j} (-4 - 0) + \mathbf{k} (1 - 0)
$$

$$
= 9 \mathbf{i} + 4 \mathbf{j} + 1 \mathbf{k}
$$

Magnitude:

$$
|\mathbf{r}_x \times \mathbf{r}_y| = \sqrt{9^2 + 4^2 + 1^2} = \sqrt{81 + 16 + 1} = \sqrt{98} = 7 \sqrt{2}
$$

Thus,

$$
dS = 7 \sqrt{2} \, dx\,dy
$$

Formulating the Surface Integral

The general form of the surface integral of a scalar function f over surface S:

$$
\iint_S f \, dS
$$

or, for a vector field $\mathbf{F}$:

$$
\iint_S \mathbf{F} \cdot \mathbf{n} \, dS
$$

In our problem, the specific integrand hasn't been explicitly provided. Assuming the task involves integrating a scalar function $f(x, y, z)$, or perhaps flux, the process would involve:

- Expressing f in terms of x and y,

- Calculating the dot product if integrating a vector field,
- Integrating over the region \(R \).

For simplicity, suppose the integrand is a scalar function \(f(x, y, z) \). Since the problem mentions "Compute the surface integral" generally, let's assume the integrand is 1, representing the surface area.

Evaluating the Surface Area (if \(f=1\))

The surface area \(A \) of the portion of the plane in the octant:

\[
A = \iint_R |\mathbf{r}_x \times \mathbf{r}_y| \, dx\,dy
\]

which simplifies to:

\[
A = 7 \sqrt{2} \int_{x=0}^{1/9} \int_{y=0}^{(1 - 9x)/4} dy\, dx
\]

Step-by-step:

1. Inner integral over \(y \):

\

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