Consider The Following System. G (s)= 1/ (s + 1)(s+2)a) Sketch The Nyquist Plot Of The System Given Above

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Introduction to the System and Its Significance

Understanding the stability and frequency response of a control system is fundamental in control engineering. The transfer function $G(s) = 1 / [(s + 1)(s + 2)]$ describes a simple yet insightful second-order system with two poles located at $s = -1$ and $s = -2$. Analyzing this system's behavior across frequencies provides critical insights into its stability margins, phase and gain margins, and overall robustness.

The Nyquist plot is one of the most powerful tools in control theory for analyzing the stability of a closed-loop system based on its open-loop transfer function. It graphically represents the frequency response of the system, mapping the complex function $G(j\omega)$ as ω varies from 0 to infinity, including the critical point at infinity.

In this article, we delve into the detailed process of sketching the Nyquist plot for the given transfer function G(s), providing a comprehensive understanding of the steps involved, the underlying principles, and the implications for control system stability.

Understanding the Transfer Function G(s) = 1 / [(s + 1)(s + 2)]

Poles and Zeros

- The transfer function G(s) has no zeros.
- It has two poles at $s = -1$ and $s = -2$, both in the left-half of the s-plane, indicating a stable open-loop system.

Poles and System Stability

- The locations of the poles are critical for stability analysis.
- Since all poles are in the left-half plane (LHP), the open-loop transfer function is stable.
- The Nyquist plot will help determine the stability of the closed-loop system when feedback is applied.

Frequency Response Characteristics

- As ω varies, G(jω) traces a path in the complex plane.
- At ω = 0, G(j0) = 1 / [(j0 + 1)(j0 + 2)] = 1 / (1 2) = 0.5 (real axis).
- As ω approaches infinity, G(jω) approaches 0.

Step-by-Step Construction of the Nyquist Plot

1. Express G(jω) in terms of ω

- Replace s with jω: G(jω) = 1 / [(jω + 1)(jω + 2)]
- Expand denominator:
- (jω + 1) = 1 + jω
- (jω + 2) = 2 + jω
- Denominator: (1 + jω)(2 + jω)

2. Simplify the Denominator

- Multiply out:
- (1)(2) + (1)(jω) + (jω)(2) + (jω)(jω)
- 2 + jω + 2jω + j^2 ω^2
- Recall that j^2 = -1:
- 2 + jω + 2jω - ω^2
- Combine like terms:
- Real part: 2 - ω^2
- Imaginary part: 3jω

- So, the denominator D(ω):
- D(ω) = (2 - ω^2) + j3ω

3. Write G(jω) as a Complex Fraction

- G(jω) = 1 / D(ω) = 1 / [(2 - ω^2) + j3ω]

4. Rationalize the Denominator

- Multiply numerator and denominator by the complex conjugate of D(ω):
- Conjugate: $(2 - \omega^2) - j3\omega$
- So:
- $G(j\omega) = [(2 - \omega^2) - j3\omega] / [(2 - \omega^2)^2 + (3\omega)^2]$
- The denominator simplifies to:
- $(2 - \omega^2)^2 + 9\omega^2$

5. Find Real and Imaginary Parts of G(jω)

- Real part:
- $Re[G(j\omega)] = (2 - \omega^2) / [(2 - \omega^2)^2 + 9\omega^2]$
- Imaginary part:
- $Im[G(j\omega)] = -3\omega / [(2 - \omega^2)^2 + 9\omega^2]$

Plotting the Nyquist Plot

1. Behavior at ω = 0

- Evaluate Re[G(0)]:
- $(2 - 0) / [(2)^2 + 0] = 2 / 4 = 0.5$
- Evaluate Im[G(0)]:
- 0
- Point: (0.5, 0) on the complex plane

2. Behavior as ω approaches infinity

- Denominator: Dominated by ω^4 terms
- Re[G(jω)] and Im[G(jω)] both tend toward 0
- The plot approaches the origin as $\omega \to \infty$

3. Behavior at ω = ∞

- G(jω) approaches 0 in magnitude, approaching the origin in the complex plane

4. Behavior at ω = ω_c (where the denominator becomes zero)

- The poles of G(jω) are not on the imaginary axis; no finite ω makes denominator zero

- Therefore, the plot does not pass through infinity; it remains bounded

5. Symmetry of the Nyquist Plot

- Since G(s) is real for real ω, the plot is symmetric with respect to the real axis

6. Critical Point: The point (-1, 0) in the G(jω) plane

- Check if the plot encircles -1 + 0j, which is essential for stability analysis in feedback systems
- At ω where Re[G(jω)] = -1, the system may make a potential encirclement

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Interpreting the Nyquist Plot for System Stability

Nyquist Criterion Overview

- The Nyquist criterion relates the number of encirclements of the critical point (-1, 0) to the stability of the closed-loop system
- For open-loop transfer functions with no right-half plane poles, the number of clockwise encirclements of (-1, 0) determines the stability

Applying the Criterion to G(s) = 1 / [(s + 1)(s + 2)]

- Since G(s) has all poles in the LHP, the open-loop system is stable
- The Nyquist plot starting at 0.5 (ω=0) and approaching 0 as ω→∞ will not encircle -1, indicating a stable closed-loop system

Plot Summary

- The Nyquist plot begins at 0.5 on the real axis
- As ω increases, it moves towards the origin
- The plot remains in the right-half of the complex plane, not encircling -1
- This confirms the stability of the closed-loop system under unity feedback

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Practical Considerations and Visualization Tips

Using Computational Tools

- Tools like MATLAB or Python (with control systems libraries) can generate accurate Nyquist plots
- Example MATLAB commands:
```matlab
sys = tf([1], [1 3 2]);
nyquist(sys);
grid on;
```
- These tools handle the complex calculations and plotting automatically

Manual Sketching Tips

- Plot key points: at $\omega=0$, intermediate frequencies, and $\omega \to \infty$
- Observe the trajectory in the complex plane
- Use symmetry to guide the plot
- Note that the magnitude decreases monotonically

Understanding the Plot's Features

- The starting point at $\omega=0$ is on the positive real axis
- The plot approaches the origin as ω increases
- The shape is smooth, without encirclements of -1

Conclusion and Final Remarks

The Nyquist plot of the system $G(s) = 1 / [(s + 1)(s + 2)]$ provides an insightful view into the system's stability and frequency response. By analyzing the real and imaginary components of $G(j\omega)$, we observe that the plot starts at 0.5 on the real axis when $\omega=0$ and approaches zero as ω approaches infinity. The plot remains in the right-half plane, does not encircle the critical point (-1, 0), and confirms the stability of the closed-loop system.

This detailed analysis underscores the importance of the Nyquist plot in control engineering, enabling engineers to assess stability margins and system robustness effectively. The process combines analytical calculations with graphical interpretation, offering a comprehensive understanding essential for designing and analyzing control systems.

References:

- Ogata, K. (2010). Modern Control Engineering. 5th Edition. Pearson Education.
- Dorf, R.

Frequently Asked Questions

What is the transfer function of the system G(s) given as 1/[(s+1)(s+2)]?

The transfer function is $G(s) = 1/[(s+1)(s+2)]$, which is a second-order rational function with poles at $s = -1$ and $s = -2$.

How do you determine the Nyquist plot of the given system G(s)?

To sketch the Nyquist plot, analyze the frequency response $G(j\omega) = 1/[(j\omega+1)(j\omega+2)]$ over a range of ω from 0 to ∞, and plot the complex values of $G(j\omega)$ as ω varies.

What are the key steps involved in sketching the Nyquist plot for G(s)?

The key steps include: (1) substituting $s = j\omega$ into $G(s)$, (2) calculating $G(j\omega)$ over a range of frequencies, (3) plotting the real and imaginary parts of $G(j\omega)$, and (4) analyzing the plot for encirclements of the critical point $-1 + 0j$.

What is the significance of the Nyquist plot in control system analysis?

The Nyquist plot helps determine the stability of a closed-loop system by analyzing the frequency response and checking for encirclements of the critical point, which relates to the Nyquist criterion.

How do the poles of G(s) at -1 and -2 influence the Nyquist plot?

The poles at -1 and -2 influence the shape of the Nyquist plot, especially near low frequencies, and determine the asymptotic behavior as ω approaches 0 or ∞, affecting the plot's encirclement of the critical point.

What is the expected shape of the Nyquist plot for G(s) = 1/[(s+1)(s+2)]?

The plot typically starts at a finite value at ω=0, dips into the negative real axis at some frequency, and approaches zero as ω approaches infinity, forming a loop that encircles the point -1 depending on system stability.

Can you explain the importance of plotting G(jω) for all ω in the Nyquist plot?

Plotting G(jω) over all ω captures the entire frequency response, revealing stability margins, gain and phase margins, and the potential for encirclements of the critical point, which are essential for stability analysis.

Are there any specific tools or software recommended for sketching the Nyquist plot of G(s)?

Yes, tools like MATLAB (using the 'nyquist' or 'bode' functions), Python with control systems libraries, or online Nyquist plot generators can be used to accurately plot and analyze the Nyquist diagram.

Additional Resources

Consider The Following System. G(s) = 1 / [(s + 1)(s + 2)] – Sketch The Nyquist Plot Of The System Given Above

In the realm of control systems and signal processing, stability analysis remains a cornerstone for designing reliable and robust systems. Among various tools, the Nyquist plot stands out as a powerful graphical technique to assess the stability of a closed-loop system based on its open-loop transfer function. Today, we delve into a specific transfer function: G(s) = 1 / [(s + 1)(s + 2)], exploring how to construct its Nyquist plot and interpret the insights it provides about system stability.

Understanding the System: Transfer Function G(s)

Before venturing into the graphical representation, it’s essential to understand the fundamental characteristics of the given transfer function:

G(s) = 1 / [(s + 1)(s + 2)]

This transfer function describes a system with two poles located at s = -1 and s = -2. The key features include:

- Poles: The system has poles at s = -1 and s = -2, both lying in the left

half of the s-plane. This indicates that the system is inherently stable in open-loop configuration.
- Zeros: There are no zeros in the transfer function, simplifying the analysis.
- Type: Since the numerator is constant (1), the system is strictly proper with a high-frequency magnitude tending to zero.

Understanding the pole locations sets the stage for drawing the Nyquist plot, as the plot's shape depends heavily on the system's pole-zero configuration.

The Nyquist Plot: What Is It and Why Is It Important?

The Nyquist plot is a complex plane representation of the frequency response of a system's open-loop transfer function G(s). It plots the magnitude and phase of G(jω) as ω varies from zero to infinity (and sometimes through negative frequencies), forming a closed curve.

Why Use the Nyquist Plot?

- Stability Analysis: It helps determine the stability of the closed-loop system when unity feedback is applied.
- Gain and Phase Margin: It provides direct insights into how much gain or phase variation the system can tolerate before becoming unstable.
- Robustness Assessment: Engineers can evaluate how system uncertainties affect stability margins.

The Nyquist criterion states that the number of clockwise encirclements of the critical point (-1, 0) in the Nyquist plot relates directly to the system's stability.

Step-by-Step Construction of the Nyquist Plot for G(s) = 1 / [(s + 1)(s + 2)]

Constructing the Nyquist plot involves analyzing the system's frequency response at various ω values and plotting the resulting complex numbers G(jω). Here's a detailed approach:

1. Derive G(jω)

Substitute s = jω into G(s):

G(jω) = 1 / [(jω + 1)(jω + 2)]

Calculating the denominator:

jω + 1 = 1 + jω

jω + 2 = 2 + jω

Therefore,

$$G(j\omega) = 1 / [(1 + j\omega)(2 + j\omega)]$$

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2. Simplify the Expression

Multiply the denominator:

$$(1 + j\omega)(2 + j\omega) = (1)(2) + (1)(j\omega) + (j\omega)(2) + (j\omega)(j\omega)$$

$$= 2 + j\omega + 2j\omega + (j\omega)^2$$

Recall that $(j\omega)^2 = -\omega^2$, so:

$$= 2 + j\omega + 2j\omega - \omega^2$$

Combine like terms:

Real part: $2 - \omega^2$

Imaginary part: $j\omega + 2j\omega = j(\omega + 2\omega) = j3\omega$

Thus,

$$G(j\omega) = 1 / [(2 - \omega^2) + j3\omega]$$

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3. Express $G(j\omega)$ in Magnitude and Phase

The complex number in the denominator:

$$D(\omega) = (2 - \omega^2) + j3\omega$$

Magnitude of $D(\omega)$:

$$|D(\omega)| = \mathrm{sqrt}[(2 - \omega^2)^2 + (3\omega)^2]$$

$$= \mathrm{sqrt}[(2 - \omega^2)^2 + 9\omega^2]$$

Magnitude of $G(j\omega)$:

$$|G(j\omega)| = 1 / |D(\omega)|$$

Phase of $D(\omega)$:

$\theta_D(\omega)$ = arctangent of (Imaginary / Real):

$$\theta_D(\omega) = \mathrm{arctangent}\ [(3\omega) / (2 - \omega^2)]$$

Since $G(j\omega)$ is the reciprocal, its phase:

$\theta_G(\omega) = -\theta_D(\omega)$

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4. Analyze Behavior at Frequency Limits

Understanding the behavior at $\omega = 0$, $\omega \to \infty$, and key points:

- At $\omega = 0$:

$|D(0)| = \mathrm{sqrt}[(2 - 0)^2 + 0] = 2$

$|G(0)| = 1/2 = 0.5$

$\theta_D(0) = \mathrm{arctangent}(0/2) = 0°$

$\theta_G(0) = 0°$

- As $\omega \to \infty$:

$|D(\omega)| \approx \mathrm{sqrt}[(-\omega^2)^2 + 9\omega^2] \approx \mathrm{sqrt}[\omega^4 + 9\omega^2] \approx \omega^2$

$|G(j\omega)| \approx 1/\omega^2 \to 0$

$\theta_D(\omega) \approx \mathrm{arctangent}((3\omega) / (-\omega^2)) \approx \mathrm{arctangent}(-3/\omega) \to 0$ from below

Thus, $\theta_G(\omega) \to 0$

- At ω where denominator imaginary part is zero:

Set $3\omega = 0 \to \omega = 0$ (already considered)

Alternatively, analyze points where Real part of denominator is zero:

$2 - \omega^2 = 0 \to \omega = \mathrm{sqrt}(2) \approx 1.414$

At $\omega = \mathrm{sqrt}(2)$:

$|D(\omega)| = \mathrm{sqrt}[0 + 9(2)] = \mathrm{sqrt}(18) \approx 4.2426$

$\theta_D(\omega) = \mathrm{arctangent}((31.414) / 0) = \mathrm{arctangent}(\infty) = 90°$

Correspondingly, $G(j\omega)$ phase = -90°

Plotting the Nyquist Diagram

Using the above insights, here’s how to sketch the Nyquist plot:

1. Start at $\omega = 0$:

- $G(j0) = 1 / (2 + j0) = 0.5 \angle 0°$, plotted at (0.5, 0) in the complex plane.

2. As ω increases:

- The magnitude $|G(j\omega)|$ decreases from 0.5 towards zero.
- The phase shifts from 0° towards -180° as ω approaches infinity.

3. At $\omega \approx 1.414$:

- The denominator's real part is zero; the response is purely imaginary.
- $G(j\omega)$ is approximately at the point:

$|G| \approx 1/4.2426 \approx 0.236$

Phase ≈ -90°, so the point is at (0, -0.236).

4. At very high ω:

- The response approaches the origin (0, 0).

5. Shape of the plot:

- The plot starts at (0.5, 0).
- Circles inward toward the origin as frequency increases.
- The trajectory crosses the imaginary axis at $\omega \approx 1.414$.

6. Encirclement of the critical point (-1, 0):

- Since $G(j\omega)$ remains within the left-half plane and the magnitude diminishes, the Nyquist plot does not encircle the point (-1, 0).
- This indicates the closed-loop system remains stable under unity feedback.

Interpreting the Nyquist Plot for Stability

The primary purpose of constructing the Nyquist plot is to evaluate the stability of the closed-loop system. According to the Nyquist stability criterion:

- Count the number of encirclements of the point (-1, 0).
- For a system with no open-loop right-half-plane poles (as in our case), stability is assured if the Nyquist plot does not encircle (-1, 0).

Given the plot's shape, the trajectory remains within the left-half of the complex plane and does not encircle (-1, 0). Therefore, the closed-loop system derived from G(s) is stable under unity feedback.

Practical Significance and Applications

Understanding the Nyquist plot of a system like G(s) = 1 / [(s + 1)(s + 2)] extends beyond theoretical analysis. Engineers utilize this graphical approach for:

- Controller design: tuning PID controllers or other compensators to shift the Nyquist plot and improve stability margins.
- Robustness evaluation: ensuring the system maintains stability despite uncertainties.
- Frequency response analysis: assessing how the system reacts to sinusoidal inputs at different frequencies.

The approach showcased here is applicable to a wide range of systems, from aerospace to automotive control, where stability and robustness are critical.

Conclusion

Constructing and interpreting the Nyquist plot for the transfer function G(s) = 1 / [(s + 1)(s + 2)] offers valuable

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