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Understanding vectors and their operations forms a fundamental part of linear algebra and vector calculus. When given specific vectors, such as v = (-4, 3, -2) and w = (8, -7.8), along with scalar multipliers like (2, -1, -1), the core mathematical challenge often revolves around expressing new vectors as linear combinations of these existing vectors or determining their relationships through dot products, cross products, and other vector operations. This article aims to provide a comprehensive explanation of how to approach such problems, emphasizing the importance of clarity in vector operations, the concept of linear combinations, and the conditions under which certain vector expressions are possible.

Understanding the Given Vectors and Scalars

Before delving into the problem, it is essential to understand the vectors and scalars involved:

Given Vectors:

- v = (-4, 3, -2)
- w = (8, -7.8)

Scalar Multipliers:

- (2, -1, -1)

Note that the vector w is given in two dimensions, whereas v is in three dimensions. This discrepancy suggests that either the problem involves embedding w into three dimensions (perhaps by adding a zero in the third coordinate) or that the problem is primarily focused on the three-dimensional vectors involving v and the scalar multipliers.

Clarifying the Objective of the Problem

The problem states:

> "Express, If Possible, The Vector of at least 1000 words."

However, this appears to be a typo or incomplete phrasing, likely intending to ask:

> "Express, if possible, the vector resulting from a specific operation involving the given vectors and scalars."

Given the context, a common interpretation is that the problem asks to:
- Express the vector as a linear combination of v and w, or
- Find a scalar multiple of v or w that equals a certain vector,
- Or determine if the vectors are linearly independent or dependent.

For clarity, this guide will focus on the most common interpretation:

"Express a given vector as a linear combination of the vectors v and w, using the specified scalar multipliers or find the vector resulting from the linear combinations."

Linear Combinations of Vectors: The Core Concept

A linear combination of vectors involves multiplying each vector by a scalar and summing the results:

$$
\text{Given vectors } \mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n \text{ and scalars } c_1, c_2, ..., c_n,
$$

the linear combination is:

$$
c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \cdots + c_n \mathbf{v}_n.
$$

Key points:
- Linear combinations are fundamental to understanding span, basis, and linear independence.
- If a vector $\mathbf{u}$ can be expressed as a linear combination of vectors v and w, then:

$$
\mathbf{u} = a \mathbf{v} + b \mathbf{w},
$$

where a and b are scalars.

Expressing a Vector as a Linear Combination of Given Vectors

Suppose we are asked to find scalars a and b such that:

$$
\mathbf{u} = a \mathbf{v} + b \mathbf{w}.
$$

Given that v is in $\mathbb{R}^3$, but w appears to be in $\mathbb{R}^2$, we must adjust for dimensional consistency:

Step 1: Extend w to three dimensions by adding a zero component:

$$
\mathbf{w} = (8, -7.8, 0).
$$

Step 2: Assume the target vector $\mathbf{u}$ is known or to be determined.

If the problem asks to express a particular vector $\mathbf{u}$, such as $\mathbf{u} = (x, y, z)$, as a linear combination of v and w, then:

$$
(x, y, z) = a(-4, 3, -2) + b(8, -7.8, 0).
$$

This leads to a system of equations:

$$
\begin{cases}
-4a + 8b = x \\
3a - 7.8b = y \\
-2a + 0b = z
\end{cases}
$$

Step 3: Solve the system for a and b.

Solving the System of Equations

Suppose, for illustration, that $\mathbf{u} = (x, y, z) = (0, 0, 0)$, i.e., the zero vector, as a basis for understanding.

The system becomes:

$$
\begin{cases}
-4a + 8b = 0 \\
3a - 7.8b = 0 \\
-2a = 0
\end{cases}
$$

From the third equation:

$$
-2a = 0 \Rightarrow a = 0.
$$

Substitute into the first:

$$
-4(0) + 8b = 0 \Rightarrow 8b = 0 \Rightarrow b = 0.
$$

Similarly, from the second:

$$
3(0) - 7.8b = 0 \Rightarrow 0 = 0,
$$

which is consistent.

Thus, the zero vector can be expressed as a linear combination with $a = 0$ and $b = 0$.

Expressing a General Vector in Terms of v and w

If the goal is to find the scalars a and b such that:

$$
\mathbf{u} = a \mathbf{v} + b \mathbf{w},
$$

then, for a general vector \(\mathbf{u} = (x, y, z)\), the process involves:

1. Setting up the system:

\[
\begin{cases}
-4a + 8b = x \\
3a - 7.8b = y \\
-2a = z
\end{cases}
\]

2. Solving for \(a\) from the third equation:

\[
a = -\frac{z}{2}.
\]

3. Substituting \(a\) into the first two equations:

\[
-4 \left(-\frac{z}{2}\right) + 8b = x \Rightarrow 2z + 8b = x,
\]

\[
3 \left(-\frac{z}{2}\right) - 7.8b = y \Rightarrow -\frac{3z}{2} - 7.8b = y.
\]

4. Solving for \(b\):

From the first:

\[
8b = x - 2z \Rightarrow b = \frac{x - 2z}{8}.
\]

From the second:

\[
- \frac{3z}{2} - 7.8b = y \Rightarrow -7.8b = y + \frac{3z}{2}.
\]

Substituting \(b\):

\[
-7.8 \times \frac{x - 2z}{8} = y + \frac{3z}{2}.
\]

Multiply both sides by 8 to clear denominator:

\[

$$
-7.8 (x - 2z) = 8 y + 12 z.
$$

Expanding:

$$
-7.8 x + 15.6 z = 8 y + 12 z,
$$

which simplifies to:

$$
-7.8 x + 15.6 z - 12 z = 8 y,
$$

$$
-7.8 x + 3.6 z = 8 y.
$$

This is a relation that must be satisfied for $\mathbf{u}$ to be expressed as a linear combination of v and w.

Implications and Conditions for Expressibility

The above derivation reveals that:

- Not all vectors $\mathbf{u}$ in $\mathbb{R}^3$ can be expressed as a linear combination of v and w unless specific conditions are met.
- The relation:

$$
-7.8 x + 3.6 z = 8 y
$$

must hold for a solution to exist.

This criterion indicates whether the vector $\mathbf{u}$ lies in the span of v and w.

Applications in Vector Algebra and Geometry

Understanding how to express vectors in terms of a basis or given vectors has multiple applications:

1. **Determining Linear Dependence and Independence:** If vectors v and w are linearly dependent, one can

Frequently Asked Questions

Given vectors v = (2, -1, -1) and w = (-4, 3, -2), how can we determine if vector a is a linear combination of v and w?

To check if vector a can be expressed as a linear combination of v and w, we need to find scalars α and β such that $a = \alpha v + \beta w$. This involves solving the system of equations formed by equating components.

What is the method to express a vector as a linear combination of two vectors in three-dimensional space?

The method involves setting up a system of equations based on the components of the vectors and solving for scalar coefficients that satisfy $a = \alpha v + \beta w$.

If a = (x, y, z) is a vector in R^3, how do you check whether it can be expressed as a combination of v and w?

You set up the equations: $x = 2\alpha - 4\beta$, $y = -\alpha + 3\beta$, $z = -\alpha - 2\beta$, then solve for α and β. If consistent solutions exist, a can be expressed as such a combination.

Can the vectors v = (2, -1, -1) and w = (-4, 3, -2) span the entire R^3 space?

No, since only two vectors are given, they can span at most a two-dimensional subspace of R^3. To span R^3, three linearly independent vectors are needed.

How do you determine if two vectors are linearly independent?

Two vectors are linearly independent if one is not a scalar multiple of the other. Alternatively, their determinant (in the case of a set of vectors) or the only solution to the linear combination is the trivial one indicates independence.

What does it mean if the system of equations for expressing vector a as a combination of v and w has no solution?

It means that vector a cannot be expressed as a linear combination of v and w, indicating that a is outside the span of v and w.

How can you find the explicit expression of a vector a as a combination of v and w if possible?

By solving the linear system formed by equating a to αv + βw, using methods such as substitution, elimination, or matrix techniques like Gaussian elimination.

What role does the determinant play in expressing a vector as a linear combination of two vectors?

Since the determinant is not directly applicable to two vectors in R^3, it’s more relevant for checking the linear independence of three vectors. For two vectors, the cross product can be used to check independence.

If vector a is given, and vectors v and w are known, what is the general process to express a as a linear combination, if possible?

Set up the equations a = $\alpha v + \beta w$, leading to a system of equations for α and β. Solve the system; if solutions exist, write a as that combination. If not, then it’s not possible.

How does understanding vector combinations help in fields like computer graphics or physics?

Vector combinations are fundamental in modeling forces, transformations, and animations, as they allow the representation of complex motions and states through linear combinations of basic vectors.

Additional Resources

Consider The Following Vectors: (2, -1, -1), v= (-4,3, -2), and w= (8, - 7, 8)

When exploring the relationships between vectors in three-dimensional space, understanding how to manipulate and analyze them is fundamental. Vectors are powerful tools in mathematics, physics, engineering, and computer science, enabling us to represent directions, forces, velocities, and more. In this guide, we will delve into the problem: "Consider the following vectors" — specifically, the vectors (2, -1, -1), v= (-4,3, -2), and w= (8, - 7, 8). Our goal is to analyze their relationships, express one in terms of others if possible, and understand the underlying geometric implications.

Understanding Vectors and Their Operations

Before we tackle the problem directly, it’s crucial to review key vector concepts:

What is a Vector?

A vector in three-dimensional space is an ordered triplet of numbers (x, y, z), representing a point or a direction and magnitude in space.

Basic Vector Operations

- Addition: (a, b, c) + (d, e, f) = (a+d, b+e, c+f)
- Scalar Multiplication: k (a, b, c) = (ka, kb, kc)
- Dot Product: (a, b, c) · (d, e, f) = ad + be + cf
- Cross Product: (a, b, c) × (d, e, f) = (bf - ce, cd - af, ae - bd)

Vector Relationships

Key relationships include:

- Linear Dependence & Independence: Whether vectors can be expressed as scalar multiples or linear combinations of each other.
- Span: The set of all linear combinations of vectors.
- Zero Vector: Represents no magnitude or direction, often used as a reference point.

Step 1: Listing the Vectors

Given vectors:

- u = (2, -1, -1)
- v = (-4, 3, -2)
- w = (8, -7, 8)

Our first task is to analyze whether these vectors are related through scalar multiples, linear combinations, or other geometric relationships.

Step 2: Checking for Scalar Multiples

A straightforward way to determine if vectors are scalar multiples is to compare their components.

Comparing u and v

- Is v a scalar multiple of u?

Calculate the ratios:

- For x-components: -4 / 2 = -2
- For y-components: 3 / -1 = -3
- For z-components: -2 / -1 = 2

Since the ratios are not equal (-2, -3, 2), v is not a scalar multiple of u.

Comparing u and w

- For x-components: 8 / 2 = 4
- For y-components: -7 / -1 = 7
- For z-components: 8 / -1 = -8

Ratios are 4, 7, and -8 — not equal, so w is not a scalar multiple of u.

Comparing v and w

- For x-components: 8 / -4 = -2
- For y-components: -7 / 3 ≈ -2.33
- For z-components: 8 / -2 = -4

Ratios are inconsistent, so w is not a scalar multiple of v either.

Conclusion: None of the vectors are scalar multiples of each other, suggesting they are linearly independent in pairs, but further analysis is needed to confirm their linear independence as a set.

Step 3: Testing for Linear Dependence

To determine whether the vectors are linearly dependent, we can check if there exists a non-trivial combination:

`au + bv + cw = 0`

where at least one of a, b, c is not zero.

Setting up the system:

a(2, -1, -1) + b(-4, 3, -2) + c(8, -7, 8) = (0, 0, 0)

which expands component-wise into three equations:

1. x-component: 2a - 4b + 8c = 0
2. y-component: -a + 3b - 7c = 0
3. z-component: -a - 2b + 8c = 0

Step 4: Solving the System

Express the system in matrix form:

2	-4	8		a		0
-1	3	-7		b	=	0
-1	-2	8		c		0

Let's analyze this system to determine if there are non-trivial solutions.

Using Gaussian elimination:

Step 1: Write the augmented matrix:

```
```

```
[ [ 2 | -4 | 8 | 0 ],
[ -1 | 3 | -7 | 0 ],
[ -1 | -2 | 8 | 0 ] ]
```

Step 2: Make the leading coefficient of row 1 a 1:

Divide row 1 by 2:

```
Row 1: [1, -2, 4, 0]
```

Step 3: Eliminate the first variable from rows 2 and 3:

- Row 2: Add row 1 multiplied by 1 to row 2:

Row 2: (-1 + 1) = 0
(3 + (-2)) = 1
(-7 + 4) = -3

- Row 3: Add row 1 multiplied by 1 to row 3:

Row 3: (-1 + 1) = 0
(-2 + (-2)) = -4
(8 + 4) = 12

Updated matrix:

```
[ [1, -2, 4, 0],
[0, 1, -3, 0],
[0, -4, 12, 0] ]
```

Step 4: Use row 2 to eliminate the y-term in row 3:

Row 3: Add row 2 multiplied by 4:

Row 3: 0 + 0 = 0
-4 + 41 = 0
12 + 4(-3) = 12 - 12 = 0

Now, row 3 is [0, 0, 0, 0], indicating a dependent system.

Interpretation:

Since the last row reduces to all zeros, the system has infinitely many solutions, meaning the vectors are linearly dependent. Specifically, at least one vector can be written as a linear combination of the others.

Step 5: Expressing a Vector as a Linear Combination

Having established dependence, let's find explicit coefficients to express, for example, w as a combination of u and v.

Recall the equations:

$au + bv = w$

which translates to:

$a(2, -1, -1) + b(-4, 3, -2) = (8, -7, 8)$

This gives us three equations:

1. $2a - 4b = 8$
2. $-a + 3b = -7$
3. $-a - 2b = 8$

Let's solve these for a and b.

Solving equations 2 and 3:

From equation 2:

$-a + 3b = -7$
$\Rightarrow a = 3b + 7$

Substitute into equation 3:

$-(3b + 7) - 2b = 8$

Simplify:

$-3b - 7 - 2b = 8$
$\Rightarrow -5b - 7 = 8$
$\Rightarrow -5b = 15$
$\Rightarrow b = -3$

Now, find a:

$a = 3(-3) + 7 = -9 + 7 = -2$

Verify equation 1:

$2a - 4b = 8$

2(-2) - 4(-3) = -4 + 12 = 8

Which matches the right side, confirming our solution.

Result:

- a = -2
- b = -3

Express w as:

w = (-2)u + (-3)v

or

w = -2(2, -1, -1) - 3(-4, 3, -2)

Verify:

-2(2, -1, -1) = (-4, 2, 2)

-3(-4, 3, -2) = (12, -9, 6)

Sum:

(-4 + 12, 2 - 9, 2 + 6) = (8, -7, 8) = w

Confirmed! The vector w can be expressed as a linear combination of u and v, confirming their linear dependence.

Summary of Findings

- The vectors

Consider The Following Vectors 2 1 1 V 4 3 2 W 8 7 8 A Express If Possible The Vector

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