Consider The Two Lines L_{1}: X=-2 T, Y=1+2 T, Z=3 T And L_{2}: X=-9+5 S, Y=2+3 S, Z=4+2 S Find The Point

Consider The Two Lines L_{1}: X=-2 T, Y=1+2 T, Z=3 T And L_{2}: X=-9+5 S, Y=2+3 S, Z=4+2 S Find The Point

---

Introduction

In the realm of analytic geometry, understanding the relationships between lines in three-dimensional space is fundamental. Whether you're solving for the intersection point, analyzing whether two lines are skew, intersecting, or parallel, mastering the concepts of parametric equations and their applications is essential. This article provides a comprehensive guide to analyzing two lines given in parametric form, specifically focusing on the lines:

- Line L1:
\(X = -2T \)
\(Y = 1 + 2T \)
\(Z = 3T \)

- Line L2:
\(X = -9 + 5S \)
\(Y = 2 + 3S \)
\(Z = 4 + 2S \)

The goal is to find the point of intersection between these two lines, if it exists, or determine that they do not intersect.

Understanding the Parametric Equations of Lines

Parametric Equations in Geometry

Parametric equations describe a line in space by expressing each coordinate as a function of one or more parameters—in this case, \(T \) and \(S \). The general form of a line in 3D space:

\[
\begin{cases}

X = X_0 + aT \\
Y = Y_0 + bT \\
Z = Z_0 + cT
\end{cases}
\]

where \((X_0, Y_0, Z_0) \) is a point on the line, and \((a, b, c) \) is the direction vector.

In the given equations:

- For L1:
- Point on the line when \(T=0 \): \((0, 1, 0) \)
- Direction vector: \(\vec{d}_1 = (-2, 2, 3) \)

- For L2:
- Point on the line when \(S=0 \): \((-9, 2, 4) \)
- Direction vector: \(\vec{d}_2 = (5, 3, 2) \)

Steps to Find the Intersection Point

1. Set Up the Equations

To find the common point between the two lines, we need to find values of \(T \) and \(S \) such that:

\[
\begin{cases}
-2T = -9 + 5S \quad \text{(X-coordinates)} \\
1 + 2T = 2 + 3S \quad \text{(Y-coordinates)} \\
3T = 4 + 2S \quad \text{(Z-coordinates)}
\end{cases}
\]

This system ensures that at the intersection point, the coordinates from both lines are equal.

2. Formulate the System of Equations

Rearranging each:

- From \(X \):
\[
-2T = -9 + 5S \implies 2T + 5S = 9
\]

- From Y:

$$
1 + 2T = 2 + 3S \implies 2T - 3S = 1
$$

- From Z:

$$
3T = 4 + 2S \implies 3T - 2S = 4
$$

Now, we have a system of three equations:

$$
\begin{cases}
2T + 5S = 9 \quad (1) \\
2T - 3S = 1 \quad (2) \\
3T - 2S = 4 \quad (3)
\end{cases}
$$

Solving the System of Equations

1. Using Equations (1) and (2)

Subtract (2) from (1):

$$
(2T + 5S) - (2T - 3S) = 9 - 1
$$

$$
2T + 5S - 2T + 3S = 8
$$

$$
8S = 8 \implies S = 1
$$

2. Find T using $S=1$

Plug $S=1$ into equation (2):

$$

$$
2T - 3(1) = 1 \implies 2T - 3 = 1 \implies 2T = 4 \implies T = 2
$$

3. Verify with Equation (3)

Substitute $T=2$ and $S=1$:

$$
3(2) - 2(1) = 6 - 2 = 4
$$

which matches the right side of equation (3). This confirms the solution's consistency.

Finding the Intersection Point

Now that we have $T=2$ and $S=1$, substitute back into the parametric equations:

- For L1:

$$
X = -2T = -2 \times 2 = -4
$$
$$
Y = 1 + 2T = 1 + 2 \times 2 = 1 + 4 = 5
$$
$$
Z = 3T = 3 \times 2 = 6
$$

- For L2:

$$
X = -9 + 5S = -9 + 5 \times 1 = -9 + 5 = -4
$$
$$
Y = 2 + 3S = 2 + 3 \times 1 = 2 + 3 = 5
$$
$$
Z = 4 + 2S = 4 + 2 \times 1 = 4 + 2 = 6
$$

The coordinates match perfectly, confirming that the lines intersect at the point:

$$
\boxed{(-4, 5, 6)}
$$

\]

Geometric Interpretation

The intersection point $(-4, 5, 6)$ indicates that the two lines cross at this point in three-dimensional space. Since they intersect at exactly one point, they are neither parallel nor skew lines but are intersecting lines.

Additional Concepts Related to Lines in 3D Space

Parallel Lines

- Two lines are parallel if their direction vectors are scalar multiples.
- For example, if $\vec{d}_1 = k \times \vec{d}_2$ for some scalar k, lines are parallel.

Skew Lines

- Lines that are neither parallel nor intersecting.
- They do not meet and are not coplanar.

Coincident Lines

- Lines that lie exactly on top of each other.
- Their parametric equations are scalar multiples of each other.

Summary

- Given two lines in parametric form, set their coordinate equations equal to find the intersection point.
- Formulate a system of equations based on the parametric forms.
- Solve the system for the parameters.
- Substitute the parameters back into the original equations to find the point of intersection.
- Confirm the solution's consistency across all equations.

Conclusion

The process of finding the intersection point between two lines in 3D space involves translating parametric equations into a system of algebraic equations and solving for the parameters. For the lines:

$$
L_{1}: X = -2T, \quad Y=1+2T, \quad Z=3T
$$
$$
L_{2}: X=-9+5S, \quad Y=2+3S, \quad Z=4+2S
$$

we determined that they intersect at the point (-4, 5, 6), with parameters $T=2$ and $S=1$. This comprehensive approach is fundamental in analytical geometry, aiding in various applications such as computer graphics, robotics, and physics.

Keywords: 3D lines intersection, parametric equations, analytic geometry, solving for intersection point, line equations in space, vector equations of lines, geometry in space, algebraic solutions in 3D, line relationships.

Frequently Asked Questions

How do you find the point of intersection between the two lines L_1 and L_2?

To find the intersection point, set the parametric equations of L_1 and L_2 equal to each other and solve for the parameters T and S. If consistent solutions exist, substitute back to find the intersection coordinates.

What are the parametric equations of lines L_1 and L_2 in the problem?

L_1: x = -2T, y = 1 + 2T, z = 3T; L_2: x = -9 + 5S, y = 2 + 3S, z = 4 + 2S.

How do you determine if the two lines are intersecting, skew, or parallel?

By solving the equations for T and S, if a common point satisfies both lines, they intersect; if not and they are not parallel, they are skew; if their direction vectors are scalar multiples, they are parallel.

What steps are involved in finding the intersection point of two lines in 3D?

Set the parametric equations equal, solve for parameters T and S, check for consistency; if consistent, substitute into equations to find the intersection point.

Can the lines L_1 and L_2 be skew lines? How to check?

Yes, if they do not intersect and are not parallel, they are skew. Check by solving for intersection; if no solution exists, they are skew lines.

What is the significance of the parameters T and S in the equations of lines L_1 and L_2?

T and S are parameters that generate points along lines L_1 and L_2 respectively, determining the position of any point on each line.

What is the final answer for the intersection point of the given lines L_1 and L_2?

By solving the equations, the intersection point is found to be at (x, y, z) = (-2, 5, 3).

Additional Resources

Consider The Two Lines L_1: X = -2T, Y = 1 + 2T, Z = 3T and L_2: X = -9 + 5S, Y = 2 + 3S, Z = 4 + 2S — Find The Point

---

Introduction: The Significance of Line Analysis in Geometry

Mathematics, particularly geometry, forms the backbone of understanding spatial relationships in both theoretical and applied contexts. Among the fundamental objects studied are lines in three-dimensional space, which can represent real-world elements such as roads, pipelines, or trajectories of moving objects. Analyzing lines involves understanding their equations, intersections, and relative positions. This insightful process is essential in fields like engineering, computer graphics, physics, and architecture.

In this article, we examine two parametric lines, L_1 and L_2, defined by their respective equations. The core challenge lies in determining whether these lines intersect — that is, whether they share a common point in space — and if so, identifying that point. The investigation involves algebraic manipulation, geometric interpretation, and critical analysis, all presented in a structured and comprehensive manner to elucidate the underlying concepts and methods.

Understanding the Given Lines: Equations and Parameterizations

Line L_1: Parametric Equations and Geometric Interpretation

The first line, L_1, is given in parametric form as:

$$
L_1:\quad
X = -2T,\quad
Y = 1 + 2T,\quad
Z = 3T
$$

Here, T is a real parameter that varies over all real numbers. Each value of T corresponds to a point on the line.

Geometric interpretation:

- The line passes through the origin when $T=0$, since then $X = 0$, $Y=1$, and $Z=0$.
- The direction vector of L_1 is derived from the coefficients of T:

$$
\vec{d_1} = \langle -2, 2, 3 \rangle
$$

- The point P_0 corresponding to $T=0$ is $(0, 1, 0)$.

Visual intuition:

- Direction vector $\vec{d_1}$ points from the origin towards a specific orientation in space.
- The line extends infinitely in both directions along this vector, passing through $(0, 1, 0)$.

Line L_2: Parametric Equations and Geometric Interpretation

The second line, L_2, is specified as:

$$
L_2:\quad
X = -9 + 5S,\quad
$$

$$
Y = 2 + 3S,\quad
Z = 4 + 2S
$$

with S a real parameter.

Geometric interpretation:

- When $S=0$, the line passes through $(-9, 2, 4)$.
- The direction vector associated with L_2 is:

$$
\vec{d_2} = \langle 5, 3, 2 \rangle
$$

- The line extends infinitely in both directions along $\vec{d_2}$.

Visual intuition:

- The line begins at point $Q = (-9, 2, 4)$, extending in the direction of $\vec{d_2}$.

Formulating the Intersection Problem

The core problem is to find the point P in space such that:

$$
P = (X, Y, Z)
$$

lies on both lines simultaneously. This requires solving the equations:

$$
\begin{cases}
X = -2T \\
Y = 1 + 2T \\
Z = 3T
\end{cases}
\quad \text{and} \quad
\begin{cases}
X = -9 + 5S \\
Y = 2 + 3S \\
Z = 4 + 2S
\end{cases}
$$

for some real parameters T and S.

Key question:

- Does there exist a pair (T, S) satisfying all six equations simultaneously?
- If yes, the corresponding (X, Y, Z) is the intersection point.
- If no, the lines are skew (neither intersecting nor parallel).

Algebraic Approach: Solving the System of Equations

To find the intersection point, we equate the parametric expressions for X, Y, Z:

$$
-2T = -9 + 5S \quad (1)
$$
$$
1 + 2T = 2 + 3S \quad (2)
$$
$$
3T = 4 + 2S \quad (3)
$$

The goal is to solve these three equations for T and S.

Step 1: Expressing T and S from equations

From equation (3):

$$
3T = 4 + 2S \Rightarrow T = \frac{4 + 2S}{3}
$$

Substitute this T into equations (1) and (2) to find S.

Step 2: Substituting T into equation (1)

Equation (1):

$$
-2T = -9 + 5S
$$

Replace T:

$$
-2 \times \frac{4 + 2S}{3} = -9 + 5S
$$

Multiply both sides by 3 to clear denominator:

$$
-2(4 + 2S) = 3(-9 + 5S)
$$

Simplify:

$$
-8 - 4S = -27 + 15S
$$

Bring all terms to one side:

$$
-8 + 27 = 15S + 4S
$$

$$
19 = 19S
$$

Solve for S:

$$
S = \frac{19}{19} = 1
$$

Step 3: Find T using S

Recall:

$$
T = \frac{4 + 2S}{3}
$$

Substitute $S=1$:

$$
T = \frac{4 + 2(1)}{3} = \frac{4 + 2}{3} = \frac{6}{3} = 2
$$

Step 4: Verify consistency with equation (2)

Equation (2):

$$
1 + 2T = 2 + 3S
$$

Substitute $T=2$, $S=1$:

$$
1 + 2(2) = 2 + 3(1)
$$
$$
1 + 4 = 2 + 3
$$
$$
5 = 5
$$

Since the equality holds, the parameters satisfy all three equations, confirming that the lines intersect at the calculated parameters.

Finding the Intersection Point

Having determined:

$$
T=2,\quad S=1
$$

we now substitute back into the parametric equations to find the coordinates of the intersection point.

Using L_1:

$$
X = -2T = -2 \times 2 = -4
$$
$$
Y = 1 + 2T = 1 + 2 \times 2 = 1 + 4 = 5
$$
$$

Z = 3T = 3 \times 2 = 6
\]

Using L_2:

\[
X = -9 + 5S = -9 + 5 \times 1 = -9 + 5 = -4
\]
\[
Y = 2 + 3S = 2 + 3 \times 1 = 2 + 3 = 5
\]
\[
Z = 4 + 2S = 4 + 2 \times 1 = 4 + 2 = 6
\]

The points match perfectly, confirming that the lines intersect at the point:

\[
\boxed{
\textbf{Intersection Point: } (-4, 5, 6)
}
\]

Geometric Significance and Implications

The analytical solution reveals that the two lines intersect at a single point in space, which is \((-4, 5, 6) \). This intersection point is a tangible geometric entity that satisfies both parametric equations simultaneously.

Understanding the intersection:

- The line L_1 passes through \((0,1,0) \) and extends infinitely along \(\vec{d_1} \).
- The line L_2 passes through \((-9,2,4) \) and extends along \(\vec{d_2} \).
- Their intersection point lies on both, confirming that the lines are neither skew nor parallel, but intersecting in space.

Additional Insights and Broader Context

1.

[Consider The Two Lines L 1 X 2 T Y 1 2 T Z 3 T And L 2 X 9 5 S Y 2 3 S Z 4 2 S Find The Point]

Consider The Two Lines L 1 X 2 T Y 1 2 T Z 3 T And L 2 X 9 5 S Y 2 3 S Z 4 2 S Find The Point

Related Articles

- [CSR Stands For Corporate Social Responsibility In This ContextHow Does CSR Relate To Law In Business?Name]
- [Decline Inc. Is Trying To Determine Its Cost Of Debt. The Firm Has A Debt Issue Outstanding With 13 Years]
- [Can Anyone Write The Research Problem On Sustainable Consumptionand Production Patterns. What Is Severely]

[Back to Home]