

# Determine The Natural Frequency In Radians Per Second For The System Shown. Neglect The Mass And Friction

Determine The Natural Frequency In Radians Per Second For The System Shown. Neglect The Mass And Friction

Understanding the natural frequency of a mechanical system is fundamental in the fields of dynamics, vibration analysis, and mechanical design. The natural frequency, often denoted as  $\omega_n$  (in radians per second), indicates the rate at which a system oscillates when disturbed from its equilibrium position, assuming no external damping or mass effects influence its motion. In this comprehensive guide, we will explore the methodology to determine the natural frequency of a given system, with the assumption that mass and friction are negligible, simplifying the analysis to focus solely on the system's stiffness characteristics.

## Overview of Natural Frequency

### What Is Natural Frequency?

Natural frequency is the intrinsic frequency at which a system tends to oscillate when displaced from its equilibrium, without any external forcing or damping. It defines the system's dynamic response and plays a critical role in designing systems to avoid resonance, which can cause excessive vibrations and potential failure.

### Relevance in Engineering and Design

- Vibration Control: Designing structures and machinery that avoid resonant conditions.
- Signal Processing: Understanding the oscillatory behavior of circuits and mechanical systems.
- Structural Analysis: Ensuring safety and longevity by analyzing vibrational modes.

## Simplified System Model for Natural Frequency Calculation

## Assumptions

To focus solely on the natural frequency, the following assumptions are made:

- Neglect Mass: The mass of the system components is considered negligible, simplifying the inertia effects.
- Neglect Friction/Damping: No energy loss occurs due to friction or damping forces.
- Linear System Behavior: The system behaves linearly, and the restoring force is proportional to displacement.

## Typical System Representation

The most common model for analyzing natural frequency is the simple harmonic oscillator, which can be represented as:

- A mass  $(m)$  attached to a spring with stiffness  $(k)$ .
- For the purpose of neglecting mass, the analysis reduces to considering the stiffness properties directly.

## Mathematical Foundations

### Equation of Motion for a Simple System

In a typical mass-spring system, the equation of motion is:

$$m \frac{d^2x}{dt^2} + kx = 0$$

where:

- $(m)$  is the mass,
- $(k)$  is the stiffness,
- $(x)$  is the displacement.

Neglecting mass simplifies the system to:

$$kx = 0$$

but to determine the natural frequency, the system's stiffness characteristics are key, and the general approach involves the second-order differential equation governing oscillations.

## Natural Frequency Formula

For a standard mass-spring system:

$$\omega_n = \sqrt{\frac{k}{m}}$$

\]

Given the assumption of negligible mass, the natural frequency depends solely on the stiffness properties of the system, often expressed in terms of the system's parameters.

## Determining the Natural Frequency for the System Shown

### Step 1: Identify the System Components

- Springs: Determine the stiffness values ( $k_1, k_2, \dots$ ) of each spring in the system.
- Configuration: Understand how springs are connected—series, parallel, or a combination.
- Constraints: Note the boundary conditions, such as fixed supports or free ends.

### Step 2: Derive the Equivalent Stiffness

For systems with multiple springs:

- Springs in Series:

\[

$$\frac{1}{k_{eq}} = \sum_i \frac{1}{k_i}$$

\]

- Springs in Parallel:

\[

$$k_{eq} = \sum_i k_i$$

\]

The equivalent stiffness ( $k_{eq}$ ) represents the overall restoring force characteristic of the system.

### Step 3: Formulate the Differential Equation

Given the system's equivalent stiffness and assuming negligible mass, the simplified differential equation resembles:

\[

$$k_{eq} x = 0$$

\]

but to find the natural frequency, we consider the dynamic behavior, which in a real system with negligible mass is theoretically infinite. However, in practical analysis, a small effective mass or the system's inertia is considered to avoid singularities.

## Step 4: Calculate the Natural Frequency

Using the relation:

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eff}}}$$

where  $(m_{eff})$  is an effective mass (which, under idealized conditions, approaches zero), the system's natural frequency tends to infinity, indicating very high frequency oscillations.

In practical terms, when neglecting mass:

- The system's natural frequency is dominated by the stiffness properties.
- For a physical system, a small mass or inertia is always present, and the calculation proceeds with that value.

Alternatively, if the problem states “neglect the mass,” it often implies focusing on the stiffness-based natural frequency, which can be derived from the system's equations of motion by setting the inertial effects to zero and solving for the oscillation condition.

## Example Calculation

Suppose the system shown consists of a single spring with stiffness  $(k)$ , attached to a massless support, and the system is constrained to oscillate about an equilibrium position.

- Spring stiffness:  $(k = 1000 \text{ N/m})$
- Effective mass: Assume a small but non-zero effective mass  $(m = 0.1 \text{ kg})$

The natural frequency in radians per second is:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{0.1}} = \sqrt{10,000} = 100 \text{ rad/sec}$$

Note: If the mass is truly negligible, the frequency approaches infinity, indicating an infinitely stiff system with oscillations at infinitely high frequency, which is physically impractical. Thus, the approximate or effective mass is essential for meaningful calculations.

## Implications and Practical Considerations

## Real-World Limitations

- Neglecting mass is an idealization; in physical systems, mass, damping, and other factors influence natural frequencies.
- Material properties and geometric configurations also affect stiffness and, consequently, the natural frequency.

## Design Applications

- Engineers use these calculations to design systems that avoid resonance.
- Adjusting stiffness parameters can shift the natural frequency away from excitation frequencies.

## Summary

- The natural frequency in radians per second is fundamentally related to the system's stiffness and mass.
- When neglecting mass and friction, the focus shifts to the stiffness properties and boundary conditions.
- The general formula  $\omega_n = \sqrt{\frac{k}{m}}$  guides the calculation when mass is considered; otherwise, the system's stiffness alone determines oscillatory characteristics.
- Practical calculations require a small but finite effective mass to avoid non-physical infinite frequencies.
- Understanding these principles helps in designing safer, more efficient mechanical systems by controlling vibrational behaviors.

## Conclusion

Determining the natural frequency of a system, especially when neglecting mass and friction, involves analyzing the system's stiffness characteristics and configuration. While idealizations simplify the theoretical understanding, real-world applications necessitate including mass and damping effects for accurate predictions. Nonetheless, the fundamental concept remains that the natural frequency is a measure of how a system inherently oscillates, governed primarily by its stiffness properties in the absence of inertial resistance. Properly understanding and calculating this frequency is crucial in designing systems that are resilient to vibrational issues and resonance phenomena.

## Frequently Asked Questions

**What is the formula to determine the natural frequency in radians per**

## **second for a simple mass-spring system?**

The natural frequency  $\omega$  is given by  $\omega = \sqrt{k/m}$ , where  $k$  is the spring constant and  $m$  is the mass. When neglecting mass, the system's natural frequency depends on the stiffness and geometry.

## **How does neglecting mass and friction affect the calculation of the natural frequency?**

Neglecting mass and friction simplifies the system to an idealized model, allowing direct calculation of natural frequency based solely on stiffness or other system parameters without damping effects.

## **Can you determine the natural frequency if only the stiffness of the system is known?**

Yes, if the system's stiffness ( $k$ ) is known and the mass is negligible, the natural frequency can be calculated using the relation  $\omega = \sqrt{k/m}$ . If mass is neglected, the system behaves as if it has infinite mass, implying the natural frequency approaches infinity; typically, a finite effective mass or other parameters are needed.

## **In a system shown with multiple springs, how do you determine the overall stiffness for natural frequency calculation?**

The overall stiffness is found by combining individual spring constants in series or parallel, depending on their configuration, then using the combined stiffness in the natural frequency formula.

## **What impact does neglecting friction have on the natural frequency of the system?**

Neglecting friction means no damping is considered, so the system's natural frequency remains unchanged by damping effects, representing an idealized undamped oscillation.

## **If the system shown includes a spring with known stiffness, how do you calculate the natural frequency in radians per second?**

Use the formula  $\omega = \sqrt{k/m}$ . With no mass considered, the natural frequency theoretically approaches infinity, but practically, a small mass or effective mass is used for calculation.

## **Why is it important to neglect mass and friction when determining the**

natural frequency in certain analyses?

Neglecting mass and friction simplifies the analysis, allowing focus on the system's inherent stiffness and geometry to understand fundamental oscillation characteristics without damping or inertial effects.

**How would you interpret the natural frequency in radians per second if the system is purely elastic and ideal?**

In an ideal, purely elastic system with no mass or damping, the natural frequency concept becomes theoretical, often approaching infinity, indicating an instantaneous response; in real systems, finite mass and damping produce a finite natural frequency.

**What additional information is needed to accurately compute the natural frequency if the system's shown diagram includes multiple elements?**

You need the stiffness values of all elements involved, the effective mass (if any), and how the elements are connected (series or parallel) to compute the overall stiffness and determine the natural frequency accurately.

## Additional Resources

Natural Frequency: The Key to Understanding Vibrational Behavior in Mechanical Systems

---

### Introduction

When analyzing mechanical systems, especially those involving oscillations or vibrations, understanding the natural frequency is crucial. This intrinsic property dictates how a system responds to external stimuli, how it resonates, and how energy is stored and transferred within the system. In this detailed exploration, we will focus on determining the natural frequency in radians per second for a specific system, emphasizing the importance of neglecting mass and friction to simplify the analysis.

Whether you're an engineer designing a bridge, a mechanical designer working on machinery, or a student delving into dynamics, grasping how to compute the natural frequency provides fundamental insights into system stability, resonance avoidance, and dynamic response.

---

### Defining Natural Frequency

Natural frequency is the frequency at which a system tends to oscillate when disturbed from its equilibrium position without any external force acting upon it. This is a fundamental characteristic of the system's structure and boundary conditions.

In physical terms, it reflects the rate at which the system "prefers" to vibrate due to the inherent stiffness and geometry, independent of mass and damping effects (when these are neglected).

Mathematically, the natural frequency (denoted as  $\omega_n$ ) in radians per second for simple systems such as springs or pendulums can be derived from the fundamental relationships governing oscillatory motion.

---

## Theoretical Foundations

### The Standard Model: The Mass-Spring System

The most common and illustrative example of a system's natural frequency involves a mass attached to an ideal spring. Its equation of motion, neglecting damping and external forces, is:

$$m \frac{d^2x}{dt^2} + kx = 0$$

Where:

- $m$  is the mass (which we neglect in this case),
- $k$  is the stiffness (spring constant),
- $x$  is the displacement.

In the absence of mass, the simplified system's natural frequency depends on the stiffness and geometry, often represented through the stiffness constant  $k$ .

---

### Deriving the Natural Frequency

For a typical mass-spring system, the natural frequency in radians per second is:

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}}$$

However, since we are instructed to neglect mass and friction, this formula adjusts accordingly. In such idealized scenarios, the system's stiffness alone governs the oscillatory behavior, especially if the system involves elastic elements like beams, rods, or springs without considering mass effects.

In many practical engineering systems, the natural frequency in the absence of mass simplifies to depend on the stiffness and geometry parameters, often expressed as:

$$\omega_n = \sqrt{\frac{\text{stiffness}}{\text{effective mass or inertia}}}$$

But with mass neglected, the primary approach is to relate the stiffness and the system's geometry directly to the oscillation characteristics.

---

## System Overview and Parameters

Suppose the system under consideration is a simplified, idealized elastic structure, such as a cantilever beam or a torsional shaft, where the key parameters include:

- Stiffness (k): The measure of the system's resistance to deformation.
- Geometry: Length, moment of inertia, or other relevant geometric factors.
- Material Properties: Modulus of elasticity (E), shear modulus (G), etc.

Given that mass and friction are neglected, the natural frequency calculation hinges primarily on the stiffness and the system's geometry.

---

## Determining the Natural Frequency: Step-by-Step Approach

### Step 1: Identify the System Type

The first step is to classify the system. Common categories include:

- Spring-mass systems (idealized)
- Torsional systems
- Flexural systems (beams, plates)

For each, the method of calculating the natural frequency differs slightly, but the core principles remain.

### Step 2: Extract the Relevant Parameters

Based on the system, determine the key parameters:

- For a spring: spring constant  $(k)$
- For a torsional shaft: torsional stiffness  $(k_{\theta})$
- For a beam or plate: flexural rigidity  $(EI)$ , length  $(L)$ , moment of inertia, etc.

### Step 3: Establish the Equation of Motion

Using classical mechanics, write down the differential equation describing the system's motion, factoring in the geometry and boundary conditions.

For example, a cantilever beam's fundamental frequency relates to its flexural rigidity and length as:

$$f_n = \frac{\beta^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}}$$

Where:

- $(\beta)$  is a mode-dependent constant (e.g., 1.875 for the first mode)
- $(E)$  is Young's modulus
- $(I)$  is the second moment of area
- $(\rho)$  is density
- $(A)$  is cross-sectional area

However, since density  $(\rho)$  and mass are neglected, focus on the elastic properties and geometry.

### Step 4: Calculate the Natural Frequency in Radians per Second

Once the relevant stiffness and geometric factors are identified, compute:

$$\boxed{\omega_n = \sqrt{\frac{\text{stiffness}}{\text{effective mass or inertia}}}}$$

In the scenario where mass is neglected, the stiffness alone (or a derived equivalent) effectively determines the oscillation rate. For systems like torsional shafts, the natural frequency becomes:

$$\boxed{\omega_n = \sqrt{\frac{k_{\theta}}{I}}}$$

\]

Where:

- $(k_{\theta})$  is the torsional stiffness
- $(I)$  is the moment of inertia

If  $(I)$  is also considered negligible or incorporated into the stiffness, the frequency simplifies further.

---

### Practical Example: Torsional System

Let's consider an example where the system is a torsional shaft with a known torsional stiffness  $(k_{\theta})$ , and we neglect the rotational inertia  $(I)$ .

Suppose:

- Torsional stiffness,  $(k_{\theta} = 150, \text{Nm/rad})$
- The system's effective inertia is negligible or included in the stiffness

Then, the natural frequency in radians per second is:

$$\omega_n = \sqrt{\frac{k_{\theta}}{I}}$$

If  $(I)$  is negligible, the frequency is primarily determined by how the system's stiffness resists angular deformation, leading to an effective natural frequency:

$$\boxed{\omega_n \approx \sqrt{k_{\theta}}}$$

which gives:

$$\omega_n \approx \sqrt{150} \approx 12.25, \text{rad/sec}$$

This value represents the intrinsic oscillation rate of the system.

---

## Special Considerations & Limitations

- Neglecting mass and friction simplifies calculations but can lead to idealized results. Real systems exhibit damping and inertial effects that modify the natural frequency.
- Boundary conditions significantly influence the natural frequency. Clamped, free, or simply supported boundary conditions alter the stiffness and mode shapes.
- Multiple modes: Systems can have several natural frequencies corresponding to different vibrational modes; here, we focus on the fundamental mode.

---

## Summary of Key Steps

1. Identify the system type and governing parameters.
2. Determine the stiffness (k or equivalent).
3. Establish the appropriate differential equation of motion, considering boundary conditions.
4. Calculate the natural frequency using  $\omega_n = \sqrt{\frac{\text{stiffness}}{\text{effective inertia}}}$ .
5. Express the result in radians per second.

---

## Final Remarks

Understanding how to determine the natural frequency in radians per second provides a foundational tool for engineers and designers. It allows them to predict how systems will behave under dynamic conditions, design for resonance avoidance, and optimize structural integrity. While neglecting mass and friction offers a simplified perspective, it serves as a valuable starting point for more complex analyses involving damping, mass distribution, and external forces.

In practice, always validate simplified calculations with experimental data or detailed finite element models, especially for critical applications where safety and performance depend on accurate vibrational analysis.

---

In essence, by carefully analyzing the system's stiffness, geometry, and boundary conditions, and applying the fundamental principles of oscillatory motion, one can reliably determine the natural frequency in radians per second — an essential step in mastering vibrational dynamics.

## **Determine The Natural Frequency In Radians Per Second For The System Shown Neglect The Mass And Friction**

Determine The Natural Frequency In Radians Per Second For The System Shown Neglect The Mass And Friction

### **Related Articles**

- [5. Because Of Pandemic, You Were Locked Up In A Flat. Your Parents Were Worried About You. Write A Letter](#)
- [A Trader Bought 30 Baskets Of Pawpaw And 100 Baskets Of Mangoes For 2450. She Sold The Pawpaw At A Profit](#)
- [A Mayor Estimates That 4,000 People Will Attend The First Day Of The County Fair. A Total Of 8,400 People](#)

[Back to Home](#)