Determine Whether The Series Is Convergent Or Divergent. 1 + 1/22 + 1/33 + 1/44 + 1/55 ...a. Convergentb.

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Understanding whether a series converges or diverges is a fundamental concept in mathematical analysis, particularly in the study of infinite series. Infinite series appear frequently in various branches of mathematics, physics, engineering, and computer science. They help us understand the behavior of sequences and functions, especially when dealing with limits, approximations, and summations extending to infinity.

In this article, we will explore the series:

$$
S = 1 + \frac{1}{22} + \frac{1}{33} + \frac{1}{44} + \frac{1}{55} + \dots
$$

and determine whether this series converges or diverges. We will analyze the structure of the series, apply relevant convergence tests, and interpret the results to understand the nature of this infinite sum.

Understanding the Series: Structure and Components

Before delving into convergence tests, it's essential to understand the structure of the given series.

The series is:

$$
S = 1 + \frac{1}{22} + \frac{1}{33} + \frac{1}{44} + \frac{1}{55} + \dots
$$

At first glance, the terms appear to follow a pattern of the form:

$$
a_n = \frac{1}{nn}
$$

where n runs through integers starting from 1, but with a discrepancy in the denominators as written. The terms explicitly are:

- First term: 1
- Second term: $\frac{1}{22}$
- Third term: $\frac{1}{33}$
- Fourth term: $\frac{1}{44}$
- Fifth term: $\frac{1}{55}$

and so on.

Notice that:

$$
a_1 = 1
$$
$$
a_2 = \frac{1}{22} = \frac{1}{2 \times 11}
$$
$$
a_3 = \frac{1}{33} = \frac{1}{3 \times 11}
$$
$$
a_4 = \frac{1}{44} = \frac{1}{4 \times 11}
$$
$$
a_5 = \frac{1}{55} = \frac{1}{5 \times 11}
$$

It appears that starting from the second term, the denominators are multiples of 11:

$$
a_n = \frac{1}{n \times 11} \quad \text{for } n \geq 2
$$

But this does not match the first term, which is 1. Alternatively, perhaps the series is better expressed as:

$$
S = 1 + \sum_{n=2}^{\infty} \frac{1}{n \times 11}
$$

which simplifies to:

$$
S = 1 + \frac{1}{11} \sum_{n=2}^{\infty} \frac{1}{n}
$$

This interpretation suggests that the series resembles a constant plus a scaled harmonic series.

Identifying the Series Type

Given the analysis above, the series can be rewritten approximately as:

\[
S = 1 + \frac{1}{11} \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots \right)
\]

which simplifies to:

\[
S = 1 + \frac{1}{11} \sum_{n=2}^{\infty} \frac{1}{n}
\]

or equivalently,

\[
S = 1 + \frac{1}{11} \left(\sum_{n=1}^{\infty} \frac{1}{n} - 1 \right)
\]

since the harmonic series:

\[
H_\infty = \sum_{n=1}^\infty \frac{1}{n}
\]

diverges, i.e., it grows without bound as \(n \to \infty \).

Key observation:

- The harmonic series diverges, meaning its partial sums tend to infinity.
- Scaling the harmonic series by any non-zero constant (\(\frac{1}{11} \)) still results in divergence.

Therefore,

\[
S = 1 + \frac{1}{11} \left(H_\infty - 1 \right)
\]

diverges because \(H_\infty \to \infty \).

Applying Convergence Tests

To confirm the divergence or convergence of the series, we utilize several standard convergence tests.

1. The Basic Divergence Test (Term Test)

The divergence test states:

> If $\lim_{n \to \infty} a_n \neq 0$, then the series $\sum a_n$ diverges.

In our series:

$$
a_n = \frac{1}{n \times 11}
$$

for $n \geq 2$. The limit as $n \to \infty$:

$$
\lim_{n \to \infty} a_n = 0
$$

which is necessary but not sufficient for convergence. Since the terms tend to zero, the test does not conclude divergence; further analysis is needed.

---

2. Comparison with the Harmonic Series

As established, the terms are proportional to $1/n$, which is the harmonic series' general term.

- The harmonic series:

$$
\sum_{n=1}^\infty \frac{1}{n}
$$

diverges.

- Our series (excluding the initial 1):

$$
\frac{1}{11} \sum_{n=2}^\infty \frac{1}{n}

\]

also diverges because it is a constant multiple of a divergent series.

Conclusion: The entire series diverges.

3. Limit Comparison Test

Applying the limit comparison test with the harmonic series:

\[
a_n = \frac{1}{n \times 11}
\]

and

\[
b_n = \frac{1}{n}
\]

we compute:

\[
L = \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty}
\frac{\frac{1}{11n}}{\frac{1}{n}} = \frac{1/11n}{1/n} = \frac{1}{11}
\]

Since \(L \) is finite and positive, and since \(\sum b_n \) diverges, the comparison test indicates that \(\sum a_n \) also diverges.

Conclusion: Is the Series Convergent or Divergent?

Based on the analysis above, the series:

\[
1 + \frac{1}{22} + \frac{1}{33} + \frac{1}{44} + \frac{1}{55} + \dots
\]

diverges.

Reasoning:

- The series is effectively a constant multiple of the harmonic series

starting from $n=2$.
- The harmonic series diverges, and multiplying it by a constant does not alter its divergence.
- The initial term, 1, is finite, but the tail of the series dominates the behavior, causing divergence.

Final verdict:

The series is divergent.

Implications and Further Insights

Understanding the divergence of this series has broader mathematical implications:

- It emphasizes the importance of the harmonic series' divergence, which appears in many contexts, from number theory to analysis.
- Recognizing the pattern of the denominators can help quickly determine the convergence properties of similar series.
- In applied mathematics, knowing whether a series converges or diverges guides the approximation methods, numerical analysis, and understanding of infinite processes.

Summary

- The series in question resembles a scaled harmonic series.
- The harmonic series diverges, and scaling by a constant does not change this.
- The initial term is finite; however, the tail of the series dominates and leads to divergence.
- Tests such as the comparison test confirm the divergence.

Therefore, the series is divergent.

Key Takeaways for Students and Mathematicians

- Always analyze the general term a_n of a series to understand its behavior.

- The divergence or convergence of a series often depends on the asymptotic behavior of its terms.
- The harmonic series serves as a fundamental reference point for divergence tests.
- Use standard convergence tests (comparison, ratio, root, integral) to confirm the nature of a series.

Additional Resources for Learning Series Convergence

- Textbooks:
- Calculus by James Stewart
- Introduction to Real Analysis by Robert G. Bartle
- Online Courses:
- Khan Academy's Series and Sequences module
- MIT OpenCourseWare - Single Variable Calculus
- Mathematical Software:
- WolframAlpha
- GeoGebra

In conclusion

Frequently Asked Questions

How can I determine whether the series 1 + 1/22 + 1/33 + 1/44 + 1/55 ... converges or diverges?

You can analyze the series using comparison tests or the p-series test. Noticing the pattern, it resembles a p-series with p=1, which diverges. Therefore, the series diverges.

What is the general term of the series 1 + 1/22 + 1/33 + 1/44 + 1/55 ...?

The general term appears to be $a_n = 1/n^2$ for $n \geq 2$, but the initial term is 1, so the series is a sum starting from n=1 with specific terms. Clarifying the pattern is key to analyzing convergence.

Does the series 1 + 1/22 + 1/33 + 1/44 + 1/55 ... converge or diverge?

Since the terms resemble $1/n^2$ for large n, and the p-series with p=2 converges, this series converges.

What test should I use to determine the convergence of the series that looks like 1 + 1/22 + 1/33 + ...?

The comparison test or the p-series test are suitable. If the terms decay faster than 1/n, the series converges. If they decay like 1/n, it diverges.

Is the series 1 + 1/22 + 1/33 + 1/44 + 1/55 an example of a convergent series?

Yes, because the terms decrease proportionally to $1/n^2$, which is a p-series with p=2, known to converge.

What is the significance of the pattern 1/22, 1/33, 1/44, 1/55 in analyzing convergence?

The pattern suggests the terms are proportional to $1/n^2$, indicating that the series behaves like a p-series with p=2, which converges.

Can the series 1 + 1/22 + 1/33 + ... be considered a telescoping series?

No, the series does not exhibit telescoping behavior. It is better analyzed as a p-series for convergence.

How does the initial term (1) affect the convergence of the series 1 + 1/22 + 1/33 + ...?

Adding a finite initial term does not affect convergence; the overall behavior depends on the tail of the series, which converges if the terms decrease sufficiently fast.

What is the conclusion about the convergence of the series 1 + 1/22 + 1/33 + 1/44 + 1/55 ...?

The series converges because its terms decrease approximately like $1/n^2$, and the p-series with p=2 converges.

Why is the series 1 + 1/22 + 1/33 + 1/44 + 1/55

considered convergent?

Because the terms decrease faster than or equal to $1/n^2$ for large n, which ensures the series converges according to the p-series test.

Additional Resources

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Understanding whether an infinite series converges or diverges is a fundamental question in calculus and mathematical analysis. Infinite series appear frequently across various fields such as physics, engineering, economics, and computer science, where they describe processes that extend indefinitely. In this article, we delve into the analysis of a specific series:

$$ 1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \frac{1}{5^5} + \dots $$

to determine whether it converges or diverges. We will explore the nature of the series, the techniques used for convergence testing, and interpret the results with clarity, making the complex concepts accessible to readers with a foundational understanding of calculus.

Understanding the Series: Structure and Significance

The series under consideration is:

$$ \sum_{n=1}^{\infty} \frac{1}{n^n} $$

This is a special type of series known for its rapidly decreasing terms. Each term in the series is of the form:

$$ a_n = \frac{1}{n^n} $$

which indicates that as n increases, the denominator grows exponentially (since it involves n raised to the power of itself), leading to an extremely quick decrease in the size of each subsequent term.

Why is this series interesting?
Because of the super-exponential decay rate, the series converges very rapidly. Nonetheless, confirming its convergence rigorously requires applying specific convergence tests. Before that, understanding the behavior of the individual terms provides intuition about the series' potential for convergence.

The Behavior of the Series Terms: Intuitive Insights

To evaluate whether the series converges or diverges, it's essential to analyze the general term:

$$ a_n = \frac{1}{n^n} $$

- For small n, the terms are relatively larger:
$$ a_1 = 1, \quad a_2 = \frac{1}{4}, \quad a_3 = \frac{1}{27} $$
- As n increases, the denominator n^n grows extremely fast, making a_n approach zero at an incredible pace.

This rapid decay suggests the series might converge; however, this must be confirmed through rigorous tests.

Convergence Tests Applicable to the Series

Several tests can be used to determine the convergence or divergence of an infinite series:

1. The Ratio Test
2. The Root Test
3. Comparison Test
4. Limit Comparison Test

Given the structure of the series, the Ratio Test and Root Test are particularly effective.

Applying the Ratio Test

The Ratio Test involves examining the limit:

$$ L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| $$

- If $L < 1$, the series converges absolutely.
- If $L > 1$, the series diverges.
- If $L = 1$, the test is inconclusive.

Let's compute this for $a_n = \frac{1}{n^n}$:

$$
a_{n+1} = \frac{1}{(n+1)^{n+1}}
$$

$$
\frac{a_{n+1}}{a_n} = \frac{1/(n+1)^{n+1}}{1/n^n} = \frac{n^n}{(n+1)^{n+1}}
$$

Expressing the denominator:

$$
(n+1)^{n+1} = (n+1)^n \times (n+1)
$$

So,

$$
\frac{a_{n+1}}{a_n} = \frac{n^n}{(n+1)^n \times (n+1)} = \left(\frac{n}{n+1} \right)^n \times \frac{1}{n+1}
$$

Now, analyze the limit:

$$
L = \lim_{n \to \infty} \left(\frac{n}{n+1} \right)^n \times \frac{1}{n+1}
$$

The term:

$$
\left(\frac{n}{n+1} \right)^n = \left(1 - \frac{1}{n+1} \right)^n
$$

As $n \to \infty$:

$$
\left(1 - \frac{1}{n+1} \right)^n \to e^{-1}
$$

and

$$
\frac{1}{n+1} \to 0
$$

Therefore,

$$
L = e^{-1} \times 0 = 0
$$

Since $L = 0 < 1$, the Ratio Test confirms that the series converges absolutely.

Confirming Convergence with the Root Test

The Root Test examines:

$$
L' = \lim_{n \to \infty} \sqrt[n]{|a_n|}
$$

Given:

$$
a_n = \frac{1}{n^n}
$$

we compute:

$$
\sqrt[n]{a_n} = \left(\frac{1}{n^n} \right)^{1/n} = \frac{1}{n}
$$

Taking the limit:

$$
L' = \lim_{n \to \infty} \frac{1}{n} = 0
$$

Again, since $L' = 0 < 1$, the series converges absolutely by the Root Test as well.

Implications of the Tests: Absolute Convergence

Both the Ratio Test and the Root Test conclusively demonstrate the series:

$$
\sum_{n=1}^{\infty} \frac{1}{n^n}
$$

is absolutely convergent. This means not only does the series converge, but so do the series of absolute values of its terms—highlighting its strong tendency toward a finite sum.

The Sum of the Series: Is There a Closed-Form Expression?

An intriguing aspect of this series is whether we can compute its sum explicitly. Unlike geometric series or certain p-series, the sum:

$$
S = \sum_{n=1}^\infty \frac{1}{n^n}
$$

does not have a simple closed-form expression involving elementary functions.

However, this series is well-studied, and its approximate value is known with high precision.

Numerical approximation:

Calculating the first few terms:

- $n=1$: 1
- $n=2$: $1/4 = 0.25$
- $n=3$: $1/27 \approx 0.0370$
- $n=4$: $1/256 \approx 0.0039$
- $n=5$: $1/3125 \approx 0.00032$

Adding these:

$$
1 + 0.25 + 0.0370 + 0.0039 + 0.00032 \approx 1.2915
$$

Including more terms rapidly approaches approximately 1.291285997..., which is recognized as the sum of the series $\sum_{n=1}^\infty \frac{1}{n^n}$. This value is sometimes called the Glaisher–Kinkelin constant in certain contexts, although that involves more advanced functions.

Broader Perspectives and Applications

The convergence of this series is more than a mathematical curiosity. It exemplifies how rapidly decreasing sequences can produce convergent infinite sums, which are useful in:

- Number theory: For example, in the study of exponential series and special constants.
- Computer science: Algorithms involving geometric decay or exponential smoothing.
- Physics: In quantum mechanics and statistical physics, series with super-exponential decay appear in partition functions and state sums.

Understanding the convergence behavior of such series aids in designing efficient algorithms, approximating functions, and solving complex problems involving infinite processes.

Summary: Is the Series Convergent or Divergent?

Based on rigorous convergence tests, the series:

$$
\sum_{n=1}^\infty \frac{1}{n^n}
$$

\]

is convergent. Both the Ratio Test and the Root Test confirm absolute convergence, and numerical approximations support this conclusion. The rapid decay of the terms ensures the sum approaches a finite value, approximately 1.2913, illustrating the elegance and power of convergence analysis.

Final Thoughts: Why It Matters

Determining whether an infinite series converges or diverges is a cornerstone of mathematical analysis, with practical implications across sciences and engineering. Series like $\sum_{n=1}^\infty 1/n^n$ highlight how certain sequences diminish so quickly that they sum up to a finite limit, despite extending infinitely. These insights deepen our understanding of infinite processes and enable us to model real-world phenomena with confidence.

In the broader scope, mastering convergence tests and their applications empowers researchers and students to analyze more complex series, unlocking

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