Evaluate D (x2+y2)^3/2 DA, Where D Is The Region Bounded By Y = X,x^2+y^2 = 1and The X-axis In The Second

Evaluate D (x^2 + y^2)^{3/2} \, dA, where D is the region bounded by y = x, x^2 + y^2 = 1, and the x-axis in the second quadrant

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Understanding the Problem: An Introduction

In the realm of multivariable calculus, evaluating double integrals over specific regions is a fundamental task that combines geometric intuition with algebraic manipulation. The problem at hand involves calculating the double integral of the function $(x^2 + y^2)^{3/2}$ over a region D, which is bounded by the curve $y = x$, the circle $x^2 + y^2 = 1$, and the x-axis in the second quadrant.

This scenario offers a rich opportunity to explore the application of coordinate transformations, particularly switching to polar coordinates, to simplify the integral. It also involves understanding the geometric boundaries of the region D, visualizing the domain, and carefully setting up the limits of integration.

Visualizing the Region D

Boundaries of Region D

The region D is defined by three conditions:

1. The circle $x^2 + y^2 = 1$: This is a unit circle centered at the origin with radius 1.
2. The line $y = x$: A straight line passing through the origin with a 45-degree angle to the axes.
3. The x-axis in the second quadrant: This is the line $y=0$, with $x \leq 0$ in the second quadrant.

Geometric Interpretation

- The circle $x^2 + y^2 = 1$ intersects the axes at $(\pm 1,0)$ and $(0,\pm 1)$.
- The line $y = x$ intersects the circle at points satisfying both equations:

$$
y = x \quad \text{and} \quad x^2 + y^2 = 1
$$

Substituting $y = x$:

$$
x^2 + x^2 = 1 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{\sqrt{2}}{2}
$$

Since we're in the second quadrant ($x \leq 0$), the intersection point is:

$$
x = -\frac{\sqrt{2}}{2}, \quad y = -\frac{\sqrt{2}}{2}
$$

- The region D in the second quadrant is bounded between:

- The arc of the circle $x^2 + y^2 = 1$ from the intersection point with the line $y=x$ to the point where the circle intersects the x-axis at $(-1,0)$.
- The line $y = x$.
- The x-axis $y=0$.

The region essentially forms a "slice" in the second quadrant between the circle and the line $y=x$, bounded below by the x-axis.

Choosing Coordinates: Cartesian vs. Polar

Since the integrand involves $x^2 + y^2$, which naturally suggests polar coordinates, and the region is circular in nature, switching to polar coordinates simplifies the process considerably.

Polar Coordinates Overview

- Conversion formulas:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

- The Jacobian determinant for the transformation:

$$
dA = r\, dr\, d\theta
$$

- The integrand becomes:

$$
(x^2 + y^2)^{3/2} = r^{3}
$$

because $x^2 + y^2 = r^2$.

This transformation turns the integral into an easier form:

$$
\iint_D r^{3} \, r\, dr\, d\theta = \iint_D r^{4} \, dr\, d\theta
$$

Determining the Limits of Integration in Polar Coordinates

To set up the integral properly, we need to establish the bounds for r and θ.

Angular Limits (θ)

- The line $y = x$ corresponds to:

$$
y = x \Rightarrow \sin \theta = \cos \theta \Rightarrow \tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}
$$

- In the second quadrant, θ ranges from $\frac{\pi}{2}$ (the positive y-axis) down to $\frac{\pi}{4}$ (the line $y=x$).

- Therefore, the angular bounds are:

$$
\theta \in \left[\frac{\pi}{2}, \frac{\pi}{4}\right]
$$

Note: Since we're integrating over the region in the second quadrant, and the line $y=x$ is at $\pi/4$, the region extends from $\pi/2$ (vertical line) down to $\pi/4$.

Radial Limits (r)

- For each fixed θ, the radius r varies from:

- The inner boundary: the circle $r=1$.
- To the boundary along the line $y=x$, which intersects the circle at $r = 1$.

But we need to find r at the boundary $x^2 + y^2=1$, and the line $y=x$:

- On the circle: $r=1$.
- On the line $y = x$:

$$
r \sin \theta = r \cos \theta \Rightarrow \sin \theta = \cos \theta \Rightarrow \theta = \pi/4
$$

But since the boundary is the circle, the maximum r in the region is $r=1$.

- At the intersection point of the circle, line, and x-axis, the maximum r is 1.

- The region in question is bounded by the circle at $r=1$ and the line $y=x$. Since r varies from 0 (origin) up to the circle's boundary $r=1$, but only in the sector between $\pi/2$ and $\pi/4$.

- For each θ, r runs from:

$$
r=0 \quad \text{(origin)} \quad \text{to} \quad r=1
$$

but constrained by the region's boundaries.

However, because we're integrating over the region bounded by the circle and the line, the actual radial limits are:

- For $\theta \in [\pi/2, \pi/4]$:

- The lower limit of r is 0.
- The upper limit of r is the minimum of the circle's radius (which is 1) and the boundary set by the line $y=x$.

But since the circle is $r=1$, and the line $y=x$ intersects the circle at $r = 1$, the maximum r is 1.

In this setup, because the region is between the circle and the line $y=x$, for each θ in $[\pi/2, \pi/4]$, the boundary given by the circle is at $r=1$, and the boundary given by the line r satisfying $y=x$:

$$
r \sin \theta = r \cos \theta \Rightarrow \text{the line is at } r \text{ arbitrary, but in the region, the boundary is at } r = \frac{\text{distance along } y=x}{\sin \theta}
$$

\]

Alternatively, considering the line \(y=x\) in polar coordinates:

\[
r \sin \theta = r \cos \theta \Rightarrow \sin \theta = \cos \theta \Rightarrow \theta = \pi/4
\]

which is the boundary line, corresponding to \(\theta = \pi/4\).

Thus, the region in the second quadrant is between:

- \(\theta=\pi/2\) and \(\theta=\pi/4\),
- For each \(\theta\), \(r\) from 0 to the boundary at \(r=1\).

Final Limits:

- \(\theta \in [\pi/2, \pi/4]\),
- \(r \in [0, 1]\).

Setting Up the Integral

Using polar coordinates, the double integral becomes:

\[
\iint_D (x^2 + y^2)^{3/2} \, dA = \int_{\theta=\pi

Frequently Asked Questions

What does the notation D represent in the given problem?

D represents the region bounded by the curves y = x, the circle x^2 + y^2 = 1, and the x-axis in the second quadrant.

How is the function to be differentiated expressed in the problem?

The function is given as D (x^2 + y^2)^{3/2}, and we need to find its differential DA with respect to the region D.

What is the significance of the region being in the second

quadrant?

The second quadrant implies $x \leq 0$ and $y \geq 0$, which affects the limits of integration and the sign conventions when evaluating the differential.

How do the boundary curves y = x and x^2 + y^2 = 1 influence the shape of the region D?

The line $y = x$ intersects the circle $x^2 + y^2 = 1$ in the second quadrant, creating a sector bounded by these curves and the x-axis, defining the region D.

What is the approach to evaluate the differential DA over the region D?

Typically, we convert the integral into polar coordinates because of the circular boundary, then integrate over the specified limits in the second quadrant.

How is the expression (x^2 + y^2)^{3/2} related to the polar coordinate variables?

In polar coordinates, $x^2 + y^2 = r^2$, so the expression becomes r^3, simplifying the integration process.

What are the limits of integration in polar coordinates for the region D?

In the second quadrant, r varies from 0 to 1, and θ varies from π/2 to π, corresponding to the boundary $y = x$ and the circle.

What are the key steps to evaluate the differential of the given function over region D?

Convert the function to polar coordinates, set the proper limits based on the boundary curves, perform the integration to find the total differential DA, considering the boundary conditions.

Additional Resources

Evaluate D $(x^2 + y^2)^{3/2}$ dA, where D is the region bounded by $y = x$, $x^2 + y^2 = 1$, and the x-axis in the second quadrant — this problem presents a fascinating integration challenge that combines geometric intuition with analytical prowess. Understanding how to evaluate such an integral requires a solid grasp of the region's geometry, the appropriate coordinate system transformation, and careful setup of the integral bounds. In this comprehensive guide, we will explore step-by-step how to approach this problem, from visualizing the region to executing the integral, ensuring clarity at every stage.

Understanding the Problem and the Region D

Before diving into calculations, it’s crucial to clearly understand what the problem asks us to do.

- Objective: Evaluate the double integral
$$
\iint_D (x^2 + y^2)^{3/2} \, dA
$$
over the region D.

- Region D: Bounded by the following curves:
- Line: $y = x$
- Circle: $x^2 + y^2 = 1$
- X-axis: $y = 0$

- In the second quadrant: This indicates the portion where $x \leq 0$ and $y \geq 0$, but the problem states the region is bounded "In the second," which suggests that the relevant portion of the circle and lines is in the second quadrant (where $x \leq 0$, $y \geq 0$).

Visualizing the Region D

Visualizing the region helps immensely:

- The unit circle $x^2 + y^2 = 1$ centered at the origin with radius 1.
- The line $y = x$ passes through the origin with a 45° angle.
- The x-axis is $y=0$.

In the second quadrant, the circle is in the area where $x \leq 0$, $y \geq 0$. The line $y = x$ in this quadrant runs from the origin to the third quadrant, but since we are in the second quadrant, it intersects the circle in some point in that region.

The region D is therefore a sector of the circle bounded by the circle arc in the second quadrant, the line $y = x$, and the x-axis, forming a triangular-like region with curved boundary.

Step 1: Setting Up the Problem

To evaluate the integral over the region D, choosing an appropriate coordinate system simplifies the process.

Polar Coordinates

Given the presence of $x^2 + y^2$ in the integrand and the circle boundary, polar coordinates are a natural choice:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

with Jacobian:

$$
dA = r \, dr \, d\theta
$$

Expressing the Boundaries in Polar Coordinates

- Circle: $r = 1$ (since the circle is $x^2 + y^2 = 1$)
- Line: $y = x \Rightarrow r \sin \theta = r \cos \theta \Rightarrow \sin \theta = \cos \theta \Rightarrow \tan \theta = 1 \Rightarrow \theta = \pi/4$

In the second quadrant:

- θ varies from $\pi/2$ (positive y-axis) down to $\pi/4$ (the line $y=x$).

- X-axis: $y=0 \Rightarrow \theta=0$, but since the region is in the second quadrant, the boundary along the x-axis is at $\theta = \pi$. However, because the region is bounded above by the circle and below by the x-axis, and in the second quadrant ($\pi/2 \leq \theta \leq \pi$), the relevant boundary along the x-axis is at $y=0$, which corresponds to $\theta= \pi$.

But, considering the specific region, it is bounded above by the circle arc from $\theta=\pi/2$ down to $\pi/4$, and below by the line $y=x$ (which has $\theta = \pi/4$). The x-axis corresponds to $y=0$, which in the second quadrant is at $\theta=\pi$, but the region between the x-axis and the circle in the second quadrant is from $\theta=\pi/2$ to π.

Since the problem specifies the region bounded by $y=x$, the circle, and the x-axis in the second quadrant, the region D is the sector between $\theta = \pi/2$ and $\theta = \pi/4$, bounded above by the circle and below by the line $y=x$, and along the outer boundary by the circle.

Step 2: Determining the Integration Limits

Angular Limits (θ)

- The region extends from the line $y=x$ at $\theta = \pi/4$ to the x-axis at $\theta = \pi/2$.
- Since the region is in the second quadrant, the angles are:

$$
\boxed{
\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]
}
$$

Radial Limits (r)

- For each θ, r varies from the inner boundary to the outer boundary:

1. Inner boundary: The line $y = x$ corresponds to $r = 0$ at the origin, but more specifically, the region is bounded inside by the circle $r=1$, and below by the line $y= x$.

2. Outer boundary: The circle $r=1$.

3. Between $\theta=\pi/4$ and $\pi/2$:

- The region is between the line $y=x$ (which passes through the origin) and the circle $r=1$. For points in this angular sector, the boundary along the radius is:

$$
r \text{ from } r = 0 \text{ (the origin) to } r = R(\theta)
$$

- But since the region is bounded by the circle and the line, the outer boundary is the circle $r=1$, and the inner boundary is the line $y=x$.

- Along each θ, the distance from the origin to the line $y=x$ is:

$$
\text{Line } y=x \Rightarrow y = x \Rightarrow r \sin \theta = r \cos \theta \Rightarrow \text{Line passes through the origin.}
$$

- The minimum r at each θ is 0 (origin), and the maximum is the circle $r=1$.

- But the region is the area between the line $y=x$ and the circle—that is, for $\theta \in [\pi/4, \pi/2]$, the radial coordinate r runs from the line $y=x$ up to the circle $r=1$.

- The distance from the origin to the line $y=x$ at angle θ is:

$$
r_{\text{line}}(\theta) = 0
$$

since the line passes through the origin.

However, for points inside the region, r ranges from the line (which is at zero in the origin) up to the circle radius $r=1$.

But in this case, the region is the sector between the line $y=x$ and the circle $r=1$. The key is that the region is bounded by the line $y = x$ (at $\theta=\pi/4$), the circle $r=1$, and the x-axis $y=0$.

Given the geometry, the region in question is the sector in the second quadrant between $\theta = \pi/4$ and $\theta = \pi/2$, bounded below by $y=0$ (the x-axis), above by the circle $r=1$, and on the side by the line $y=x$.

Thus, the radial limits are:

- For $\theta \in [\pi/4, \pi/2]$,
$$
r \in [0, R(\theta)]
$$

where $R(\theta)$ is the minimum of:

- \(r=1 \) (the circle)
- and the line \(y=x \), which passes through the origin, so at \(\theta=\pi/4 \), the line intersects the circle at \(r= \frac{1}{\cos \

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