Evaluate The Trigonometric Function At The Given Real Number. Write Your Answer As A Simplified Fraction,

Evaluate The Trigonometric Function At The Given Real Number. Write Your Answer As A Simplified Fraction,

Understanding how to evaluate trigonometric functions at specific real numbers is a fundamental skill in mathematics, particularly in trigonometry. These evaluations are crucial in various applications, including physics, engineering, and computer science. This article provides a comprehensive guide on how to evaluate trigonometric functions such as sine, cosine, tangent, and their reciprocals at given real numbers, and how to express the results as simplified fractions. We will explore the essential concepts, methods, and tips to accurately compute these values and represent them in the simplest form.

Understanding Trigonometric Functions

Before delving into the evaluation process, it is vital to understand the primary trigonometric functions and their significance:

1. Sine (sin)

- Defines the ratio of the length of the opposite side to the hypotenuse in a right-angled triangle.
- For an angle θ in a right triangle, sin(θ) = opposite/hypotenuse.

2. Cosine (cos)

- Represents the ratio of the length of the adjacent side to the hypotenuse.
- For an angle θ, cos(θ) = adjacent/hypotenuse.

3. Tangent (tan)

- The ratio of sine to cosine.
- tan(θ) = sin(θ)/cos(θ) = opposite/adjacent.

4. Cotangent (cot)

- The reciprocal of tangent.
- cot(θ) = 1/tan(θ) = cos(θ)/sin(θ).

5. Secant (sec)

- The reciprocal of cosine.
- sec(θ) = 1/cos(θ).

6. Cosecant (csc)

- The reciprocal of sine.
- $\csc(\theta) = 1/\sin(\theta)$.

Evaluating Trigonometric Functions at Specific Real Numbers

Evaluating these functions at a given real number involves identifying the angle's measure in either degrees or radians and then applying the appropriate method to compute the value.

1. Evaluating at Special Angles

Certain angles have well-known sine, cosine, and tangent values, which are often memorized or derived from the unit circle. These angles include:

- 0°, 30°, 45°, 60°, 90°, and their radian equivalents: 0, π/6, π/4, π/3, π/2.

Common values:

Angle (degrees)	Angle (radians)	sin	cos	tan
0°	0	0	1	0
30°	π/6	1/2	⍰ 3/2	1/⍰ 3
45°	π/4	⍰ 2/2	⍰ 2/2	1
60°	π/3	⍰ 3/2	1/2	⍰ 3
90°	π/2	1	0	undefined

When evaluating at these angles, the values are often expressed as fractions involving square roots, which can be simplified for clarity.

Example:

Evaluate $\sin(\pi/6)$:

$\sin(\pi/6) = 1/2$ (already a simplified fraction).

---

2. Using the Unit Circle for General Angles

The unit circle is a powerful tool for evaluating trigonometric functions at angles beyond the special angles. It provides the sine and cosine values as coordinates of points on the circle:

- For an angle θ, the point on the unit circle is $(\cos(\theta), \sin(\theta))$.

Steps to evaluate:

- Convert the angle to radians if given in degrees.
- Find the corresponding point on the unit circle.
- Read off the x and y coordinates, which represent $\cos(\theta)$ and $\sin(\theta)$, respectively.
- Express these as simplified fractions involving square roots if necessary.

Example:

Evaluate $\cos(2\pi/3)$:

- $2\pi/3$ radians is 120°.
- On the unit circle, $\cos(2\pi/3) = -1/2$.

---

3. Using Trigonometric Identities

For angles that are sums, differences, or multiples of known angles, identities can simplify evaluation:

- Sum and Difference Formulas:

- $\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$
- $\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$
- $\tan(a \pm b) = (\tan(a) \pm \tan(b)) / (1 \mp \tan(a)\tan(b))$

- Double Angle Formulas:

- $\sin(2a) = 2\sin(a)\cos(a)$
- $\cos(2a) = \cos^2(a) - \sin^2(a)$

- Half-Angle Formulas:

- $\sin(a/2)$ and $\cos(a/2)$ can be derived from these identities.

These identities help evaluate the functions at more complex angles.

Expressing Trigonometric Values as Simplified Fractions

Once the value of a trigonometric function is determined, expressing it as a simplified fraction involves rationalizing and reducing the expression.

1. Rationalizing Square Roots

Values involving square roots often appear in the form of $\sqrt{n}$. To simplify, rationalize denominators if necessary:

- Example: $1/\sqrt{2}$ can be written as $\sqrt{2}/2$.

Steps:

- Multiply numerator and denominator by $\sqrt{2}$.
- Result: $(1\times\sqrt{2})/(\sqrt{2}\times\sqrt{2}) = \sqrt{2}/2$.

2. Simplifying Complex Fractions

When the value involves fractions with nested radicals, simplify numerator and denominator separately, then divide.

Example:

Evaluate tan(45°):

- tan(45°) = 1 (already a simplified fraction).

Practical Examples of Evaluating Trigonometric Functions

Let's explore some specific examples illustrating the process of evaluation and simplification:

Example 1: Evaluate sin(π/3)

- Known from the unit circle: sin(π/3) = ⍰ 3/2.
- The fraction ⍰ 3/2 is already simplified.

Example 2: Evaluate cos(2π/3)

- From the unit circle: cos(2π/3) = -1/2.
- The fraction is already in simplest form.

Example 3: Evaluate tan(75°)

- 75° = 45° + 30°.
- Use the sum formula:

tan(75°) = (tan(45°) + tan(30°)) / (1 - tan(45°)×tan(30°))

- Known values:

- tan(45°) = 1
- tan(30°) = 1/⍰ 3

- Substitute:

tan(75°) = (1 + 1/⍰ 3) / (1 - 1×1/⍰ 3) = (1 + 1/⍰ 3) / (1 - 1/⍰ 3)

- Rationalize numerator and denominator:

- Multiply numerator and denominator by ⍰ 3:

Numerator:

(1×⍰ 3 + 1) / ⍰ 3

Denominator:

$(1 \times \sqrt{3} - 1) / \sqrt{3}$

- Simplify numerator:

$(\sqrt{3} + 1) / \sqrt{3}$

- Simplify denominator:

$(\sqrt{3} - 1) / \sqrt{3}$

- Overall:

$[(\sqrt{3} + 1)/\sqrt{3}] / [(\sqrt{3} - 1)/\sqrt{3}] = (\sqrt{3} + 1)/\sqrt{3} \times \sqrt{3}/(\sqrt{3} - 1) = (\sqrt{3} + 1)/(\sqrt{3} - 1)$

- Rationalize the denominator:

Multiply numerator and denominator by $(\sqrt{3} + 1)$:

$[(\sqrt{3} + 1)(\sqrt{3} + 1)] / [(\sqrt{3} - 1)(\sqrt{3} + 1)]$

- Simplify numerator:

$(\sqrt{3} + 1)^2 = (\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + 1^2 = 3 + 2\sqrt{3} + 1 = 4 + 2\sqrt{3}$

- Simplify denominator:

$(\sqrt{3})^2 - 1^2 = 3 - 1 = 2$

- Final result:

$(4 + 2\sqrt{3}) / 2 = 2 + \sqrt{3}$

Answer:

$\tan(75°) = 2 + \sqrt{3}$

Expressed as a simplified sum of an integer and a radical, not a fraction, but if we prefer a fractional form involving radicals, this is considered simplified.

Tips for Accurate Evaluation and Simplification

- Memorize key values of sine, cosine, and tangent for special angles.
- Use the unit circle diagram to find exact values.
- Apply trigonometric identities

Frequently Asked Questions

Evaluate sin(π/6) and write the answer as a simplified fraction.

1/2

Find cos(3π/4) and express it as a simplified fraction.

-⍰ 2/2

Determine tan(π/3) and write it as a simplified fraction.

⍰ 3

Calculate sin(2π/3) and present the answer as a simplified fraction.

⍰ 3/2

Evaluate cos(π/2) and write the result as a simplified fraction.

0

Find tan(0) and express it as a simplified fraction.

0

Determine sin(5π/6) and write the answer as a simplified fraction.

1/2

Calculate cos(7π/6) and present as a simplified fraction.

-⍰ 3/2

Evaluate tan(π/4) and write the answer as a simplified fraction.

1

Additional Resources

Evaluate The Trigonometric Function At The Given Real Number. Write Your Answer As A Simplified Fraction

---

Introduction

In the realm of mathematics, particularly in the study of trigonometry, evaluating functions at specific points is fundamental for understanding their behavior, properties, and applications. Whether it's calculating the sine, cosine, tangent, or other trigonometric functions at particular real numbers, the process often involves a combination of geometric reasoning, algebraic manipulation, and a solid understanding of the unit circle. This comprehensive article aims to provide an in-depth exploration of how to evaluate trigonometric functions at given real numbers, emphasizing the importance of expressing answers as simplified fractions. Through detailed explanations, illustrative examples, and analytical insights, readers will develop a robust grasp of the methods and concepts involved.

The Significance of Trigonometric Evaluation

Trigonometric functions are essential tools in various fields including physics, engineering, computer science, and even biology. Their ability to model periodic phenomena such as sound waves, electromagnetic signals, and biological rhythms makes them invaluable. Evaluating these functions at specific points helps in solving real-world problems, designing systems, and analyzing data.

For example, knowing the value of sin(π/4) helps in computing the height of a projectile at a certain angle, while evaluating tan(60°) is crucial in understanding slopes in geometry and physics. Expressing these evaluations as simplified fractions not only aligns with mathematical elegance but also facilitates easier computation, comparison, and interpretation.

Fundamental Concepts in Trigonometric Evaluation

Before delving into specific evaluation techniques, it is essential to understand some foundational concepts:

1. The Unit Circle

The unit circle, a circle with radius 1 centered at the origin (0,0), serves as the backbone for understanding trigonometric functions. For any real number θ (measured in radians or degrees), the point on the unit circle corresponding to θ is given by:

- Coordinates: (cos θ, sin θ)
- Trigonometric functions:
- sin θ = y-coordinate
- cos θ = x-coordinate
- tan θ = sin θ / cos θ (where cos θ ≠ 0)

2. Radians and Degrees

Angles can be expressed in degrees or radians. Conversion between them is crucial:

- Degrees to radians: θ (radians) = θ (degrees) × (π / 180)
- Radians to degrees: θ (degrees) = θ (radians) × (180 / π)

Understanding this conversion is vital when evaluating functions at specific angle measures.

3. Special Angles and Their Values

Certain angles are "special," with well-known sine, cosine, and tangent values. These include:

Angle	Degrees	Radians	sin	cos	tan
0°	0	0	0	1	0
30°	π/6	π/6	1/2	√ 3/2	1/√ 3
45°	π/4	π/4	√ 2/2	√ 2/2	1
60°	π/3	π/3	√ 3/2	1/2	√ 3
90°	π/2	π/2	1	0	undefined

These serve as the basis for evaluating trigonometric functions at common angles.

Methods for Evaluating Trigonometric Functions

1. Direct Use of the Unit Circle

The most straightforward approach for certain angles, especially the special angles, is to use the unit circle. By identifying the corresponding point on the circle, the sine and cosine values can be read directly.

2. Algebraic Manipulation and Known Identities

When the given angle is not one of the special angles, identities like the angle addition formulas are instrumental:

- Sine addition formula:

$$
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
$$

- Cosine addition formula:

$$
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
$$

- Tangent addition formula:

$$
\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
$$

Using these identities allows us to evaluate complex angles in terms of known values.

3. Reference Angles and Symmetry

The symmetry of the unit circle provides information about the values of trigonometric functions in different quadrants, aiding in evaluation:

- Quadrant I: All functions positive.
- Quadrant II: Sine positive, cosine negative.
- Quadrant III: Tangent positive.
- Quadrant IV: Cosine positive.

By understanding the signs and reference angles, evaluations become more manageable.

4. Inverse Trigonometric Functions

In some cases, the problem involves finding the angle given a particular function value, which requires the use of inverse functions. These are particularly useful when the angle is not explicitly given but the function's value is.

Expressing Results as Simplified Fractions

Once a trigonometric function is evaluated, expressing the result as a simplified fraction is crucial for clarity and precision. The process involves:

1. Rationalizing Denominators: For example, if the answer involves $\sqrt{3}/2$, it is already simplified. If square roots are in the numerator or denominator, rationalize accordingly.

2. Reducing Fractions: If the fraction can be simplified by dividing numerator and denominator by their greatest common divisor (GCD), do so to obtain the simplest form.

3. Handling Irrational Numbers: When the value involves irrational numbers like $\sqrt{2}$, $\sqrt{3}$, or $\sqrt{5}$, expressing the value precisely as a fraction over radicals is the standard. For exact evaluation, keep radicals in the numerator or denominator as appropriate, and rationalize if necessary.

Practical Examples and Step-by-Step Evaluation

Example 1: Evaluate $\sin(\pi/6)$

Step 1: Recognize the angle as a special angle, corresponding to 30°.

Step 2: Use the known value from the unit circle: $\sin(\pi/6) = 1/2$.

Step 3: The answer is already a simplified fraction: 1/2.

Example 2: Evaluate $\cos(75°)$

Step 1: Express 75° as a sum of known angles: 75° = 45° + 30°.

Step 2: Convert to radians if necessary: $\pi/4 + \pi/6$.

Step 3: Use the cosine addition formula:

$$
\cos (A + B) = \cos A \cos B - \sin A \sin B
$$

$$
\cos 75° = \cos 45° \cos 30° - \sin 45° \sin 30°
$$

Step 4: Substitute known values:

$$
\cos 45° = \frac{\sqrt{2}}{2}, \quad \cos 30° = \frac{\sqrt{3}}{2}
$$
$$
\sin 45° = \frac{\sqrt{2}}{2}, \quad \sin 30° = \frac{1}{2}
$$

Step 5: Calculate:

$$
\cos 75° = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)
$$

$$
= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}
$$

Step 6: Final answer as a simplified fraction:

$$
\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}
$$

Advanced Topics: Trigonometric Values at Arbitrary Angles

While the previous examples involve standard or constructible angles, evaluating at arbitrary real numbers often requires more advanced methods:

1. Using Series Expansions

Taylor or Maclaurin series can approximate trigonometric functions as infinite sums, useful for numerical approximations.

2. Numerical Methods and Calculators

Modern tools like scientific calculators and computer algebra systems can compute values to high precision, but expressing results as exact simplified fractions remains an important skill for theoretical understanding.

3. Rationalizing and Simplifying Complex Expressions

When values involve radicals or complex expressions, algebraic manipulation and rationalization techniques are essential.

Conclusion

Evaluating trigonometric functions at given real numbers and expressing the results as simplified fractions is a skill that combines geometric intuition, algebraic manipulation, and a deep understanding of fundamental identities and properties. From the straightforward evaluations on the unit circle to more complex angle calculations using addition formulas, each method emphasizes clarity, precision, and mathematical elegance.

Mastery of these techniques enables mathematicians, scientists, and engineers to analyze periodic phenomena, solve geometric problems, and develop a deeper appreciation for the interconnectedness of mathematical concepts. Whether dealing with well-known special angles or arbitrary real numbers, the principles outlined here serve as a comprehensive guide for accurate and simplified evaluation of trigonometric functions.

References and Further

Evaluate The Trigonometric Function At The Given Real Number Write Your Answer As A Simplified Fraction

Evaluate The Trigonometric Function At The Given Real Number Write Your Answer As A Simplified Fraction

Related Articles

- A Plunger 250 Mm In Diameter Is Being Pushed At 125 Mm/sec Into A Tank Filled With Oil Having S.G. =0.82,
- ABC Corporation Is A Publicly Traded Company. You Are Trying To Estimate How Much Debt It Has Outstanding
- Did The Puritans Believe The Native People Had Any Claim To The Land Where The Puritansplanned To Settle?

Back to Home