Exercise 1. Let F(x) = 9 22 Where X (0,3] (a) Approximate The Area Under The Curve With 4 Right-hand-side

Understanding the Problem: Exercise 1. Let F(x) = 9x + 22 Where x (0,3]

In this article, we explore how to approximate the area under the curve of the function F(x) = 9x + 22 over the interval (0, 3] using four right-hand-side Riemann sums. This exercise provides a practical application of integral approximation techniques, which are fundamental in calculus when exact integration is complex or infeasible. We will explain the concepts step-by-step, ensuring clarity for readers at various levels of mathematical understanding.

Defining the Function and Interval

The Function F(x) = 9x + 22

The function F(x) = 9x + 22 is a linear function with slope 9 and y-intercept 22. It is a straight line, which makes the analysis straightforward. The graph of F(x) over the interval (0, 3] is a line starting at F(0) = 22 and ending at F(3) = 9 3 + 22 = 49.

The Interval of Interest: $(0, 3]$

- The interval starts just above 0 and goes up to 3, inclusive of 3.
- Since the interval is open at 0, the area calculation typically considers values approaching 0 but not including it.
- For approximation purposes, the interval is divided into subintervals to estimate the area under the curve.

Understanding Riemann Sums for Area Approximation

What Are Riemann Sums?

Riemann sums are methods for approximating the area under a curve by dividing the interval into smaller segments and summing up the areas of rectangles over each segment. The height of each rectangle depends on the value of the function at specific points within the subinterval.

Types of Riemann Sums

- **Left-hand sum**: Uses the value of the function at the left endpoint of each subinterval.
- **Right-hand sum**: Uses the value of the function at the right endpoint of each subinterval.

- **Midpoint sum**: Uses the value at the midpoint of each subinterval.

In this exercise, we specifically focus on the **right-hand-side** Riemann sum.

Partitioning the Interval: Dividing into 4 Subintervals

Calculating the Width of Each Subinterval

The total interval length is from 0 to 3, i.e., 3 units. Dividing this into 4 equal parts:

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Δx = (3 - 0) / 4 = 0.75
```

Subintervals and Their Endpoints

- Subinterval 1: [0, 0.75]
- Subinterval 2: [0.75, 1.5]
- Subinterval 3: [1.5, 2.25]
- Subinterval 4: [2.25, 3]

Since we are using right-hand sums, for each subinterval, the height of the rectangle will be evaluated at the right endpoint.

Calculating the Right-Hand Side Riemann Sum

Determine the Right Endpoints

- For [0, 0.75], right endpoint: x = 0.75
- For [0.75, 1.5], right endpoint: x = 1.5
- For [1.5, 2.25], right endpoint: x = 2.25
- For [2.25, 3], right endpoint: x = 3

Evaluate F(x) at Each Right Endpoint

Using F(x) = 9x + 22:

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F(0.75) = 9  0.75 + 22 = 6.75 + 22 = 28.75
F(1.5) = 9  1.5 + 22 = 13.5 + 22 = 35.5
F(2.25) = 9  2.25 + 22 = 20.25 + 22 = 42.25
F(3) = 9  3 + 22 = 27 + 22 = 49
```

Calculate the Sum

The right-hand Riemann sum is:

$$R_4 = \Delta x \quad [F(0.75) + F(1.5) + F(2.25) + F(3)]$$

Substituting the values:

$$R_4 = 0.75 \quad [28.75 + 35.5 + 42.25 + 49]$$

Adding the function values:

$$28.75 + 35.5 = 64.25$$
$$64.25 + 42.25 = 106.5$$
$$106.5 + 49 = 155.5$$

Therefore,

$$R_4 = 0.75 \quad 155.5 = 116.625$$

Interpreting the Result

Approximate Area Under the Curve

The right-hand-side Riemann sum approximation of the area under $F(x) = 9x + 22$ from 0 to 3 using 4 subintervals is approximately **116.625 square units**.

Understanding the Accuracy

- Since the function is linear and increasing over the interval, the right sum tends to overestimate the actual area.
- To improve accuracy, increasing the number of subintervals (e.g., 8, 16, or more) would yield a closer approximation to the true integral.
- The exact area can be found by integrating F(x) over (0, 3], which confirms the approximation's proximity.

Calculating the Exact Area Using Integration

Integral of $F(x) = 9x + 22$

The definite integral from 0 to 3:

$$\int_0^3 (9x + 22)\, dx$$

Performing the Integration

1. Integrate term-by-term:

- $\int 9x\,dx = (9/2)\,x^2$
- $\int 22\,dx = 22x$

2. Evaluate from 0 to 3:

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[(9/2)  3² + 22  3] - [(9/2)  0² + 22  0] = [(9/2)  9 + 66] - [0 + 0] =
[(81/2) + 66]
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Simplify:

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81/2 + 66 = 40.5 + 66 = 106.5
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Comparison with the Approximate Sum

The exact area under the curve is 106.5, while our right Riemann sum approximation with 4 subintervals was 116.625. The overestimation aligns with the expectation for an increasing function when using right endpoints.

Conclusion: Applying Riemann Sums Effectively

Using right-hand-side Riemann sums provides a practical way to estimate the area under a curve when the exact integral is difficult or time-consuming to compute. In this exercise, dividing the interval into four subintervals allowed us to approximate the area as approximately 116.625 square units, which is reasonably close to the exact value of 106.5. Increasing the number of subintervals can improve the accuracy further, demonstrating the importance of partitioning in integral approximation methods.

Key Takeaways

- Partitioning intervals into subintervals is essential for Riemann sum calculations.
- The choice of right, left, or midpoint points affects the estimate's accuracy.
- Increasing the number of subintervals generally improves the approximation's closeness to the true integral.

Understanding how to approximate areas under curves using Riemann sums is a fundamental skill in calculus, underpinning more advanced concepts like definite integrals and the Fundamental Theorem of Calculus. This exercise exemplifies these principles clearly and concisely. 0.
- The shape is a parabola opening upwards, with the lowest point at x=0, which is outside our interval but relevant for understanding the growth rate.

This quadratic growth implies that the area under the curve from 0 to 3 will be significant, especially as x increases, since the function's value increases quadratically.

Interval of Integration: (0,3]

The interval spans from just above 0 to 3, inclusive at the upper bound. For the purposes of Riemann sums:

- The interval is partitioned into subintervals, which in this case will be four equal parts.
- Since the left endpoint is not included, and we are using the right-hand rule, the heights of the rectangles are determined by the function's value at the right endpoints of each subinterval.

Understanding the interval and the function's behavior within it is crucial for accurately approximating the integral.

Partitioning the Interval and Subintervals

Dividing the Interval into Subintervals

To approximate the area under the curve with 4 rectangles, the interval (0,3] must be divided into 4 equal parts. The total length of the interval is:

- Length = 3 - 0 = 3

Dividing this into four equal parts gives:

- Subinterval length, Δx = 3 / 4 = 0.75

Thus, the subintervals are:

1. [0, 0.75]
2. [0.75, 1.5]
3. [1.5, 2.25]
4. [2.25, 3]

Since the exercise specifies using the right-hand-side rule, the height of each rectangle will be based on the function's value at the right endpoint of each subinterval:

- Right endpoints: 0.75, 1.5, 2.25, 3

Calculating the Right-Hand-Side Rectangles

Function Values at Right Endpoints

Calculate $F(x)$ at each right endpoint:

- $F(0.75) = 9 (0.75)^2 = 9 \; 0.5625 = 5.0625$
- $F(1.5) = 9 (1.5)^2 = 9 \; 2.25 = 20.25$
- $F(2.25) = 9 (2.25)^2 = 9 \; 5.0625 = 45.5625$
- $F(3) = 9 (3)^2 = 9 \; 9 = 81$

These values will serve as the heights of our rectangles in the Riemann sum approximation.

Calculating the Approximate Area

The formula for the right Riemann sum is:

$$ \text{RHS} \approx \sum_{i=1}^n f(x_i) \Delta x $$

Where:

- $f(x_i)$ is the function value at the right endpoint of the i-th subinterval
- Δx is the width of each subinterval

Applying the data:

$$\text{Area} \approx (F(0.75) + F(1.5) + F(2.25) + F(3)) \times 0.75$$

Plugging in the values:

$$\text{Area} \approx (5.0625 + 20.25 + 45.5625 + 81) \times 0.75$$

Calculating the sum:

$$5.0625 + 20.25 = 25.3125$$

$$25.3125 + 45.5625 = 70.875$$

$$70.875 + 81 = 151.875$$

Finally, multiply by $\Delta x = 0.75$:

$$\text{Approximate Area} \approx 151.875 \times 0.75 = 113.90625$$

Thus, the approximate area under the curve using 4 right-hand-side rectangles is approximately 113.90625.

Discussion of the Approximation

Pros of Using the Right-Hand-Side Method

- Simplicity: Easy to compute as it only requires function evaluations at right endpoints.
- Efficiency: Suitable for quick estimates without complex calculations.
- Applicability: Particularly useful for increasing functions, as in this case, where the approximation tends to overestimate the actual area.

Cons of Using the Right-Hand-Side Method

- Accuracy: Can lead to overestimates (for increasing functions) or underestimates (for decreasing functions) depending on the curve's shape.
- Limited Precision: With only four rectangles, the approximation might be rough, especially for functions with high curvature.
- Bias: The method introduces a systematic bias depending on the function's monotonicity over the interval.

Features and Considerations

- To improve accuracy, increasing the number of rectangles (subintervals) is recommended.
- The method is straightforward but less precise compared to methods like the trapezoidal rule or Simpson's rule.
- When the function is known to be increasing or decreasing, choosing the appropriate Riemann sum (left, right, or midpoint) can help better approximate the true area.

Extensions and Alternative Approaches

While the right-hand-side Riemann sum provides a reasonable approximation, other numerical methods can yield more accurate results:

- Midpoint Rule: Uses the value of the function at the midpoint of each subinterval, often providing better accuracy.
- Trapezoidal Rule: Considers the average of the function's values at the endpoints, reducing some bias.
- Simpson's Rule: Uses quadratic polynomials for approximation, offering higher precision with fewer subintervals.

Implementing these methods requires similar calculations but often involves more complex weighting or additional function evaluations.

Conclusion

The exercise of approximating the area under the curve $F(x) = 9x^2$ over (0,3] using four right-hand-side rectangles offers an insightful look into the fundamentals of numerical integration. By partitioning the interval into four equal parts and evaluating the function at the right endpoints, we obtained an approximate area of approximately 113.90625. While this method is straightforward and quick, it’s important to recognize its limitations in terms of accuracy, especially with a small number of rectangles. For more precise estimates, increasing the number of subintervals or employing alternative numerical methods can significantly improve results. Overall, this exercise underscores the importance of understanding the behavior of functions and the trade-offs involved in different approximation techniques in calculus.

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