F(x)=2x^(2)-7 G(x)=-2x+9 (a) Find ((f)/(g))(1) (b) Find All Values That Are NOT In The Domain Of (f)/(g).

F(x)=2x^(2)-7 G(x)=-2x+9 (a) Find ((f)/(g))(1) (b) Find All Values That Are NOT In The Domain Of (f)/(g).

Understanding how to evaluate and analyze functions, especially when dealing with ratios of functions, is a fundamental skill in algebra and calculus. In this article, we will explore the functions $F(x) = 2x^2 - 7$ and $G(x) = -2x + 9$, focusing on evaluating the combined function $\frac{F(x)}{G(x)}$ at specific points and identifying the restrictions on its domain. Whether you're a student preparing for exams or someone looking to strengthen your understanding of functions, this comprehensive guide will walk you through each step with clear explanations.

Understanding the Given Functions

Before diving into calculations, it's important to understand the individual functions involved and what their expressions mean.

Function Definitions

- F(x): This is a quadratic function, $F(x) = 2x^2 - 7$. Quadratic functions graph as parabolas, opening upwards or downwards depending on the coefficient of x^2. Here, since the coefficient is positive (2), the parabola opens upward.

- G(x): This is a linear function, $G(x) = -2x + 9$. Linear functions graph as straight lines, with the slope indicating the steepness and the intercept indicating where it crosses the y-axis.

Understanding these functions' forms helps anticipate their behavior, such as where they might intersect or where the ratio might be undefined.

Part (a): Calculating $\left(\frac{F}{G}\right)(1)$

This part involves evaluating the ratio of the two functions at $x=1$. The ratio function is defined as:

$$
\left(\frac{F}{G}\right)(x) = \frac{F(x)}{G(x)}
$$

Step-by-step process:

Step 1: Find $F(1)$

Substitute $x=1$ into $F(x)$:

$$
F(1) = 2(1)^2 - 7 = 2(1) - 7 = 2 - 7 = -5
$$

Step 2: Find $G(1)$

Substitute $x=1$ into $G(x)$:

$$
G(1) = -2(1) + 9 = -2 + 9 = 7
$$

Step 3: Compute the ratio $\frac{F(1)}{G(1)}$

Now, divide $F(1)$ by $G(1)$:

$$
\left(\frac{F}{G}\right)(1) = \frac{-5}{7}
$$

Answer for part (a):

$$
\boxed{\left(\frac{F}{G}\right)(1) = -\frac{5}{7}}
$$

This value indicates that at $x=1$, the ratio of the two functions is $-\frac{5}{7}$. The key here is to ensure that the denominator $G(1)$ is not zero; otherwise, the ratio would be undefined.

Part (b): Identifying Values Not in the Domain of $\frac{F}{G}$

The domain of a ratio of functions $\frac{F}{G}$ is all real numbers where:

1. Both $F(x)$ and $G(x)$ are defined.
2. The denominator $G(x)$ is not zero (since division by zero is undefined).

Step 1: Domain of $F(x)$ and $G(x)$

- $F(x) = 2x^2 - 7$: This is a polynomial, which is defined for all real numbers, so the domain of F is $\mathbb{R}$.

- $G(x) = -2x + 9$: Also a polynomial, thus defined for all real numbers, with domain $\mathbb{R}$.

Step 2: Find where $G(x) = 0$

Since the only restriction on the domain of $\frac{F}{G}$ comes from the denominator, identify all x values where $G(x) = 0$:

$$
-2x + 9 = 0
$$

Solve for x:

$$
-2x = -9 \implies x = \frac{-9}{-2} = \frac{9}{2}
$$

$$
x = 4.5
$$

Conclusion:

- The ratio $\frac{F}{G}(x)$ is undefined at $x = \frac{9}{2}$ because the denominator becomes zero.

- Domain of $\frac{F}{G}$: All real numbers except $x = \frac{9}{2}$.

Summary and Key Takeaways

To summarize the important points from this problem:

- Evaluating the ratio at a specific point involves substituting the value into the functions and simplifying.
- The ratio $\frac{F}{G}$ is undefined where $G(x) = 0$, which in this case is at $x= \frac{9}{2}$.
- The domain of the ratio excludes any x where the denominator is zero.

Additional Insights on Function Ratios and Domain Restrictions

Understanding ratios of functions is crucial for many areas of mathematics, including calculus, algebra, and applied sciences. Here are some additional insights:

Why are ratios important?

Ratios often represent rates, slopes, or proportional relationships. For example, in physics, velocity is a ratio of distance over time. In economics, cost per unit is a ratio.

Common restrictions on domain

The primary restriction arises from division by zero. Always check the denominator of a ratio to find potential points of discontinuity.

Graphical interpretation

Graphing the ratio $\frac{F}{G}$ involves plotting both functions and identifying vertical asymptotes at points where the denominator equals zero. Such asymptotes indicate where the function is undefined.

Practice Problems for Reinforcement

1. Given $H(x) = \frac{x^2 - 4}{x - 2}$, find the domain.
2. Evaluate $\left(\frac{H}{F}\right)(3)$ using the functions $H(x) = \frac{x^2 - 4}{x - 2}$ and $F(x) = 2x^2 - 7$.
3. Identify all x values where $\frac{F}{H}$ is undefined.

Conclusion

Mastering the evaluation of function ratios and understanding their domain restrictions are essential skills in mathematics. In this specific case, we've shown how to evaluate $\left(\frac{F}{G}\right)(1) = -\frac{5}{7}$ and identified that the ratio is undefined at $x=\frac{9}{2}$. Remember, always analyze the denominator when working with ratios to determine where the function is valid. These skills lay the foundation for more advanced topics like limits, derivatives, and integrals, where understanding points of discontinuity and function behavior is crucial.

By practicing these concepts regularly, you'll develop a strong intuition for functions, their behaviors, and how to work with complex expressions involving ratios.

Frequently Asked Questions

What is the function $(f/g)(x)$ in terms of $f(x)$ and $g(x)$?

$(f/g)(x)$ is the quotient of the functions, defined as $f(x)$ divided by $g(x)$, or $(f/g)(x) = f(x) / g(x)$, provided $g(x) \neq 0$.

How do you evaluate $(f/g)(1)$ given $f(x)=2x^2 - 7$ and $g(x)=-2x + 9$?

Calculate $f(1)$ and $g(1)$ separately: $f(1)=2(1)^2 - 7=2 - 7= -5$; $g(1)= -2(1)+9= -2+9=7$. Then, $(f/g)(1)=f(1)/g(1)= -5/7$.

What is the value of $(f/g)(1)$?

The value of $(f/g)(1)$ is $-5/7$.

How do you determine the domain of $(f/g)(x)$?

The domain of $(f/g)(x)$ includes all real values of x where both $f(x)$ and $g(x)$ are defined, and $g(x) \neq 0$ to avoid division by zero.

Which values are NOT in the domain of $(f/g)(x)$?

Values where $g(x)=0$ are NOT in the domain. For $g(x)=-2x+9$, set $-2x+9=0$, solving for x gives $x=4.5$. So, $x=4.5$ is excluded.

What is the domain of $(f/g)(x)$ for the given functions?

The domain is all real numbers except $x=4.5$, since at that point $g(x)=0$ and division is undefined.

Are there any other restrictions on the domain besides $g(x) \neq 0$?

No, the only restriction comes from $g(x)$ being non-zero, as $f(x)$ is defined for all real x.

How would you write the set of all x-values not in the domain?

All real x such that x=4.5, because at x=4.5, g(x)=0.

Can the functions f(x) and g(x) be simplified further before evaluation?

In this case, f(x)=2x^2 - 7 and g(x)=-2x + 9 are already in simplified form for evaluation.

What is the importance of checking g(x) □ 0 when working with (f/g)(x)?

Because division by zero is undefined, it's essential to identify where g(x)=0 to determine the domain restrictions for (f/g)(x).

Additional Resources

Comprehensive Analysis of the Function Composition and Domain Considerations for \(\frac{f(x)}{g(x)} \)

Understanding the behavior of functions, especially when involved in ratios or compositions, is fundamental in algebra and calculus. The functions provided:

\[f(x) = 2x^2 - 7 \]
\[g(x) = -2x + 9 \]

are quadratic and linear functions, respectively. Analyzing their ratio \(\frac{f(x)}{g(x)} \) requires a nuanced approach, including evaluating specific points and determining the domain restrictions. This review delves into the detailed process of solving the given problems, exploring not only the immediate solutions but also the underlying principles governing such functions.

Understanding the Functions $f(x)$ and $g(x)$

Before tackling the specific questions, it is essential to thoroughly understand the nature of the functions involved.

Properties of $f(x) = 2x^2 - 7$

- Type: Quadratic function
- Shape: Parabola opening upwards (since coefficient of x^2 is positive)
- Vertex: Located at the minimum point
- For $f(x)$, the vertex occurs at $x = 0$ because the quadratic term dominates, and the linear term is absent.
- $f(0) = 2(0)^2 - 7 = -7$
- Range: All values $y \geq -7$, since the parabola opens upward and the minimum value is at -7
- Domain: All real numbers $\mathbb{R}$, because quadratic functions are defined everywhere

Properties of $g(x) = -2x + 9$

- Type: Linear function
- Graph: Straight line with a slope of -2 and y-intercept at 9
- Domain: All real numbers $\mathbb{R}$
- Range: All real numbers $\mathbb{R}$
- Zero of $g(x)$: Find x such that $g(x) = 0$:

$$
-2x + 9 = 0 \Rightarrow -2x = -9 \Rightarrow x = \frac{9}{2} = 4.5
$$

This zero is critical for understanding domain restrictions in ratios involving $g(x)$.

Part (a): Evaluating $\left(\frac{f}{g}\right)(1)$

The first task is straightforward: compute the value of the ratio $\frac{f(x)}{g(x)}$ at $x=1$.

Step 1: Calculate $f(1)$

$$
f(1) = 2(1)^2 - 7 = 2(1) - 7 = 2 - 7 = -5
$$

Step 2: Calculate $g(1)$

$$
g(1) = -2(1) + 9 = -2 + 9 = 7
$$

Step 3: Compute $\frac{f(1)}{g(1)}$

$$
\left(\frac{f}{g}\right)(1) = \frac{-5}{7}
$$

Result:

\[
\boxed{
\left(\frac{f}{g}\right)(1) = -\frac{5}{7}
}
\]

This calculation is straightforward but highlights an important aspect: division by zero must be avoided. Since \(g(1) \neq 0\), the evaluation is valid, and the result is \(-\frac{5}{7}\).

Part (b): Identifying All Values Not in the Domain of \(\frac{f}{g} \)

Understanding the domain restrictions of \(\frac{f}{g} \) involves examining the points where the denominator \(g(x)\) equals zero, as division by zero is undefined in mathematics.

Step 1: Find where \(g(x) = 0\)

\[
-2x + 9 = 0
\]
\[
-2x = -9
\]
\[

x = \frac{9}{2} = 4.5

\]

At \(x = 4.5\), the function \(g(x)\) becomes zero, causing the ratio \(\frac{f}{g} \) to be undefined.

Step 2: Domain of \(\frac{f}{g} \)

- The domain of \(\frac{f}{g} \) is all real numbers except where \(g(x) = 0\).
- Therefore, the domain is:

\[

\boxed{

\mathbb{R} \setminus \left\{ \frac{9}{2} \right\}

}

\]

In other words:

\[

\text{All real numbers except } x = 4.5

\]

Step 3: Are there other restrictions?

- Since \(f(x)\) is defined everywhere (quadratic), no restrictions emerge from the numerator.

- The only restriction stems from the denominator being zero.

Hence, the only value not in the domain of $\frac{f}{g}$ is $x=4.5$.

Deeper Insights and Additional Considerations

While the problem at hand is straightforward, it opens the door to understanding more complex aspects of functions involving ratios and their domains.

Why is the domain restriction critical?

- The domain restriction at $x=4.5$ exemplifies a fundamental principle: division by zero is undefined.

- When working with rational functions, always identify points where the denominator equals zero to delineate the domain precisely.
- Ignoring these restrictions can lead to incorrect conclusions or undefined expressions.

Impact of such restrictions in calculus and real-world applications

- In calculus, such restrictions are vital when determining limits, continuity, and differentiability.
- In real-world models, these restrictions may correspond to physical impossibilities or conditions where the model breaks down.

Additional explorations: Domain of composite functions

- If one considers \(f(g(x))\) or \(g(f(x))\), the domain restrictions become more intricate.
- For \(f(g(x))\), the domain is the set of \(x\) such that \(g(x)\) is

within the domain of f. Since f is defined everywhere, the only concern remains the domain of g.

Summary and Final Remarks

This detailed exploration of the functions $f(x) = 2x^2 - 7$ and $g(x) = -2x + 9$ within the context of their ratio $\frac{f(x)}{g(x)}$ underscores several essential principles.

- Evaluation at a specific point: At $x=1$,

$$
\left(\frac{f}{g}\right)(1) = -\frac{5}{7}
$$

- Domain restrictions: The ratio is undefined where the denominator is zero, i.e., at $x=4.5$.

- Domain set: All real numbers except \(x=4.5\).

This analysis not only provides the solutions but also emphasizes the importance of understanding the behavior and restrictions of functions, particularly when dealing with ratios and compositions. Such foundational knowledge is crucial for advanced studies in calculus, differential equations, and applied mathematics, where domain considerations influence the validity of models and solutions.

In conclusion, mastering the evaluation of functions at specific points and identifying their domain restrictions are vital skills in mathematics. They form the backbone of more complex concepts and applications, from analyzing limits and continuity to solving real-world problems involving ratios and rates.

F X 2x 2 7 G X 2x 9 A Find F G 1 B Find All Values That Are Not In The Domain Of F G

F X 2x 2 7 G X 2x 9 A Find F G 1 B Find All Values That Are Not In The Domain Of F G

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