Find Both The Vector Equation And The Parametric Equations Of The Line Through (,,) That Is Perpendicular

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When studying lines in three-dimensional space, understanding how to derive both the vector and parametric equations of a line is essential, especially when specific geometric conditions, such as perpendicularity, are involved. This guide provides a comprehensive explanation of how to find these equations for a line passing through a given point and perpendicular to a specified vector or plane. Whether you're a student preparing for exams or a professional working on a geometric problem, mastering these concepts is vital for a solid grasp of vector calculus and 3D geometry.

Understanding the Basics of Line Equations in 3D Space

Before diving into the problem specifics, it's important to understand the foundational concepts involved in line equations in three-dimensional space.

Point and Direction Vector

- Point: The specific point through which the line passes. Denoted as $P_0 (x_0, y_0, z_0)$.
- Direction Vector: A vector indicating the line's direction, denoted as $\vec{d}$. It determines the slope and orientation of the line.

Line Equations in 3D

- Vector Equation: Combines the point and the direction vector.

$$
\vec{r} = \vec{r}_0 + t \vec{d}
$$

where:
- $\vec{r}_0$ is the position vector of the point P_0,
- t is a scalar parameter,
- $\vec{d}$ is the direction vector.

- Parametric Equations: Express the coordinates explicitly as functions of the parameter t:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

where $\vec{d} = \langle a, b, c \rangle$.

Problem Setup: Given Data and Goals

Suppose you are provided with:

- A point $P_0 (x_0, y_0, z_0)$,
- A vector or a condition specifying the line's perpendicularity (e.g., perpendicular to a given vector or plane).

Goals:
1. Find the vector equation of the line passing through P_0 that satisfies the perpendicularity condition.
2. Derive the parametric equations corresponding to this line.

Step-by-Step Approach to Find the Line Equation

To systematically find the equations, follow these steps:

Step 1: Identify the Given Point and Perpendicular Condition

- Point: $P_0 (x_0, y_0, z_0)$
- Perpendicular condition: Usually, the line is perpendicular to a given vector $\vec{v}$ or a plane with a normal vector $\vec{n}$.

Step 2: Determine the Direction Vector $\vec{d}$

- Perpendicular to a vector $\vec{v}$:
- The line's direction vector $\vec{d}$ must be orthogonal to $\vec{v}$.
- This implies $\vec{d} \cdot \vec{v} = 0$.

- Perpendicular to a plane:
- The line's direction vector $\vec{d}$ is parallel to the plane's normal vector $\vec{n}$, since the

line is perpendicular to the plane.

Step 3: Find the Appropriate Direction Vector $\vec{d}$

- If the normal vector $\vec{n}$ of the plane is given, then $\vec{d}$ can be chosen as $\vec{n}$ itself.
- If only a vector $\vec{v}$ is given, then find $\vec{d}$ such that $\vec{d} \cdot \vec{v} = 0$.

Example:
- To find $\vec{d}$ perpendicular to $\vec{v} = \langle v_x, v_y, v_z \rangle$, choose any vector $\vec{d}$ satisfying:

$$
v_x a + v_y b + v_z c = 0
$$

- For simplicity, pick some values for two components and solve for the third to satisfy the orthogonality condition.

Step 4: Write the Vector Equation

- Once $\vec{d}$ is determined, the vector equation becomes:

$$
\vec{r} = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle
$$

where $\langle a, b, c \rangle$ is the direction vector satisfying the perpendicularity condition.

Step 5: Derive the Parametric Equations

- From the vector form, extract the parametric equations:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

Worked Example: Finding the Line Perpendicular to a

Vector

Let’s consider a concrete example for clarity.

Given:
- Point \(P_0 (1, 2, 3) \),
- The line must be perpendicular to \(\vec{v} = \langle 4, 1, -2 \rangle \).

Objective:
- Find the vector and parametric equations of the line passing through \(P_0 \), perpendicular to \(\vec{v} \).

Step 1: Determine the Direction Vector \(\vec{d} \)

- Since the line is perpendicular to \(\vec{v} \), the dot product must be zero:

\[
\vec{d} \cdot \vec{v} = 0
\]

- Let \(\vec{d} = \langle a, b, c \rangle \). Then:

\[
4a + 1b - 2c = 0
\]

- To find specific \(a, b, c \), choose arbitrary values for two components and solve for the third.

Choosing:
- \(a = 1 \),
- \(b = 0 \).

Then:

\[
4(1) + 1(0) - 2c = 0 \Rightarrow 4 - 2c = 0 \Rightarrow c = 2
\]

Direction vector:
- \(\vec{d} = \langle 1, 0, 2 \rangle \).

Step 2: Write the Vector Equation

\[
\boxed{
\vec{r} = \langle 1, 2, 3 \rangle + t \langle 1, 0, 2 \rangle

}
\]

Step 3: Derive the Parametric Equations

$$
\begin{cases}
x = 1 + t \\
y = 2 + 0 \cdot t = 2 \\
z = 3 + 2t
\end{cases}
$$

These equations fully describe the line passing through $(1, 2, 3)$ and perpendicular to $\vec{v}$.

Additional Considerations for Special Cases

Lines Perpendicular to Planes
- If the line must be perpendicular to a plane, then its direction vector $\vec{d}$ is parallel to the plane's normal vector $\vec{n}$.
- The process simplifies to choosing $\vec{d} = \vec{n}$ and substituting into the point to form equations.

Finding a Line Perpendicular to Both a Vector and a Plane
- When the line must be perpendicular to a vector $\vec{v}$ and lie within a plane with normal $\vec{n}$, the direction vector $\vec{d}$ can be found using the cross product:

$$
\vec{d} = \vec{v} \times \vec{n}
$$

- The cross product yields a vector perpendicular to both $\vec{v}$ and $\vec{n}$, satisfying the perpendicularity conditions.

Summary and Key Takeaways

- The vector equation of a line in 3D space combines a point and a direction vector.
- The parametric equations explicitly express the line's coordinates as functions of a parameter.
- To find a line perpendicular to a vector $\vec{v}$, the line's direction vector $\vec{d}$

Frequently Asked Questions

How do I find the vector and parametric equations of a line passing through a point and perpendicular to a given vector?

To find the vector and parametric equations of a line passing through a point and perpendicular to a given vector, use the point as a point on the line and the given vector as the direction vector. The vector equation is $r = r0 + t v$, where $r0$ is the point and v is the perpendicular direction vector. The parametric equations are then derived component-wise from this vector equation.

What is the process to determine a line's equation that passes through a specific point and is perpendicular to a given vector?

First, identify the point through which the line passes and the vector to which it is perpendicular. Use the point as the position vector $r0$ and the perpendicular vector as the direction vector v. The vector equation is $r = r0 + tv$. Then, break it down into parametric equations for x, y, and z coordinates.

Can you provide an example of finding the line through (1, 2, 3) perpendicular to the vector (4, -2, 5)?

Yes. The point is (1, 2, 3) and the perpendicular vector is (4, -2, 5). The vector equation is $r = (1, 2, 3) + t(4, -2, 5)$. The parametric equations are: $x = 1 + 4t$, $y = 2 - 2t$, $z = 3 + 5t$.

What are the steps to convert the vector equation of a line into parametric equations when the line is perpendicular to a given vector?

Starting with the vector equation $r = r0 + tv$, where $r0$ is a point on the line and v is the perpendicular vector, extract each component to write the parametric equations: $x = x0 + tv_x$, $y = y0 + tv_y$, $z = z0 + tv_z$. These parametric equations describe the line in space.

How do I verify that a line is perpendicular to a given vector using its parametric equations?

To verify perpendicularity, find the direction vector of the line from its parametric equations and check if its dot product with the given vector is zero. If the dot product equals zero, the line is perpendicular to the vector.

What if the point through which the line passes is missing coordinates? How do I find the equations of the line then?

You need the specific point coordinates to define the line's position vector. Without the coordinates, you cannot uniquely determine the line's equations. If only the direction vector is known, the line can be described parametrically or vectorially once a point is provided.

Are there special considerations when the perpendicular vector is a zero vector?

Yes. A zero vector cannot define a direction for a line, so if the perpendicular vector is zero, it indicates no specific direction. In such cases, the concept of a line perpendicular to a zero vector is invalid, and additional information is needed to define the line.

Additional Resources

Find Both The Vector Equation And The Parametric Equations Of The Line Through (,,) That Is Perpendicular — this is a common problem in vector algebra and analytic geometry that often challenges students and professionals alike. Whether you're working through a homework assignment, preparing for an exam, or applying these concepts in a real-world context such as engineering or computer graphics, understanding how to find both the vector and parametric equations of a line given specific conditions is essential. This guide aims to walk you through the entire process step-by-step, providing clarity and insight into the geometric principles involved.

Understanding the Problem

Before diving into the solution, it’s important to understand what the problem entails:

- You are given a point (which, for the sake of illustration, we will denote as $P_0 = (x_0, y_0, z_0)$).
- You are asked to find the line passing through this point.
- The line must be perpendicular to a certain vector or plane (the problem statement may specify this explicitly).
- Your goal is to derive:
- The vector equation of the line.
- The parametric equations of the line.

Clarifying the Details

Since the original problem statement was incomplete ("through (,,)"), we will assume a typical scenario:

- The line passes through a point $P_0 = (x_0, y_0, z_0)$.
- The line is perpendicular to a given vector $\vec{v} = \langle a, b, c \rangle$ or a plane.

The most common case involves the line being perpendicular to a vector, which simplifies the process. If your problem involves a plane, the process is similar but involves the plane's normal vector, which is perpendicular to the plane.

Step 1: Understanding Perpendicular Lines and Vectors

What Does "Perpendicular" Mean in 3D?

In three-dimensional space, a line is perpendicular to a vector if the direction vector of the line is orthogonal to that vector. Specifically, if:

- The line has a direction vector $\vec{d}$,
- The vector $\vec{v}$ (or the normal to a plane) is given,

then the line is perpendicular to $\vec{v}$ if and only if:

$$
\vec{d} \cdot \vec{v} = 0
$$

where $\cdot$ denotes the dot product.

Implication for Our Line

- If the line is perpendicular to $\vec{v}$, then the direction vector $\vec{d}$ of the line must be orthogonal to $\vec{v}$.
- Therefore, the direction vector $\vec{d}$ can be any vector satisfying $\vec{d} \cdot \vec{v} = 0$.

Step 2: Choosing the Direction Vector

Finding a Direction Vector Perpendicular to $\vec{v}$

Suppose $\vec{v} = \langle a, b, c \rangle$. To find a direction vector $\vec{d}$ such that:

$$
\vec{d} \cdot \vec{v} = 0
$$

we can:

- Choose two arbitrary values for two components of $\vec{d}$,
- Solve for the third component using the dot product condition.

Method 1: Use a General Approach

Let:

$$
\vec{d} = \langle d_x, d_y, d_z \rangle
$$

and impose:

$$
a d_x + b d_y + c d_z = 0
$$

- Select values for two of d_x, d_y, d_z arbitrarily.
- Solve for the third.

Example:

If $\vec{v} = \langle 2, 3, 4 \rangle$, then:

$$
2 d_x + 3 d_y + 4 d_z = 0
$$

Choose $d_x = 1$, $d_y = 0$:

$$
2(1) + 3(0) + 4 d_z = 0 \Rightarrow 2 + 0 + 4 d_z = 0 \Rightarrow 4 d_z = -2 \Rightarrow d_z = -\frac{1}{2}
$$

Thus, a valid direction vector is:

$$
\vec{d} = \langle 1, 0, -\frac{1}{2} \rangle
$$

which can be scaled to clear fractions if needed.

Step 3: Constructing the Vector Equation of the Line

The General Form

Given:

- A point $P_0 = (x_0, y_0, z_0)$,
- A direction vector $\vec{d} = \langle d_x, d_y, d_z \rangle$,

the vector equation of the line is:

$$
\boxed{
\vec{r}(t) = \vec{r}_0 + t \vec{d}
}
$$

where:

- $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$,
- t is a real parameter.

Example

Suppose the point is \(P_0 = (1, 2, 3) \), and the direction vector found earlier is \(\langle 1, 0, -\frac{1}{2} \rangle \).

The vector equation:

\[
\vec{r}(t) = \langle 1, 2, 3 \rangle + t \langle 1, 0, -\frac{1}{2} \rangle
\]

which expands to:

\[
\boxed{
\vec{r}(t) = (1 + t, \quad 2 + 0 \cdots, \quad 3 - \frac{t}{2})
}
\]

Step 4: Deriving Parametric Equations

The parametric equations are obtained by expressing each coordinate component from the vector equation:

\[
x(t) = x_0 + d_x t
\]
\[
y(t) = y_0 + d_y t
\]
\[
z(t) = z_0 + d_z t
\]

Continuing the example:

\[
x(t) = 1 + t
\]
\[
y(t) = 2 + 0 \Rightarrow y(t) = 2
\]
\[
z(t) = 3 - \frac{1}{2} t
\]

These parametric equations describe the line passing through \((1, 2, 3) \), perpendicular to vector \(\langle 2, 3, 4 \rangle \).

Additional Considerations

Handling Special Cases

- If the vector $\vec{v}$ is the zero vector, the problem becomes invalid as the direction of the line cannot be determined.
- If the point coincides with the origin, the equations simplify accordingly.
- Multiple solutions exist: because any scalar multiple of the direction vector that satisfies the perpendicular condition can serve as the line’s direction vector.

Verifying Perpendicularity

Always verify that the chosen direction vector $\vec{d}$ is orthogonal to $\vec{v}$:

$$
\vec{d} \cdot \vec{v} = 0
$$

This step ensures correctness.

Summary: Step-by-Step Procedure

1. Identify the given point P_0 and the vector $\vec{v}$ (or plane normal).
2. Find a direction vector $\vec{d}$ perpendicular to $\vec{v}$:
- Choose arbitrary components for two entries.
- Solve for the third using the dot product condition.
3. Write the vector equation:
$$
\vec{r}(t) = \vec{r}_0 + t \vec{d}
$$
4. Extract the parametric equations:
$$
x(t), y(t), z(t)
$$
5. Verify perpendicularity by checking $\vec{d} \cdot \vec{v} = 0$.

Final Example for Clarity

Suppose:

- Point: $P_0 = (2, -1, 4)$,
- Vector: $\vec{v} = \langle 1, -2, 2 \rangle$.

Step 1: Find $\vec{d}$ such that:

$$
1 \cdot d_x + (-2) \cdot d_y + 2 \cdot d_z = 0

\]

Step 2: Choose \(d_x = 2 \), \(d_y = 0 \):

\[
1(2) + (-2)(0) + 2 d_z = 0 \Rightarrow 2 + 0 + 2 d_z = 0 \Rightarrow 2 d_z = -2 \Rightarrow d_z = -1
\]

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