Find The Coordinate Vector [x]B Of The Vector X Relative To The Given Basis B. 25 4) B1 = And B = {b1,b2}

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Introduction to Coordinate Vectors and Basis

Understanding how vectors relate to different bases is fundamental in linear algebra. The process of finding the coordinate vector [x]_B of a vector X relative to a basis B involves expressing X as a linear combination of the basis vectors. This allows us to represent the vector in a coordinate system that simplifies many operations such as transformation, projection, and change of basis.

In this article, we will explore the method of determining the coordinate vector [x]_B given a vector X and a basis B, where B consists of vectors {b□, b□}. We will also clarify the notation, assumptions, and step-by-step procedures to compute the coordinates effectively.

Understanding the Components of the Problem

What is a Basis B?

- A basis B of a vector space V is a set of vectors $\{b_1, b_2, ..., b_n\}$ that are linearly independent and span V.
- In our case, $B = \{b_1, b_2\}$, which indicates a 2-dimensional basis in a 2D vector space (such as $\mathbb{R}^2$).

What is a Vector X?

- The vector X is an element of the vector space V.
- Our goal is to find its coordinate vector relative to basis B, denoted as [x]_B.

What is the Coordinate Vector [x]_B?

- The coordinate vector $[x]_B = (c_1, c_2)$ represents the scalars such that:

$X = c_1 b_1 + c_2 b_2$

- These scalars, c_1 and c_2, form the coordinate vector of X relative to basis B.

Step-by-Step Procedure to Find [x]_B

Step 1: Express X as a Linear Combination of B

- Write X in terms of basis vectors:

$X = c_1 b_1 + c_2 b_2$

- Our goal is to find the scalars c□ and c□.

Step 2: Write the Vectors in Coordinate Form

- Represent the basis vectors and X in coordinate form (assuming □²):

- b□ = (b□□, b□□)
- b□ = (b□□, b□□)
- X = (x□, x□)

- The linear combination becomes:

(x□, x□) = c□(b□□, b□□) + c□(b□□, b□□)

Step 3: Set Up the System of Equations

- Equate components:

x□ = c□b□□ + c□b□□

x□ = c□b□□ + c□b□□

- These two equations form a linear system:

\[
\begin{cases}
b□□ c□ + b□□ c□ = x□ \\
b□□ c□ + b□□ c□ = x□
\end{cases}
\]

Step 4: Solve the System for c_1 and c_2

- Use methods such as substitution, elimination, or matrix algebra (Cramer's rule, inverse matrix) to find c_1 and c_2.
- For example, in matrix form:

$$
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
=
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
$$

- The coefficient matrix is invertible if $\{b_1, b_2\}$ are linearly independent.

Step 5: Calculate the Inverse or Use Cramer's Rule

- If the matrix is invertible, find its inverse:

$$
C = A^{-1} \times X
$$

where

$$
A = \begin{bmatrix}
b_{\square\square} & b_{\square\square} \\
b_{\square\square} & b_{\square\square}
\end{bmatrix}
$$

and

$$
X = \begin{bmatrix}
x_{\square} \\
x_{\square}
\end{bmatrix}
$$

- Alternatively, compute determinants and apply Cramer's rule for each variable:

$$
c_{\square} = \frac{\det(A_{c_{\square}})}{\det(A)}, \quad c_{\square} = \frac{\det(A_{c_{\square}})}{\det(A)}
$$

where $A_{c_{\square}}$ and $A_{c_{\square}}$ are matrices formed by replacing the relevant columns with the vector X.

Worked Example

Suppose:

- b_1 = (2, 1)
- b_2 = (1, 3)
- X = (5, 7)

Find [x]_B.

Step 1: Set Up Equations

- Write the system:

\[
\begin{cases}
2c_1 + 1c_2 = 5 \\
1c_1 + 3c_2 = 7
\end{cases}
\]

Step 2: Solve the System

- Use substitution or matrix methods:

Matrix A:

\[
\begin{bmatrix}
2 & 1 \\
1 & 3

\end{bmatrix}

\]

- Calculate determinant:

\[

\det(A) = 2 \times 3 - 1 \times 1 = 6 - 1 = 5

\]

- Compute determinants for c□ and c□:

\[

A_{c□} = \begin{bmatrix}

5 & 1 \\

7 & 3

\end{bmatrix}

\Rightarrow \det(A_{c□}) = 5 \times 3 - 1 \times 7 = 15 - 7 = 8

\]

\[

A_{c□} = \begin{bmatrix}

2 & 5 \\

1 & 7

\end{bmatrix}

\Rightarrow \det(A_{c□}) = 2 \times 7 - 5 \times 1 = 14 - 5 = 9

\]

- Find the coordinates:

\[

c□ = \frac{8}{5} = 1.6

\]

$$
c_2 = \frac{9}{5} = 1.8
$$

Result:

- The coordinate vector of X relative to B is:

$$
[x]_B = (1.6, 1.8)
$$

Implications and Applications

- Change of Basis: Once the coordinate vector is known, it becomes easier to perform transformations or switch between different basis representations.
- Linear Transformations: Coordinates facilitate understanding how vectors change under linear maps.
- Computer Graphics & Engineering: Coordinate vectors are essential in rendering, modeling, and system simulations.

Summary and Key Takeaways

- To find [x]_B, express X as a linear combination of basis vectors.
- Set up and solve the corresponding linear system.
- Use matrix algebra or determinants for efficient calculation.
- The process hinges on the linear independence of the basis vectors to ensure invertibility.

Conclusion

Finding the coordinate vector of a vector relative to a given basis is a foundational skill in linear algebra that enables deeper understanding of vector spaces and their transformations. The method involves expressing the vector as a combination of basis vectors, setting up a system of equations, and solving for the scalar coefficients. Whether using algebraic methods or matrix operations, mastering this process enhances your ability to analyze and manipulate vectors in various mathematical and practical contexts.

Note: Always verify the basis vectors are linearly independent before solving, as dependent vectors do not form a valid basis, and the coefficient matrix would be singular.

Frequently Asked Questions

What is the process to find the coordinate vector [x]_B of a vector x

relative to a basis B?

To find [x]_B, express x as a linear combination of the basis vectors in B, then solve the resulting system of equations to determine the coefficients, which form the coordinate vector.

If basis B = {b1, b2} and vector x is given, how do I set up the equations to find [x]_B?

Write x as x = c1 b1 + c2 b2, then equate the components to form a system of equations. Solving for c1 and c2 yields the coordinate vector [x]_B = [c1, c2].

What if the basis vectors are not orthogonal? Can I still find the coordinate vector [x]_B?

Yes. The basis vectors do not need to be orthogonal. You can still find [x]_B by solving the system of equations derived from expressing x as a linear combination of the basis vectors.

How does knowing the basis B help in finding the coordinate vector of x?

Knowing the basis B allows you to express x uniquely as a linear combination of the basis vectors, enabling you to determine the coordinate vector directly from the coefficients of this combination.

Can you provide a step-by-step example of finding [x]_B given specific vectors?

Certainly. For example, if B = {b1 = (1, 0), b2 = (0, 1)} and x = (3, 4), then x = 3b1 + 4b2, so [x]_B = [3, 4].

What are common mistakes to avoid when calculating the coordinate

vector [x]_B?

Common mistakes include mixing up the basis vectors and the vector components, not solving the system correctly, or assuming the basis is orthogonal when it is not. Always verify your solution by reconstructing x from the found coordinates.

How does the basis B = {b1, b2} affect the uniqueness of the coordinate vector [x]_B?

Since B is a basis, it provides a unique linear combination for any vector x in the span, ensuring that [x]_B is uniquely determined by the coefficients in that combination.

Additional Resources

Understanding How to Find the Coordinate Vector [x]_B of a Vector X Relative to a Given Basis B

When working with vectors in linear algebra, one of the fundamental tasks is expressing a vector in terms of a basis. This process involves finding the coordinate vector [x]_B of a vector X relative to a basis B. The ability to compute these coordinates accurately is crucial in various applications, including change of basis, linear transformations, and coordinate system conversions. In this detailed exploration, we will examine the problem of finding the coordinate vector [x]_B of a vector X relative to a given basis B, specifically focusing on a basis B = {b_1, b_2} in a 2-dimensional vector space, and a vector X given with certain components, such as (25, 4).

Understanding the Basics: Vectors, Bases, and Coordinates

What is a Vector?

- A vector in a vector space (like $\mathbb{R}^2$) is an object characterized by both magnitude and direction.
- In $\mathbb{R}^2$, vectors are often represented as ordered pairs: $X = (x_1, x_2)$.
- For example, $X = (25, 4)$ is a point or vector in the 2-dimensional space.

What is a Basis?

- A basis B of a vector space is a set of vectors that are:
- Linearly independent: No vector in the basis can be written as a linear combination of the others.
- Spanning: Any vector in the space can be expressed as a linear combination of basis vectors.
- In $\mathbb{R}^2$, a basis typically consists of two vectors, say b_1 and b_2.

Coordinate Vectors Relative to a Basis

- The coordinate vector $[x]_B$ of a vector X relative to basis B is a unique set of scalars (c_1, c_2) such that:

$$
X = c_1 b_1 + c_2 b_2
$$

- These scalars c_1 and c_2 are called the coordinates of X relative to B.

Given Data and Objective

Suppose the problem provides:

- A basis $B = \{b_1, b_2\}$ in $\mathbb{R}^2$.
- A vector X with components (25, 4).
- The basis vectors b_1 and b_2 are known or specified (though the initial prompt is somewhat incomplete, so we will assume typical forms or general representations).

The goal:

- Find the coordinate vector [x]_B of X relative to basis B.

Step-by-Step Approach to Find [x]_B

Step 1: Express the Vector as a Linear Combination

- Write X as:

$$
X = c_1 b_1 + c_2 b_2
$$

- Our task reduces to solving for c_1 and c_2.

Step 2: Set Up the System of Equations

- Depending on the form of b_1 and b_2, write each as an ordered pair:

$$
b_1 = (b_{11}, b_{12}), \quad b_2 = (b_{21}, b_{22})
$$

- Then, the linear combination becomes:

$$
(x_1, x_2) = c_1 (b_{11}, b_{12}) + c_2 (b_{21}, b_{22})
$$

- Which leads to the component-wise system:

$$
\begin{cases}
x_1 = c_1 b_{11} + c_2 b_{21} \\
x_2 = c_1 b_{12} + c_2 b_{22}
\end{cases}
$$

- For the specific X = (25, 4):

$$
\begin{cases}
25 = c_1 b_{11} + c_2 b_{21} \\
4 = c_1 b_{12} + c_2 b_{22}
\end{cases}
$$

Note: To proceed concretely, explicit basis vectors are necessary.

Step 3: Solve the System of Equations

- Typically, this is a 2x2 linear system, which can be solved using:

- Substitution method
- Elimination method
- Matrix algebra (using inverse matrices)

- Expressed in matrix form:

$$
\begin{bmatrix}
b_{11} & b_{21} \\
b_{12} & b_{22}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}
=
\begin{bmatrix}
25 \\
4
\end{bmatrix}
$$

- Denote the matrix as B and the vector as X:

\[

B = \begin{bmatrix}

b_{11} & b_{21} \\

b_{12} & b_{22}

\end{bmatrix}

, \quad

X = \begin{bmatrix}

25 \\

4

\end{bmatrix}

\]

- Then, the coordinates [x]_B are:

\[

[x]_B = B^{-1} X

\]

- Note: The invertibility of B (i.e., its determinant is non-zero) is essential for the existence of a unique solution.

An Explicit Example: Assuming Specific Basis Vectors

Suppose the basis vectors are:

\[

b_1 = (1, 0), \quad b_2 = (0, 1)

\]

which is the standard basis in $\mathbb{R}^2$.

- Then, the coordinate vector [x]_B of X = (25, 4) relative to this basis is simply:

\[

[x]_B = (25, 4)

\]

- In this trivial case, the coordinates are the same as the components of X.

More General Case: Non-Standard Basis

Suppose the basis vectors are:

\[

b_1 = (2, 1), \quad b_2 = (1, 3)

\]

- The matrix B becomes:

\[

B = \begin{bmatrix}

2 & 1 \\

1 & 3

\end{bmatrix}

\]

- The vector X is (25, 4), so:

\[
X = \begin{bmatrix}
25 \\
4
\end{bmatrix}
\]

- To find [x]_B, compute B^{-1}:

- Determinant:

\[
\det(B) = (2)(3) - (1)(1) = 6 - 1 = 5
\]

- Inverse:

\[
B^{-1} = \frac{1}{\det(B)} \begin{bmatrix}
3 & -1 \\
-1 & 2
\end{bmatrix}
= \frac{1}{5}
\begin{bmatrix}
3 & -1 \\
-1 & 2
\end{bmatrix}

\]

- Compute $[x]_B$:

$$
[x]_B = B^{-1} X = \frac{1}{5}
\begin{bmatrix}
3 & -1 \\
-1 & 2
\end{bmatrix}
\begin{bmatrix}
25 \\
4
\end{bmatrix}
$$

- Calculations:

$$
c_1 = \frac{1}{5}(3 \times 25 + (-1) \times 4) = \frac{1}{5}(75 - 4) = \frac{71}{5} = 14.2
$$

$$
c_2 = \frac{1}{5}((-1) \times 25 + 2 \times 4) = \frac{1}{5}(-25 + 8) = \frac{-17}{5} = -3.4
$$

- Therefore,

$$
[x]_B = (14.2, -3.4)
$$

This process illustrates the general method: forming the matrix, computing its inverse, and multiplying by the vector.

Interpreting the Result: What Does the Coordinate Vector Tell Us?

The coordinate vector [x]_B provides a unique representation of X in the basis B:

- The scalar c_1 indicates how much of b_1 is needed to construct X.
- The scalar c_2 indicates how much of b_2 is

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