Find The Curvature Of The Following: Give Exact Values Only A) R(t) =(2t, T^3, T^2), At T=1 B) F(x)=sin^3

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Introduction

Curvature is a fundamental concept in differential geometry that measures how sharply a curve bends at a particular point. It provides insight into the local geometric properties of curves in space and is essential in various fields such as physics, engineering, computer graphics, and more. Calculating the curvature involves understanding how the tangent vector to a curve changes direction as one moves along the curve, which can reveal whether a curve is straight, gently curved, or sharply bent.

In this article, we will explore how to find the exact curvature values of two different types of functions: a space curve represented parametrically and a scalar function. Specifically, we will analyze:

- Part A: The curvature of the vector function $R(t) = (2t, t^3, t^2)$ at $t=1$.
- Part B: The curvature of the scalar function $f(x) = \sin^3 x$, which is a planar curve, at a specific point.

By working through these examples, readers will gain a comprehensive understanding of the methods used to compute curvature in various contexts, including parametric and explicit functions. We will cover the core formulas, step-by-step calculations, and interpret the results, ensuring a thorough grasp of the subject.

Understanding Curvature in Differential Geometry

Before diving into the specific problems, it is important to review the fundamental concepts and formulas used to compute curvature.

What Is Curvature?

Curvature (κ) quantifies how rapidly a curve deviates from a straight line at a particular point. In simple terms, it measures the "bendiness" of the curve. For a plane curve, the curvature at a point can be visualized as the reciprocal of the radius of the osculating circle—the circle that best fits the curve at that point.

Curvature of Space Curves

For curves in three-dimensional space, the curvature is defined similarly but involves derivatives of the position vector with respect to the parameter. The general formula for the curvature $\kappa(t)$ of a space curve $R(t)$ is:

$$
\boxed{
\kappa(t) = \frac{\| R'(t) \times R''(t) \|}{\| R'(t) \|^3}
}
$$

where:

- $R'(t)$ is the first derivative of $R(t)$ with respect to t,
- $R''(t)$ is the second derivative,
- $\times$ denotes the cross product,
- $\| \cdot \|$ denotes the vector magnitude.

This formula is valid when the derivatives exist and $R'(t) \neq 0$.

Curvature of Plane Curves

For a scalar function $y = f(x)$, the curvature at a point x is given by:

$$
\boxed{
\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}
}
$$

This formula measures how sharply the graph of the function bends at x.

Part A: Calculating the Curvature of $R(t) = (2t, t^3, t^2)$ at $t=1$

Step 1: Find the First Derivative $R'(t)$

Given:

$$
R(t) = (2t, t^3, t^2)
$$

Differentiate each component:

$$
R'(t) = \left(\frac{d}{dt} 2t, \frac{d}{dt} t^3, \frac{d}{dt} t^2 \right) = (2, 3t^2, 2t)
$$

At $t=1$:

$$
R'(1) = (2, 3(1)^2, 2(1)) = (2, 3, 2)
$$

Step 2: Find the Second Derivative $R''(t)$

Differentiate $R'(t)$:

$$
R''(t) = \left(\frac{d}{dt} 2, \frac{d}{dt} 3t^2, \frac{d}{dt} 2t \right) = (0, 6t, 2)
$$

At $t=1$:

$$
R''(1) = (0, 6(1), 2) = (0, 6, 2)
$$

Step 3: Compute the Cross Product $R'(t) \times R''(t)$ at $t=1$

Recall:

$$
R'(1) = (2, 3, 2), \quad R''(1) = (0, 6, 2)
$$

Calculate:

$$
R'(1) \times R''(1) =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 3 & 2 \\
0 & 6 & 2
\end{vmatrix}
$$

Expanding:

$$
= \mathbf{i} (3 \times 2 - 2 \times 6) - \mathbf{j} (2 \times 2 - 2 \times 0) + \mathbf{k} (2 \times 6 - 3 \times 0)
$$

Compute each component:

- $\mathbf{i}$: $(3 \times 2) - (2 \times 6) = 6 - 12 = -6$

- $\mathbf{j}$: $(2 \times 2) - (2 \times 0) = 4 - 0 = 4$, note the minus sign in the formula: -4
- $\mathbf{k}$: $(2 \times 6) - (3 \times 0) = 12 - 0 = 12$

Thus:

$$
R'(1) \times R''(1) = (-6, -4, 12)
$$

Step 4: Find the Magnitude of the Cross Product

$$
\| R'(1) \times R''(1) \| = \sqrt{(-6)^2 + (-4)^2 + 12^2} = \sqrt{36 + 16 + 144} = \sqrt{196} = 14
$$

Step 5: Find the Magnitude of $R'(1)$

$$
\| R'(1) \| = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{4 + 9 + 4} = \sqrt{17}
$$

Step 6: Calculate the Curvature $\kappa(1)$

Using the formula:

$$
\kappa(1) = \frac{\| R'(1) \times R''(1) \|}{\| R'(1) \|^3} = \frac{14}{(\sqrt{17})^3}
$$

Since $(\sqrt{17})^3 = 17 \times \sqrt{17}$:

$$
\kappa(1) = \frac{14}{17 \sqrt{17}}
$$

Final exact value:

$$
\boxed{
\kappa(1) = \frac{14}{17 \sqrt{17}}
}
$$

This is the exact curvature of the space curve $R(t)$ at $t=1$.

Part B: Calculating the Curvature of $f(x) = \sin^3 x$

Step 1: Find the First Derivative $f'(x)$

Given:

$$
f(x) = \sin^3 x
$$

Using chain rule:

$$
f'(x) = 3 \sin^2 x \cdot \cos x
$$

Step 2: Find the Second Derivative $f''(x)$

Differentiate $f'(x)$:

$$
f''(x) = \frac{d}{dx} \left(3 \sin^2 x \cos x \right)
$$

Apply the product rule:

$$
f''(x) = 3 \left[\frac{d}{dx} (\sin^2 x) \cdot \cos x + \sin^2 x \cdot \frac{d}{dx} (\cos x) \right]
$$

Calculate derivatives:

- $\frac{d}{dx} \sin^2 x = 2 \sin x \cos x$
- $\frac{d}{dx} \cos x = - \sin x$

Substitute:

$$
f''(x) = 3 \left[2 \sin x \cos x \cdot \cos x + \sin^2 x \cdot (- \sin x) \right]
$$

Simplify:

$$
f''(x) = 3 \left[2 \sin x \cos^2 x - \sin
$$

Frequently Asked Questions

How do you find the curvature of the vector function $R(t) = (2t, t^3, t^2)$ at $t = 1$?

First, compute $R'(t)$ and $R''(t)$, then use the curvature formula $\kappa(t) = |R'(t) \times R''(t)| / |R'(t)|^3$. For $t = 1$, substitute to find the exact curvature value.

What are the derivatives $R'(t)$ and $R''(t)$ for $R(t) = (2t, t^3, t^2)$?

$R'(t) = (2, 3t^2, 2t)$ and $R''(t) = (0, 6t, 2)$.

How do you evaluate the curvature of $R(t)$ at $t = 1$ using the derivatives?

Plug $t=1$ into $R'(t)$ and $R''(t)$: $R'(1) = (2, 3, 2)$ and $R''(1) = (0, 6, 2)$. Then compute the cross product and the magnitude to find the curvature.

What is the exact curvature value of $R(t)$ at $t = 1$?

The exact curvature is $\kappa(1) = 2/9$.

How do you find the derivative of $F(x) = \sin^3(x)$?

Use the chain rule: $d/dx [\sin^3(x)] = 3 \sin^2(x) \cos(x)$.

What is the simplified form of the derivative of $F(x) = \sin^3(x)$?

The derivative simplifies to $3 \sin^2(x) \cos(x)$.

Additional Resources

Find The Curvature Of The Following is a fundamental problem in differential geometry and vector calculus, often encountered in both academic coursework and practical applications such as computer graphics, physics, and engineering. Curvature provides a measure of how sharply a curve bends at a particular point, offering insights into the geometric and physical properties of the curve. In this comprehensive review, we will explore the methods to find the curvature for two distinct types of functions: a vector-valued function $R(t) = (2t, t^3, t^2)$ at $t=1$, and a scalar function $F(x) = \sin^3 x$. Each case involves different techniques and conceptual approaches, and understanding these examples will deepen your grasp of curvature analysis.

Understanding Curvature: A Brief Overview

Before delving into specific examples, it’s essential to understand what curvature signifies mathematically. The curvature κ of a curve quantifies how rapidly the curve changes direction at a particular point. For a smooth curve parameterized by a variable t, the curvature is given by the formula:

$$
\kappa(t) = \frac{\| R'(t) \times R''(t) \|}{\| R'(t) \|^3}
$$

for space curves, where:
- $R(t)$ is the position vector,
- $R'(t)$ is the first derivative (velocity vector),
- $R''(t)$ is the second derivative (acceleration vector),
- $\times$ denotes the cross product,
- $\| \cdot \|$ indicates the vector norm.

For scalar functions, when considering the graph of $y = f(x)$, the curvature at a point is given by:

$$
\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}
$$

which measures how sharply the graph bends at x.

Part A: Finding the Curvature of $R(t) = (2t, t^3, t^2)$ at $t=1$

Step 1: Compute the First Derivative $R'(t)$

Given:

$$
R(t) = (2t, t^3, t^2)
$$

Differentiate each component with respect to t:

$$
R'(t) = \left(\frac{d}{dt} 2t, \frac{d}{dt} t^3, \frac{d}{dt} t^2 \right) = (2, 3t^2, 2t)
$$

At $t=1$:

$$
R'(1) = (2, 3 \times 1^2, 2 \times 1) = (2, 3, 2)
$$

Step 2: Compute the Second Derivative $R''(t)$

Differentiate $R'(t)$:

$$
R''(t) = \left(0, 6t, 2 \right)
$$

At $t=1$:

$$
R''(1) = (0, 6 \times 1, 2) = (0, 6, 2)
$$

Step 3: Calculate the Cross Product $R'(t) \times R''(t)$ at $t=1$

Using the vectors:

$$
R'(1) = (2, 3, 2), \quad R''(1) = (0, 6, 2)
$$

The cross product:

$$
R'(1) \times R''(1) =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 3 & 2 \\
0 & 6 & 2
\end{vmatrix}
$$

Calculating the determinant:

\[

$$
\mathbf{i} (3 \times 2 - 2 \times 6) - \mathbf{j} (2 \times 2 - 2 \times 0) + \mathbf{k} (2 \times 6 - 3 \times 0)
$$

$$
= \mathbf{i} (6 - 12) - \mathbf{j} (4 - 0) + \mathbf{k} (12 - 0) = (-6, -4, 12)
$$

Step 4: Find the Norms $\| R'(1) \|$ and $\| R'(1) \times R''(1) \|$

$$
\| R'(1) \| = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{4 + 9 + 4} = \sqrt{17}
$$

$$
\| R'(1) \times R''(1) \| = \sqrt{(-6)^2 + (-4)^2 + 12^2} = \sqrt{36 + 16 + 144} = \sqrt{196} = 14
$$

Step 5: Compute the Curvature $\kappa(1)$

Applying the formula:

$$
\kappa(1) = \frac{\| R'(1) \times R''(1) \|}{\| R'(1) \|^3} = \frac{14}{(\sqrt{17})^3}
$$

Since $(\sqrt{17})^3 = 17 \sqrt{17}$, the exact value is:

$$
\boxed{\kappa(1) = \frac{14}{17 \sqrt{17}}}
$$

This is the exact curvature of the space curve $R(t)$ at $t=1$.

Part B: Finding the Curvature of $F(x) = \sin^3 x$

Step 1: Compute the First Derivative \(f'(x) \)

Given:

\[
f(x) = \sin^3 x
\]

Using the chain rule:

\[
f'(x) = 3 \sin^2 x \times \cos x
\]

Expressed explicitly:

\[
f'(x) = 3 \sin^2 x \cos x
\]

Step 2: Compute the Second Derivative \(f''(x) \)

Differentiate \(f'(x) \):

\[
f''(x) = \frac{d}{dx} [3 \sin^2 x \cos x]
\]

Apply the product rule:

\[
f''(x) = 3 \left[\frac{d}{dx} (\sin^2 x) \times \cos x + \sin^2 x \times \frac{d}{dx} (\cos x) \right]
\]

Calculate derivatives:

\[
\frac{d}{dx} (\sin^2 x) = 2 \sin x \cos x
\]
\[
\frac{d}{dx} (\cos x) = - \sin x
\]

Substitute back:

\[

f''(x) = 3 [2 \sin x \cos x \times \cos x + \sin^2 x \times (- \sin x)] = 3 [2 \sin x \cos^2 x - \sin^3 x]
\]

Factor out \(\sin x \):

\[
f''(x) = 3 \sin x (2 \cos^2 x - \sin^2 x)
\]

Using the identity \(\sin^2 x + \cos^2 x = 1 \), rewrite:

\[
f''(x) = 3 \sin x (2 \cos^2 x - (1 - \cos^2 x)) = 3 \sin x (2 \cos^2 x - 1 + \cos^2 x) = 3 \sin x (3 \cos^2 x - 1)
\]

Step 3: Apply the curvature formula for \(y = f(x) \)

\[
\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}
\]

Compute \((f'(x))^2 \):

\[
f'(x) = 3 \sin^2 x \cos x
\]
\[
(f'(x))^2 = 9 \sin^4 x \cos^2 x
\]

The denominator:

\[
D = [1 + 9 \sin^4 x \cos^2 x]^{3/2}
\]

The numerator:

\[
N = |f''(x)| = |3 \sin x (3 \cos^2 x - 1)| = 3 |\sin x| |3 \cos^2 x - 1|
\]

Final exact expression for curvature \(\kappa(x) \)

\[
\boxed{
\kappa(x) =

Find The Curvature Of The Following Give Exact Values Only A R T 2t T 3 T 2 At T 1 B F X Sin 3

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