Find The Distance From The Point $(-1,6)$ To The Line $y=-2x$. Round To The Nearest Tenth, If Necessary.

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Understanding how to find the distance between a point and a line is a fundamental concept in coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or someone working on a mathematics project, mastering this skill is essential. In this article, we will explore the step-by-step process to calculate the shortest distance from the point $(-1,6)$ to the line $y=-2x$. We will also discuss related concepts, formulas, and practical tips to ensure you grasp the topic thoroughly. Additionally, the calculated distance will be rounded to the nearest tenth, as specified.

Understanding the Basics: Point-Line Distance in Coordinate Geometry

Before diving into calculations, it’s important to understand the core concepts involved:

- Point: A specific location in the coordinate plane, represented as (x, y).
- Line: A straight one-dimensional figure extending infinitely in both directions, which can be represented algebraically.
- Distance: The shortest length between a point and a line, often called the perpendicular distance.

The problem at hand involves finding the perpendicular (shortest) distance from the point $(-1,6)$ to the line $y=-2x$.

Mathematical Foundations and Formulas

The key to solving this problem lies in understanding and applying the formula for the distance from a point to a line in the coordinate plane.

Standard Form of a Line Equation

Any line can be expressed in the general form:

$$
Ax + By + C = 0
$$

where:
- $A, B,$ and C are constants.

Distance Formula from a Point to a Line

Given a point $P(x_0, y_0)$ and a line $Ax + By + C = 0$, the shortest distance d from the point to the line is given by:

$$
d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}
$$

This formula is derived from the perpendicular distance between a point and a line in the coordinate plane.

Step-by-Step Solution to the Problem

Let's now apply these concepts to find the distance from the point $(-1,6)$ to the line $y=-2x$.

Step 1: Rewrite the Line Equation in Standard Form

Given line:

$$
y = -2x
$$

Rearranged to the standard form:

$$
y + 2x = 0
$$

or equivalently:

$$
2x + y + 0 = 0
$$

Here, $A=2$, $B=1$, and $C=0$.

Step 2: Identify the Coordinates of the Point

$$
P(x_0, y_0) = (-1, 6)
$$

Step 3: Plug into the Distance Formula

Using:

$$
d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}
$$

Substitute the known values:

$$
d = \frac{|2 \times (-1) + 1 \times 6 + 0|}{\sqrt{(2)^2 + (1)^2}}
$$

Calculate numerator:

$$
| -2 + 6 + 0 | = |4| = 4
$$

Calculate denominator:

$$
\sqrt{4 + 1} = \sqrt{5} \approx 2.236
$$

Step 4: Compute the Distance

$$
d = \frac{4}{\sqrt{5}} \approx \frac{4}{2.236} \approx 1.79
$$

Step 5: Round to the Nearest Tenth

$$
\boxed{1.8}
$$

Thus, the shortest distance from the point $(-1,6)$ to the line $y = -2x$ is approximately 1.8 units when rounded to the nearest tenth.

Additional Insights and Tips

While the above steps provide a straightforward method, here are some additional tips and insights to enhance your understanding:

1. Visualize the Problem

Plotting the point and the line on a graph can help you intuitively understand the shortest distance—it's the length of the perpendicular segment from the point to the line.

2. Confirm the Line Equation

Ensure the line is written in standard form before applying the formula. Sometimes lines are given in slope-intercept form ($y=mx + b$); converting to standard form simplifies calculations.

3. Use of Software Tools

Graphing calculators or software like Desmos, GeoGebra, or WolframAlpha can be used to verify the calculation visually or numerically.

4. Practice with Different Lines and Points

Applying this process to various points and lines will enhance your understanding and fluency.

Related Topics and Applications

Understanding point-to-line distances is foundational in various fields:

- Physics: Calculating the shortest path or minimal distance between objects.
- Engineering: Designing components with precise tolerances.
- Computer Graphics: Collision detection and rendering.
- Navigation: Determining the closest point of approach in geographic information systems (GIS).

Practical Example

Suppose you are designing a bridge and need to determine how far a support point is from a boundary line. Using these formulas ensures accurate measurements and efficient planning.

Summary and Key Takeaways

- The shortest distance from a point (x_0, y_0) to a line $Ax + By + C=0$ is calculated using the formula:

$$
d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}
$$

- Convert the line to standard form to identify $A, B,$ and C.

- Substitute the point's coordinates into the formula and compute.

- Round the final answer to the desired precision—in this case, to the nearest tenth.

- For the specific problem, the distance from $(-1,6)$ to $y=-2x$ is approximately 1.8 units.

By mastering these steps, you can confidently solve similar problems involving distances between points and lines in the coordinate plane.

Conclusion

Calculating the distance from a point to a line is a vital skill in coordinate geometry, blending algebraic manipulation with geometric intuition. By converting the line to standard form, applying the distance formula, and carefully performing calculations, you can accurately determine the shortest distance between any point and line. Remember to verify your work visually when possible and practice with various problems to strengthen your understanding. Whether you're tackling academic homework, preparing for exams, or applying these concepts professionally, these methods form a core part of your mathematical toolkit.

Keywords for SEO Optimization:
Distance from point to line, point-line distance formula, coordinate geometry, shortest distance, line equation, standard form of a line, perpendicular distance, math problems, geometry calculation, rounding to nearest tenth

Frequently Asked Questions

How do I find the distance from the point (-1, 6) to the line y = -2x?

Use the point-to-line distance formula: Distance = $|Ax + By + C| / \sqrt{A^2 + B^2}$, where the line is written in standard form $Ax + By + C = 0$.

What is the standard form of the line y = -2x?

Rewrite as $2x + y = 0$, so A=2, B=1, and C=0.

How do I plug in the point (-1, 6) into the distance formula?

Calculate $|2(-1) + 1(6) + 0| = |-2 + 6| = 4$.

What is the denominator in the point-to-line distance formula for this line?

It is $\sqrt{A^2 + B^2} = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5}$.

What is the approximate value of √5, and how do I round the final answer?

$\sqrt{5} \approx 2.236$. Divide the numerator by this value and round the result to the nearest tenth.

What is the calculated distance from the point to the line before rounding?

Distance = $4 / \sqrt{5} \approx 4 / 2.236 \approx 1.79$.

What is the final answer rounded to the nearest tenth?

The distance is approximately 1.8 units.

Can I verify the answer using a graph or coordinate geometry tools?

Yes, plotting the point and the line on a graphing tool can visually confirm the shortest distance, and coordinate geometry calculators can also verify your calculation.

Additional Resources

Finding the Distance from a Point to a Line: An In-Depth Exploration

Understanding how to calculate the shortest distance from a point to a line is a fundamental concept in coordinate geometry, with applications spanning engineering, physics, computer graphics, and navigation. In this comprehensive review, we will explore the process of finding the distance from the point \((-1,6)\) to the line \(y = -2x \). We will delve into the geometric principles, algebraic methods, step-by-step calculations, and practical considerations involved in solving such problems. By the end, you will gain a clear understanding of the techniques and reasoning necessary to determine this distance accurately, including how to round the result to the nearest tenth.

Understanding the Geometric Foundations

The Concept of Distance in Coordinate Geometry

In the Cartesian plane, the shortest distance between a point and a line is represented by a perpendicular segment connecting the point to the line. This minimal length is crucial because it directly measures how "far" the point lies from the line, regardless of the line's orientation or position.

Key points:
- The shortest distance is always along a line perpendicular to the given line.
- This distance can be computed algebraically using the point's coordinates and the line's equation.
- Geometrically, it can be visualized as dropping a perpendicular from the point onto the line and measuring its length.

The Line Equation and Its Implications

The line in question, \(y = -2x \), can be rewritten in standard form to facilitate calculations:
\[

$$
y = -2x \implies 2x + y = 0
$$
Standard form of a line equation: $Ax + By + C = 0$. For our line:
- $A = 2$
- $B = 1$
- $C = 0$

This form is essential because it allows us to utilize a well-established formula for point-to-line distance.

Mathematical Approach to Find the Distance

The Distance Formula from a Point to a Line

Given a point $P(x_0, y_0)$ and a line $Ax + By + C = 0$, the perpendicular distance d from the point to the line is given by:
$$
d = \frac{|A x_0 + B y_0 + C|}{\sqrt{A^2 + B^2}}
$$

This formula is derived from the general equation of a line and the concept of perpendicular distance in Euclidean geometry.

Applying the Formula to Our Specific Problem

Our data:
- Point: $P(-1, 6)$
- Line: $2x + y = 0$ (which corresponds to $A=2, B=1, C=0$)

Substituting into the formula:
$$
d = \frac{|2 \times (-1) + 1 \times 6 + 0|}{\sqrt{2^2 + 1^2}}
$$

Calculating numerator:
$$
|2 \times (-1) + 6| = |-2 + 6| = |4| = 4
$$

Calculating denominator:
$$
\sqrt{4 + 1} = \sqrt{5} \approx 2.2360679775
$$

Thus,
$$
d \approx \frac{4}{2.2360679775} \approx 1.7888543829
$$

Step-by-Step Calculation and Explanation

Step 1: Write the line in standard form

The original line $y = -2x$ is rewritten as:

$$
2x + y = 0
$$

This transformation simplifies the calculation process and aligns with the point-to-line distance formula.

Step 2: Identify the coefficients A, B, C

From the standard form:

- $A = 2$
- $B = 1$
- $C = 0$

Step 3: Plug in the point coordinates

- $x_0 = -1$
- $y_0 = 6$

Calculate numerator:

$$
|A x_0 + B y_0 + C| = |2 \times (-1) + 1 \times 6 + 0| = |-2 + 6| = 4
$$

Calculate denominator:

$$
\sqrt{A^2 + B^2} = \sqrt{(2)^2 + (1)^2} = \sqrt{4 + 1} = \sqrt{5} \approx 2.236
$$

Therefore:

$$
d \approx \frac{4}{2.236} \approx 1.789
$$

Step 4: Rounding to the nearest tenth

Rounding 1.789 to the nearest tenth:

$$
\boxed{1.8}
$$

Practical Considerations and Alternative

Methods

Graphical Interpretation

Plotting the point and the line can provide a visual understanding of the problem. Drawing the perpendicular from the point to the line confirms the shortest distance visually, though this method lacks precision without tools.

Vector Approach

Another method involves vector calculus:
- Find a vector perpendicular to the line.
- Use the dot product to project the vector from a point on the line to the given point.
- Compute the magnitude of this projection to determine the shortest distance.

This approach, while more involved algebraically, offers deeper geometric insight and is useful in higher-dimensional problems.

Using the Normal Vector

The normal vector to the line $2x + y = 0$ is $\vec{n} = (2, 1)$. The magnitude of this vector is $\sqrt{5}$.

The shortest distance from the point $(-1,6)$ to the line can also be seen as the magnitude of the projection of the vector from the point to a point on the line, along the normal vector.

Verifying the Calculation

Physical and Geometrical Checks

- The computed distance should be positive and less than the maximum possible distance between the point and the line.
- If the point is on the line, the distance should be zero.
- Since the point $(-1,6)$ clearly does not lie on $y = -2x$, the distance should be positive.

Alternative Calculation via Coordinates

Suppose we find a point Q on the line closest to P. The line is $y = -2x$, so any point Q on the line can be expressed as:

$$
Q(x, -2x)
$$

The goal is to find x such that the segment PQ is perpendicular to the line. The slope of the line is -2; thus, the perpendicular slope is $1/2$.

The line through $P(-1,6)$ with slope $1/2$:
$$
y - 6 = \frac{1}{2}(x + 1)
$$
$$
y = \frac{1}{2}x + \frac{1}{2} + 6 = \frac{1}{2}x + \frac{13}{2}
$$

Set this equal to the line $y = -2x$:
$$
-2x = \frac{1}{2}x + \frac{13}{2}
$$
Multiply both sides by 2 to clear fractions:
$$
-4x = x + 13
$$
$$
-4x - x = 13
$$
$$
-5x = 13
$$
$$
x = -\frac{13}{5} = -2.6
$$

Calculate y:
$$
y = -2x = -2 \times (-2.6) = 5.2
$$

The closest point Q on the line is $(-2.6, 5.2)$.

Calculate the distance from $P(-1,6)$ to $Q(-2.6, 5.2)$:
$$
d = \sqrt{(-1 + 2.6)^2 + (6 - 5.2)^2} = \sqrt{(1.6)^2 + (0.8)^2} = \sqrt{2.56 + 0.64} = \sqrt{3.2} \approx 1.789
$$

Again, rounding to the nearest tenth:
$$
\boxed{1.8}
$$

This confirms the previous calculation, illustrating the consistency and reliability of the methods.

Summary and Final Remarks

This detailed examination of the problem emphasizes the importance of understanding both algebraic and geometric perspectives when calculating distances in coordinate geometry. The key steps involve transforming the line

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into standard form, applying the point-to-line distance formula, and
verifying the result through alternative approaches.

Key takeaways include:
- The standard form of a line simplifies the calculation of the shortest
distance.
- The
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