Find The Equation For (a) The Tangent Plane And (b) The Normal Line At The Point P0(2,0,2) On The Surface

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Understanding how to find the tangent plane and the normal line to a surface at a specific point is a fundamental concept in multivariable calculus. These tools are essential for analyzing the local behavior of surfaces, optimizing functions, and solving geometric problems. In this article, we will walk through the step-by-step process of deriving both the tangent plane and the normal line equations at the point P0(2,0,2) on a given surface.

Understanding the Surface and Its Implicit Equation

Before diving into the calculations, it's important to clarify the nature of the surface. Typically, surfaces are given either explicitly or implicitly:

- Explicit form: $z = f(x, y)$
- Implicit form: $G(x, y, z) = 0$

For the purpose of this article, we'll assume the surface is defined implicitly by a function $G(x, y, z) = 0$. The process described here applies generally, with adjustments based on the specific surface.

Suppose the surface is defined by the implicit equation:

$G(x, y, z) = 0$

where G is differentiable at the point of interest.

Given the point P0(2, 0, 2), our goal is to find:

a) The equation of the tangent plane at P0

b) The equation of the normal line passing through P0

Part A: Finding the Equation of the Tangent Plane at P0(2,0,2)

1. Compute the Gradient of G at P0

The gradient vector, $\nabla G(x, y, z)$, is a vector of partial derivatives:

$\nabla G(x, y, z) = (\partial G/\partial x, \partial G/\partial y, \partial G/\partial z)$

This vector is perpendicular to the surface at any point where G is differentiable.

Steps:

- Calculate the partial derivatives of G with respect to x, y, and z.
- Evaluate these derivatives at the point P0(2, 0, 2).

Example:

Suppose the surface is given by:

$G(x, y, z) = x^2 + y^2 + z^2 - 8$

This describes a sphere of radius √8 centered at the origin.

- $\partial G/\partial x = 2x$
- $\partial G/\partial y = 2y$
- $\partial G/\partial z = 2z$

At P0(2, 0, 2):

- $\partial G/\partial x = 22 = 4$
- $\partial G/\partial y = 20 = 0$
- $\partial G/\partial z = 22 = 4$

Thus, $\nabla G(2, 0, 2) = (4, 0, 4)$

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2. Formulate the Equation of the Tangent Plane

The tangent plane at P0 is given by the equation:

$\nabla G(P0) \cdot (r - r0) = 0$

where:

- $\nabla G(P0)$ is the gradient vector at P0
- $r = (x, y, z)$ is a generic point on the plane
- $r0 = (x0, y0, z0) = P0$

Using the example:

$(4, 0, 4) \cdot ((x, y, z) - (2, 0, 2)) = 0$

Expanding:

$4(x - 2) + 0(y - 0) + 4(z - 2) = 0$

Simplify:

$4x - 8 + 4z - 8 = 0$

$4x + 4z - 16 = 0$

Divide through by 4:

$x + z - 4 = 0$

This is the equation of the tangent plane at P0.

Note: The actual equation will depend on the specific surface G(x, y, z). The key is evaluating the gradient at P0 and applying the point-normal form of the plane.

Part B: Finding the Equation of the Normal Line at P0(2,0,2)

1. Use the Gradient as the Direction Vector

The normal line to the surface at P0 is a line passing through P0 and directed along the gradient vector ∇G(P0):

- Direction vector: ∇G(P0)

Continuing the example:

Direction vector: (4, 0, 4)

2. Write the Parametric Equations of the Normal Line

The parametric equations are:

$x = x0 + t\ (\partial G/\partial x)|P0$

$y = y0 + t\ (\partial G/\partial y)|P0$

$z = z0 + t\ (\partial G/\partial z)|P0$

Using the example:

$x = 2 + 4t$

$y = 0 + 0\ t = 0$

$z = 2 + 4t$

Where t is a real parameter.

This set of parametric equations describes the line passing through P0 in the direction of the gradient vector.

General Procedure for Finding the Tangent Plane and Normal Line

To summarize, here is a step-by-step process applicable to any surface defined implicitly:

1. **Identify the implicit surface equation:** $G(x, y, z) = 0$
2. **Compute the gradient:** Find $\nabla G(x, y, z) = (\partial G/\partial x, \partial G/\partial y, \partial G/\partial z)$
3. **Evaluate the gradient at P0:** Calculate $\nabla G(P0)$
4. **Equation of the tangent plane:** Use the point-normal form:

   $$(\partial G/\partial x)(x - x0) + (\partial G/\partial y)(y - y0) + (\partial G/\partial z)(z - z0) = 0$$

5. **Equation of the normal line:** Use the point and the gradient vector:

   $$x = x0 + t(\partial G/\partial x)$$
   $$y = y0 + t(\partial G/\partial y)$$
   $$z = z0 + t(\partial G/\partial z)$$

Additional Tips and Considerations

- **Surface differentiability:** Ensure the surface is smooth and differentiable at P0 to apply these methods.
- **Multiple surfaces:** If dealing with parametric surfaces, partial derivatives with respect to parameters are used instead.
- **Visualization:** Sketching the surface and the tangent plane can aid understanding of the geometric relationships.
- **Special cases:** If the gradient at P0 is zero, the surface may have a tangent plane that is not well-defined at that point, indicating a possible singularity or a point of non-smoothness.

Conclusion

Finding the equation of the tangent plane and the normal line at a specific point on a surface is a crucial skill in multivariable calculus. By calculating the gradient vector of the surface's implicit function at the point of interest, you can directly derive both the tangent plane and the normal line equations. Remember that the gradient is always perpendicular to the surface, making it an invaluable tool for understanding surface geometry.

Whether you're analyzing physical surfaces, optimizing functions, or exploring geometric concepts, mastering these methods will enhance your ability to interpret and work with multivariable surfaces effectively. Practice with different surface equations and points to build confidence and deepen your understanding of these fundamental calculus concepts.

Frequently Asked Questions

How do you find the equation of the tangent plane to a surface at a given point?

To find the tangent plane equation at a point P_0 on a surface, compute the gradient vector of the surface's function at P_0. Then, use the point-normal form of the plane: $(r - r_0) \cdot \nabla F(r_0) = 0$, where $r = (x, y, z)$.

What is the formula for the normal line to a surface at a specific point?

The normal line at a point P_0 on a surface is given parametrically by: $r(t) = P_0 + t \cdot \nabla F(P_0)$, where $\nabla F(P_0)$ is the gradient vector at P_0, which serves as the direction vector.

Given a surface defined implicitly by F(x, y, z) = 0, how do you compute the gradient vector at a point?

Calculate the partial derivatives: $\nabla F = (\partial F/\partial x, \partial F/\partial y, \partial F/\partial z)$, and evaluate at the point P_0. This vector is normal to the surface at that point.

How do you determine the tangent plane equation at the point $P_0(2,0,2)$ on a surface F(x, y, z) = 0?

First, find ∇F at P_0. Then, use the plane equation: $(x - x_0, y - y_0, z - z_0) \cdot \nabla F(P_0) = 0$, substituting the point coordinates and the gradient vector.

What is the step-by-step process to find the normal line at $P_0(2,0,2)$?

1. Calculate ∇F at P_0. 2. Use the point P_0 as the base point. 3. Write the parametric equations: $x = 2 + t\cdot(\partial F/\partial x)$, $y = 0 + t\cdot(\partial F/\partial y)$, $z = 2 + t\cdot(\partial F/\partial z)$.

How does the gradient vector relate to the tangent plane and normal line at a point on a surface?

The gradient vector is perpendicular (normal) to the surface at that point. It serves as the normal vector for the tangent plane and the direction vector for the normal line.

Can the tangent plane be parallel to the surface? How does that affect finding its equation?

The tangent plane is always tangent at a specific point on the surface. If the surface has a point of singularity or a flat region, the gradient might be zero, and the tangent plane may be indeterminate at that point.

What are common mistakes to avoid when calculating the tangent plane and normal line equations?

Common mistakes include forgetting to evaluate the gradient at the given point, mixing up the point coordinates, or misapplying the point-normal form. Always double-check derivative computations and substitutions.

How do you interpret the geometric meaning of the tangent plane and normal line at a point on a surface?

The tangent plane provides the best linear approximation of the surface near the point, while the normal line indicates the direction perpendicular to the surface at that point, representing the surface's orientation.

Additional Resources

Find The Equation For (a) The Tangent Plane And (b) The Normal Line At The Point $P_0(2,0,2)$ On The Surface is a fundamental problem in multivariable calculus, often encountered in the study of surfaces and their local behaviors. Understanding how to derive the tangent plane and the normal line at a specific point provides critical insights into the surface's geometry, aiding in applications like optimization, approximation, and modeling in physics and engineering. This article offers a comprehensive, step-by-step guide to solving such problems, including detailed explanations, formulas, and illustrative examples to facilitate mastery of the concepts.

Understanding the Surface and the Point $P_0(2,0,2)$

Before diving into the derivation of the tangent plane and the normal line, it is essential to understand the nature of the surface involved. Typically, such problems are based on a given surface equation, often expressed implicitly as:

$$ F(x, y, z) = 0 $$

or explicitly as:

$$ z = f(x, y) $$

In this context, the point $P_0(2,0,2)$ lies on the surface, meaning it satisfies the surface's equation. To proceed, one must confirm that P_0 is indeed on the surface and understand the surface's form.

For example, suppose the surface is given implicitly by:

$$ F(x, y, z) = z - x^2 - y^2 = 0 $$

which represents a paraboloid. Checking whether $P_0(2,0,2)$ lies on this surface involves substituting into the equation:

$$ 2 - (2)^2 - (0)^2 = 2 - 4 - 0 = -2 \neq 0 $$

Thus, P_0 is not on this surface, implying a different surface should be considered. Alternatively, if the surface is explicitly given as:

$$ z = x^2 + y^2 $$

then at $P_0(2,0,2)$:

$$ z = (2)^2 + (0)^2 = 4 + 0 = 4 \neq 2 $$

Again, P_0 does not lie on this surface. Therefore, the actual surface equation must be specified first to proceed accurately.

Part A: Finding the Equation of the Tangent Plane at $P_0(2,0,2)$

Step 1: Identify the Surface Equation

The initial step is to write down the explicit or implicit surface equation $F(x, y, z) = 0$. The choice of method hinges on how the surface is given:

- Explicit form: $z = f(x, y)$

- Implicit form: $F(x, y, z) = 0$

For illustration, assume the surface is explicitly defined as:

$$ z = x^2 + y^2 $$

and $P_0(2, 0, 2)$. Checking P_0:

$$ z = 2 \quad \text{and} \quad x^2 + y^2 = 4 + 0 = 4 \neq 2 $$

which indicates P_0 is not on this surface. To proceed, suppose the surface is given by:

$$ z = x + y $$

At $P_0(2, 0, 2)$:

$$ z = 2 + 0 = 2 $$

which matches P_0. Thus, P_0 lies on this surface, and it is suitable for our purposes.

Step 2: Find the Gradient ∇F

If the surface is given implicitly as $F(x, y, z) = 0$, then the gradient vector:

$$ \nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) $$

evaluated at P_0, is normal to the surface at that point.

For the explicit function $z = x + y$, we can define:

$$ F(x, y, z) = z - x - y = 0 $$

Calculating the partial derivatives:

$$ \frac{\partial F}{\partial x} = -1 $$
$$ \frac{\partial F}{\partial y} = -1 $$
$$ \frac{\partial F}{\partial z} = 1 $$

Evaluating at $P_0(2, 0, 2)$:

$$ \nabla F(2, 0, 2) = (-1, -1, 1) $$

This vector is normal to the surface at P_0.

Step 3: Write the Equation of the Tangent Plane

The general equation of the tangent plane at P_0 is given by:

$$ \nabla F(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0 $$

or explicitly:

$$ \left(\frac{\partial F}{\partial x} \big|_{P_0} \right)(x - x_0) + \left(\frac{\partial F}{\partial y} \big|_{P_0} \right)(y - y_0) + \left(\frac{\partial F}{\partial z} \big|_{P_0} \right)(z - z_0) = 0 $$

Plugging in the values:

$$ -1 \times (x - 2) -1 \times (y - 0) + 1 \times (z - 2) = 0 $$

Simplify:

$$ -(x - 2) - y + (z - 2) = 0 $$
$$ -x + 2 - y + z - 2 = 0 $$
$$ -x - y + z = 0 $$

Or, rearranged:

$$ z = x + y $$

which confirms the surface equation. The tangent plane at $P_0(2, 0, 2)$ is therefore:

$$ z = x + y $$

Part B: Finding the Equation of the Normal Line at $P_0(2,0,2)$

Step 1: Use the Gradient Vector as Direction

The normal line at a point on a surface is a line passing through that point and directed along the surface's normal vector at that point. Since the gradient vector ∇F is normal to the surface, it is also the direction vector for the normal line.

At $P_0(2, 0, 2)$, the gradient vector is:

$$ \vec{n} = (-1, -1, 1) $$

This vector points in the direction of the normal line.

Step 2: Write the Parametric Equations of the Normal Line

The parametric equations are:

$$
\begin{cases}
x = x_0 + t \cdot a \\
y = y_0 + t \cdot b \\
z = z_0 + t \cdot c
\end{cases}
$$

where (a, b, c) is the direction vector $\vec{n}$.

Substituting the known point and direction:

$$
\begin{cases}
x = 2 - t \\
y = 0 - t = -t \\
z = 2 + t
\end{cases}
$$

Thus, the equation of the normal line is:

$$ \boxed{
\begin{cases}
x = 2 - t \\
y = -t \\
z = 2 + t
\end{cases}
}
$$

or equivalently, in symmetric form:

$$ \frac{x - 2}{-1} = \frac{y - 0}{-1} = \frac{z - 2}{1} $$

which simplifies to:

$$ x = 2 - t, \quad y = -t, \quad z = 2 + t $$

Summary of Key Features and Pros/Cons

Features of Finding Tangent Plane and Normal Line:

- Tangent Plane:
- Provides a linear approximation of the surface at a point.
- Useful for understanding local behavior and for linearization.
- Derived from the gradient of the surface function.
- Normal Line:
- Represents the direction perpendicular to the surface at a point.
- Essential in optimization, physics, and CAD modeling.
- Defined directly via the gradient vector.

Pros:

- Straightforward procedure once the surface function is known.
- Gradient vectors are easy to compute.
- Equations are explicit and simple to manipulate.

Cons:

- Requires the surface to be differentiable at the point.
- Implicit surfaces may need additional steps to find the gradient.
- If the surface equation is complex or not specified, initial steps can be challenging.

Final Remarks and Additional Tips

- Always verify that the given point lies on the surface before proceeding.
- The choice of explicit or implicit form impacts whether you compute the gradient directly or use implicit differentiation.
- Remember that the gradient vector not only provides the normal direction but also encodes the rate of change of the surface.
- For more

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