Find The Equation Of The Line That Passes Through (-3,5) And Is Perpendicular To The Line Passing Through

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Understanding how to find the equation of a line given specific conditions is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams, a teacher designing lesson plans, or someone working on a mathematical problem, mastering this concept will enhance your analytical abilities. In particular, finding a line that passes through a particular point and is perpendicular to another line involves several key steps, including calculating slopes, understanding perpendicularity, and applying the point-slope form of a line equation.

This article provides a comprehensive guide to solving such problems, focusing on the scenario where you need to determine the equation of a line passing through the point (-3, 5) and perpendicular to a given line passing through a specific point. We will explore the foundational concepts, step-by-step procedures, and practical tips to ensure you can confidently approach similar problems.

Understanding the Basics of Line Equations and Slopes

Before diving into the problem-solving process, it’s essential to review some fundamental concepts related to lines in coordinate geometry.

What is the Equation of a Line?

The equation of a line in the Cartesian plane can be expressed in various forms, but the most common are:

- Slope-intercept form: $y = mx + b$
where m is the slope, and b is the y-intercept.

- Point-slope form: $y - y_1 = m(x - x_1)$
where (x_1, y_1) is a point on the line, and m is the slope.

- Standard form: $Ax + By = C$

For the purpose of this guide, the point-slope form is particularly useful when the slope and a point on the line are known.

Understanding the Slope of a Line

The slope (m) of a line measures its steepness and is calculated as:

$$ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} $$

where (x_1, y_1) and (x_2, y_2) are two distinct points on the line.

Perpendicular Lines and Their Slopes

A key concept when dealing with lines perpendicular to each other is the relationship between their slopes.

The Slope of a Perpendicular Line

If two lines are perpendicular, their slopes are negative reciprocals of each other. Mathematically:

$$ m_1 \times m_2 = -1 $$

or equivalently:

$$ m_2 = -\frac{1}{m_1} $$

This means:

- If one line has slope m, the perpendicular line has slope -1/m.

- Special case: a vertical line has an undefined slope, and its perpendicular is a horizontal line with a slope of zero.

Understanding this relationship is crucial because it allows you to determine the slope of the line you need to find.

Step-by-Step Guide to Find the Equation of the Perpendicular Line

Suppose we are given:

- A point through which the new line passes: (-3, 5)

- The original line passes through a certain point, say (x_0, y_0), with a known slope m_0, or through two points, from which you can determine m_0.

The goal is to find the equation of the line passing through (-3, 5) and perpendicular to the original line.

Step 1: Determine the Slope of the Original Line

If the original line is given explicitly, find its slope m_0. For example, if the line passes through points (x_1, y_1) and (x_2, y_2):

$$m_0 = \frac{y_2 - y_1}{x_2 - x_1}$$

If the original line is described by an equation, such as $y = 2x + 3$, then the slope $m_0 = 2$.

Step 2: Calculate the Slope of the Perpendicular Line

Using the relationship between perpendicular slopes:

$$m_{perp} = -\frac{1}{m_0}$$

- For example, if $m_0 = 2$, then:

$$m_{perp} = -\frac{1}{2}$$

- If the original line is horizontal (slope 0), then the perpendicular line is vertical, with an undefined slope.

Step 3: Write the Equation of the Perpendicular Line in Point-Slope Form

Using the point (-3, 5) and the slope m_{perp}:

$$y - y_1 = m_{perp} (x - x_1)$$
$$y - 5 = m_{perp} (x + 3)$$

This is the equation of the line passing through (-3, 5) and perpendicular to the original line.

Step 4: Convert to Slope-Intercept or Standard Form (Optional)

Depending on your needs, you can expand and rearrange the equation:

- Slope-intercept form:

$$y = m_{perp} (x + 3) + 5$$

- Standard form:

$$y - m_{perp} x = m_{perp} \times 3 + 5$$

This provides the final equation in the desired form.

Practical Examples

Let’s consider real-world examples to solidify understanding.

Example 1: Original line y = 3x + 1

- Step 1: The slope of the original line is $m_0 = 3$.

- Step 2: Slope of the perpendicular line:

$$m_{perp} = -\frac{1}{3}$$

- Step 3: Equation passing through (-3, 5):

$$y - 5 = -\frac{1}{3}(x + 3)$$

- Step 4: Simplify:

$$y - 5 = -\frac{1}{3}x - 1$$

$$y = -\frac{1}{3}x - 1 + 5$$

$$y = -\frac{1}{3}x + 4$$

This is the equation of the line passing through (-3, 5) and perpendicular to y = 3x + 1.

Example 2: Original line passes through points (2, 4) and (5, 10)

- Step 1: Calculate slope:

$$m_0 = \frac{10 - 4}{5 - 2} = \frac{6}{3} = 2$$

- Step 2: Slope of the perpendicular line:

$$m_{perp} = -\frac{1}{2}$$

- Step 3: Equation through (-3, 5):

$$y - 5 = -\frac{1}{2}(x + 3)$$

- Step 4: Simplify:

$$y - 5 = -\frac{1}{2}x - \frac{3}{2}$$

$$y = -\frac{1}{2}x - \frac{3}{2} + 5$$

$$y = -\frac{1}{2}x + \frac{7}{2}$$

or

$$ y = -\frac{1}{2}x + 3.5 $$

Special Cases and Additional Considerations

While the above steps cover most scenarios, some special cases may arise:

Vertical and Horizontal Lines

- If the original line is vertical (undefined slope), then the perpendicular line is horizontal (slope zero).

- Conversely, if the original line is horizontal (slope zero), the perpendicular line is vertical, which cannot be expressed in slope-intercept form but can be written as x = constant.

Example:

- Original line: x = 2 (vertical line)

- Perpendicular line passing through (-3, 5): since perpendicular to a vertical line is horizontal:

$$ y = 5 $$

- Similarly, if the original line: y = 4 (horizontal line), the perpendicular line is:

$$ x = -3 $$

Tips for Solving Line Perpendicular Problems Effectively

To enhance your problem-solving skills, consider these practical tips:

- Always determine the slope of the original line first before attempting to find the perpendicular slope.

- Be cautious with vertical and horizontal lines, as their slopes are undefined or zero, respectively.

- Use the point-slope form to write the line equation quickly once you have the slope and a point.

- Check your work by verifying whether the slopes are negative reciprocals.

- Practice with various problems, including those where the original line is given in different forms, to build versatility.

Conclusion

Finding the equation of a line passing through a specific point and perpendicular to another line is a crucial skill in coordinate geometry. The process involves understanding the relationship between slopes of perpendicular lines, calculating the original line's slope, determining the perpendicular slope, and then applying the point-slope form to derive the new line's equation.

By mastering these steps, you can

Frequently Asked Questions

How do I find the equation of a line passing through (-3, 5) and perpendicular to a given line?

First, determine the slope of the given line, then find the negative reciprocal of that slope for the perpendicular line. Use the point-slope form with point (-3, 5) to write the equation.

What is the formula for the slope of a line perpendicular to another line?

If the original line has slope m, the perpendicular line has slope -1/m, provided $m \neq 0$.

How do I find the slope of the line passing through two points?

Use the slope formula: $m = (y_2 - y_1) / (x_2 - x_1)$.

Can you give an example of finding the perpendicular line passing through a point?

Yes. For example, if the original line has slope 2, then the perpendicular line has slope -1/2. Using point (-3, 5), the equation is $y - 5 = -1/2(x + 3)$.

What is the point-slope form of a line equation?

The point-slope form is $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope.

How do I convert the point-slope form to slope-intercept form?

Expand and simplify the point-slope equation to solve for y, resulting in $y = mx + b$.

What should I do if the original line's slope is zero?

If the original line is horizontal with slope 0, the perpendicular line is vertical, which has an equation $x = x_1$, passing through the given point.

Are there any special cases to consider when finding perpendicular lines?

Yes, if the original line is vertical (undefined slope), the perpendicular line will be horizontal, with an equation $y = y_1$ passing through the given point.

Additional Resources

Find The Equation Of The Line That Passes Through (-3, 5) And Is Perpendicular To The Line Passing Through is a common problem in coordinate geometry that tests your understanding of slopes, perpendicular lines, and the point-slope form of a line's equation. Whether you're a student brushing up for an exam or a professional reviewing foundational concepts, mastering how to determine the equation of a line given specific conditions is essential. This guide aims to walk you through the process step-by-step, providing clarity, examples, and tips to ensure you can confidently tackle such problems.

Understanding the Problem

Before diving into calculations, it’s crucial to understand what the problem asks for:

- You are given a specific point: (-3, 5).
- You are told that the line you need to find is perpendicular to another line passing through some given points or with a known equation.

The core task is to find the equation of the line that:

1. Passes through the point (-3, 5).
2. Is perpendicular to a given line (or a line passing through specific points).

Note: Since the initial problem statement is incomplete regarding the line passing through which points or equation, let's consider a common scenario: Suppose the line passes through points (x_1, y_1) and (x_2, y_2), and we need to find the line through (-3, 5) perpendicular to that line.

Step 1: Find the Slope of the Given Line

The first step in the process is to determine the slope of the original line:

When given two points (x_1, y_1) and (x_2, y_2)

$$ m_{original} = \frac{y_2 - y_1}{x_2 - x_1} $$

Example:

Suppose the original line passes through points (2, 3) and (4, 7).

$$ m_{original} = \frac{7 - 3}{4 - 2} = \frac{4}{2} = 2 $$

This slope represents the steepness of the initial line.

Step 2: Determine the Slope of the Perpendicular Line

Lines that are perpendicular have slopes that are negative reciprocals of each other.

How to find the negative reciprocal:

- Take the slope $ m_{original} $.
- Find its reciprocal: $ \frac{1}{m_{original}} $.
- Change the sign: positive becomes negative, negative becomes positive.

For the example:

$$ m_{perpendicular} = -\frac{1}{m_{original}} = -\frac{1}{2} $$

Important: If the original line is horizontal (slope 0), the perpendicular line is vertical (undefined slope). Conversely, if the original line is vertical (slope undefined), the perpendicular line is horizontal (slope 0).

Step 3: Write the Equation of the Perpendicular Line

Now, with the point (-3, 5) and the perpendicular slope $ m_{perpendicular} $, we can write the equation of the new line.

Using point-slope form:

$$ y - y_1 = m (x - x_1) $$

Plugging in the point and the slope:

$$ y - 5 = -\frac{1}{2}(x + 3) $$

Simplify:

$$ y - 5 = -\frac{1}{2}x - \frac{3}{2} $$

$$ y = -\frac{1}{2}x - \frac{3}{2} + 5 $$

$$ y = -\frac{1}{2}x + \frac{7}{2} $$

This is the slope-intercept form of the required line.

Additional Considerations and Variations

1. When the original line is given in different forms

- Standard form: $Ax + By + C = 0$

To find the slope, rearrange the equation into slope-intercept form or extract the slope directly:

$$ y = -\frac{A}{B}x - \frac{C}{B} $$

- Vertical or horizontal lines:

- Vertical line: $x = a$, slope undefined.

- Horizontal line: $y = b$, slope 0.

In such cases, the perpendicular line will be:

- Vertical if original is horizontal: $x = -3$ (passes through the point).

- Horizontal if original is vertical: $y = 5$.

2. Handling special cases

- Original line is vertical

- Its slope is undefined.

- The perpendicular line passing through (-3, 5) will be horizontal, with slope 0:

$$ y = 5 $$

- Original line is horizontal

- Its slope is 0.

- The perpendicular line will be vertical, passing through x = -3:

$$ x = -3 $$

Step 4: Final Equation in Different Forms

Depending on the context or preference, you might want the line in different forms:

Slope-intercept form:

$$ y = mx + b $$

Point-slope form:

$$ y - y_1 = m (x - x_1) $$

Standard form:

$$ Ax + By = C $$

Practical Example: Summarized Walkthrough

Suppose the original line passes through (1, 2) and (3, 6), and you need to find the line passing through (-3, 5) that is perpendicular to it.

Step 1: Find the slope of the original line:

$$ m_{original} = \frac{6 - 2}{3 - 1} = \frac{4}{2} = 2 $$

Step 2: Find the perpendicular slope:

$$ m_{perpendicular} = -\frac{1}{2} $$

Step 3: Write the equation using point (-3, 5):

$$ y - 5 = -\frac{1}{2}(x + 3) $$

Step 4: Simplify:

$$ y - 5 = -\frac{1}{2}x - \frac{3}{2} $$

$$ y = -\frac{1}{2}x - \frac{3}{2} + 5 $$

$$ y = -\frac{1}{2}x + \frac{7}{2} $$

Final Equation:

$$ y = -\frac{1}{2}x + \frac{7}{2} $$

Tips for Success

- Always identify the slope of the original line before attempting to find the perpendicular slope.
- Remember the negative reciprocal rule for perpendicular lines.
- Be mindful of special cases where the original line is vertical or horizontal.
- Use the appropriate form for the final answer depending on the problem's context or instructions.
- Double-check your calculations by verifying if the point satisfies the final equation.

Additional Resources

- Practice problems: Work through various examples involving different types of lines.
- Graphing tools: Use graphing calculators or software to visualize lines and their perpendicular counterparts.
- Interactive tutorials: Many educational websites offer step-by-step guides and quizzes.

Conclusion

Finding the equation of a line passing through a specific point and perpendicular to another line is a fundamental skill in coordinate geometry. By understanding how to calculate slopes, identify negative reciprocals, and utilize point-slope or slope-intercept forms, you can confidently solve these problems. Practice with different scenarios will strengthen your grasp and improve your ability to handle more complex geometric problems efficiently. Remember, the key steps involve determining the original slope, finding its negative reciprocal, and then applying the point-slope formula with the given point. With these tools in hand, you're well-equipped to tackle any similar questions with clarity and precision.

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