Find The General Form Equation Of The Plane Through The Origin And Perpendicular To The Vector (3, -5,4).

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When studying three-dimensional geometry, understanding how to find the equation of a plane based on certain conditions is essential. One common problem involves determining the equation of a plane that passes through the origin and is perpendicular to a given vector. In this case, the vector (3, -5, 4) serves as the normal vector to the plane. This article will guide you through the process of finding the general form equation of such a plane, exploring relevant concepts, formulas, and step-by-step procedures to enhance your understanding of this important geometric topic.

Understanding the Concept of Plane Equations in 3D Space

Before delving into the specifics of this problem, it is crucial to review the foundational principles regarding the equation of a plane in three-dimensional space.

The Equation of a Plane in 3D

The general equation of a plane in three dimensions can be expressed in the form:

$$ Ax + By + Cz + D = 0 $$

where:
- (A, B, C) are the coefficients that represent the normal vector to the plane.
- D is the constant term.

The normal vector to the plane, $\vec{n} = (A, B, C)$, is perpendicular to every line lying within the plane.

Normal Vectors and Their Significance

The normal vector plays a pivotal role in defining the orientation of the plane. If the plane is perpendicular to a given vector, then that vector is, by definition, the normal vector of the plane. This relationship simplifies the process of finding the plane's equation when the normal vector is known.

Given Condition: Plane Through the Origin and Perpendicular to (3, -5, 4)

In our problem, the plane passes through the origin, meaning that the point $(0, 0, 0)$ lies on the plane. Additionally, the plane is perpendicular to the vector $\vec{v} = (3, -5, 4)$, which indicates that $\vec{v}$ is the normal vector to the plane.

Why Is the Normal Vector Important Here?

Since the plane is perpendicular to $\vec{v}$, it follows that:
- The normal vector of the plane, $\vec{n}$, is parallel or equal to $\vec{v}$.
- Consequently, the coefficients A, B, C in the plane's equation are proportional to the components of $\vec{v}$.

This simplifies the process because the normal vector directly provides the coefficients for the plane's equation.

Deriving the Equation of the Plane

To find the general form of the plane's equation, follow these steps:

Step 1: Identify the Normal Vector

- Given vector $\vec{v} = (3, -5, 4)$
- Since the plane is perpendicular to $\vec{v}$, the normal vector $\vec{n}$ is $(3, -5, 4)$.

Step 2: Write the General Equation Using the Normal Vector

- The general form is:

$$ 3x - 5y + 4z + D = 0 $$

- Here, D is the constant to be determined.

Step 3: Use the Point Through Which the Plane Passes

- The plane passes through the origin $(0, 0, 0)$.
- Substitute $(x, y, z) = (0, 0, 0)$ into the equation:

$$ 3(0) - 5(0) + 4(0) + D = 0 \Rightarrow D = 0 $$

- Therefore, the equation simplifies to:

$$3x - 5y + 4z = 0$$

Final Equation of the Plane in General Form

The general form of the plane passing through the origin and perpendicular to the vector $(3, -5, 4)$ is:

$$3x - 5y + 4z = 0$$

This equation succinctly captures the plane's orientation and position in three-dimensional space.

Additional Insights and Applications

Understanding how to derive the equation of a plane based on its normal vector and a point it passes through is fundamental in various fields, including engineering, computer graphics, physics, and architecture.

Applications of Plane Equations

- **Computer Graphics:** Rendering 3D objects requires defining planes to simulate surfaces and shadows.
- **Physics:** Calculating the orientation of surfaces and analyzing forces acting on planes.
- **Engineering:** Designing structures with specific orientations and stability considerations.
- **Mathematics Education:** Developing spatial reasoning and understanding geometric relationships in 3D space.

Variations and Complex Scenarios

- If the plane does not pass through the origin but still remains perpendicular to a given vector, the process involves substituting a known point (x_0, y_0, z_0) into the general equation to find D.
- The equation can be adapted for planes with different orientations or additional constraints.

Summary

In summary, finding the general form equation of a plane through the origin and perpendicular to a specific vector involves understanding the relationship between the normal vector and the plane's orientation. Since the plane passes through the origin, the constant term D becomes zero, simplifying the process. The key steps include:

- Identifying the normal vector from the given perpendicular vector.
- Writing the general plane equation using the normal vector components.
- Substituting known points to determine the constant term, which is zero in this case.

The resulting equation:

$$3x - 5y + 4z = 0$$

accurately describes the plane's position and orientation in 3D space.

Conclusion

Mastering the process of deriving plane equations from normal vectors and points is an essential skill in geometry and related disciplines. By understanding how the normal vector influences the plane's orientation and utilizing simple substitution techniques, you can efficiently find the general form equation of any plane, especially those passing through the origin and perpendicular to a specified vector like $(3, -5, 4)$. Remember, the key is recognizing the relationship between the normal vector and the plane's coefficients, which simplifies what might initially seem like a complex problem.

Frequently Asked Questions

What is the general form equation of a plane passing through the origin and perpendicular to the vector (3, -5, 4)?

The general form is $3x - 5y + 4z = 0$.

How do you find the equation of a plane given a normal vector and passing through the origin?

The equation is given by $n \cdot (r - r_0) = 0$, which simplifies to $n \cdot r = 0$ when passing through the origin, where n is the normal vector. For the vector (3, -5, 4), the equation is $3x - 5y + 4z = 0$.

Why does the equation of the plane take the form 3x - 5y + 4z = 0 in this case?

Because the plane is perpendicular to the vector (3, -5, 4), which serves as its normal vector. The general form of a plane through the origin with normal vector (a, b, c) is $ax + by + cz = 0$.

Can you explain how the normal vector relates to the plane's equation?

Yes, the normal vector provides the coefficients in the plane's equation. Specifically, if the plane's normal vector is (a, b, c), then the plane's equation through the origin is $ax + by + cz = 0$.

Is the point (0, 0, 0) on the plane defined by the vector (3, -5, 4)?

Yes, since the plane passes through the origin, the point (0, 0, 0) lies on the plane.

What is the significance of the plane being perpendicular to the vector (3, -5, 4)?

It means the vector (3, -5, 4) is normal to the plane, and the plane's equation is directly derived from this normal vector.

How would the equation change if the plane passed through a point other than the origin?

If the plane passes through a point $P(x_1, y_1, z_1)$, the equation becomes $3(x - x_1) - 5(y - y_1) + 4(z - z_1) = 0$, using the normal vector (3, -5, 4).

Additional Resources

Find The General Form Equation Of The Plane Through The Origin And Perpendicular To The Vector (3, -5, 4)

Introduction

In the realm of vector calculus and analytic geometry, the study of planes in three-dimensional space is fundamental. One of the core challenges involves determining the equation of a plane given certain geometric constraints, such as its position relative to a specified point or its orientation relative to a normal vector. This article explores the process of finding the general form equation of the plane through the origin and perpendicular to the vector (3, -5, 4), illustrating the concepts with comprehensive explanations, step-by-step procedures, and contextual insights.

Understanding the Geometric Foundations

The Nature of Planes in 3D Space

A plane in three-dimensional space is a flat, two-dimensional surface extending infinitely in all directions. It can be uniquely defined by several parameters, such as:

- A point through which it passes.
- A normal vector orthogonal to its surface.

The general form of a plane's equation, in vector form, is given as:

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

where:

- $\vec{n}$ is the normal vector to the plane.
- $\vec{r} = (x, y, z)$ represents an arbitrary point on the plane.
- $\vec{r}_0$ is a specific point on the plane.

Implications of the Plane Passing Through the Origin

When the plane passes through the origin, the point $\vec{r}_0$ simplifies to $(0, 0, 0)$. Therefore, the equation reduces to:

$$\vec{n} \cdot \vec{r} = 0$$

This form emphasizes that any point (x, y, z) lying on the plane satisfies the dot product with the normal vector being zero.

The Normal Vector and Its Significance

The problem states that the plane is perpendicular to the vector (3, -5, 4). In geometric terms, this indicates that:

- The vector $\vec{n} = (3, -5, 4)$ is normal (perpendicular) to the plane.
- Any point (x, y, z) satisfying the plane's equation must be orthogonal to this normal vector.

Deriving the Plane's Equation

Step 1: Recognize the Normal Vector

Given that the plane's normal vector is $\vec{n} = (3, -5, 4)$, the general form of the plane passing through the origin is:

$$3x - 5y + 4z = 0$$

This is derived directly from the point-normal form, where the point $(0, 0, 0)$ is the origin, and the normal vector provides the coefficients of the plane's equation.

Step 2: Confirm the Plane's Position

Since the plane passes through the origin, plugging $(0, 0, 0)$ into the equation confirms:

$$3 \times 0 - 5 \times 0 + 4 \times 0 = 0$$

which holds true, ensuring the correctness of the equation.

General Form Equation: Complete Analysis

The general form of a plane's equation is expressed as:

$$Ax + By + Cz + D = 0$$

where:

- A, B, C are the components of the normal vector.
- D is a constant term, which adjusts the plane's position relative to the origin.

In our case:

- $A = 3$
- $B = -5$
- $C = 4$
- $D = 0$

Thus, the general form of the plane's equation is:

$$
\boxed{
3x - 5y + 4z = 0
}
$$

Validating the Equation

Check 1: Normality to the Vector

The normal vector $(3, -5, 4)$ is orthogonal to the plane's surface, and the equation explicitly incorporates this by the coefficients.

Check 2: Passes Through the Origin

Substituting $(0, 0, 0)$ confirms the plane passes through the origin:

$$
3 \times 0 - 5 \times 0 + 4 \times 0 = 0
$$

which satisfies the plane's equation.

Additional Context: Alternative Forms of the Plane Equation

While the problem specifies the general form, other forms include:

1. Point-Normal Form:

$$
\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0
$$

which, for the origin, simplifies to the previous form.

2. Parametric Form:

Expressing the plane using parameters s and t:

$$
\begin{cases}
x = s \\
y = t \\
z = \frac{3x - 5y}{-4} \quad \text{(derived from the plane equation)}
\end{cases}
$$

but this is less direct for the current problem.

Applications and Significance

The ability to determine the equation of a plane given a normal vector and a point is crucial in various fields such as:

- Computer Graphics: for rendering and object modeling.
- Engineering: in structural analysis and design.
- Physics: for analyzing planes of motion and electromagnetic fields.
- Mathematics: in solving systems involving planes and vectors.

Understanding this process enhances spatial reasoning and analytical skills, vital for advanced studies and practical applications.

Summary and Conclusion

The problem of finding the general form equation of the plane through the origin and perpendicular to the vector (3, -5, 4) reduces to a straightforward application of vector and plane principles.

Recognizing that the normal vector directly defines the plane's orientation, and that the plane passes through the origin, leads to the concise equation:

\[
\boxed{
3x - 5y + 4z = 0
}
\]

This equation succinctly encapsulates the geometric constraints, serving as a fundamental example of the interplay between vectors and planes in three-dimensional space.

Final Remarks

Mastering the derivation of plane equations from normal vectors and positional constraints is essential for deeper comprehension of spatial geometry. The clarity of the process, as demonstrated in this example, illustrates the power of vector calculus in solving geometric problems efficiently and accurately.

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