Find The Indicated Derivative Or Antiderivative (a)d/dx X2+4xx1(b)x2+4xx1dx(c)d/dx(x+5)(x2)(d)(x+5)(x2)dx

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In calculus, derivatives and antiderivatives are fundamental concepts that help us understand the behavior of functions, analyze rates of change, and compute areas under curves. This article aims to provide a comprehensive explanation of how to find derivatives and antiderivatives for the given expressions, ensuring clarity and depth for learners and enthusiasts alike.

Understanding Derivatives and Antiderivatives

Before delving into specific problems, it is essential to establish a clear understanding of what derivatives and antiderivatives are.

What is a Derivative?

A derivative represents the rate at which a function changes with respect to its variable. Geometrically, it is the slope of the tangent line to the graph of the function at a specific point. Mathematically, the derivative of a function \(f(x)\) with respect to \(x\) is denoted as \(f'(x)\) or \(\frac{d}{dx}f(x)\).

Key rules for finding derivatives include:
- Power rule
- Product rule
- Quotient rule
- Chain rule

What is an Antiderivative?

An antiderivative, also known as an indefinite integral, is a function \(F(x)\) whose derivative is the original function \(f(x)\). In notation:
\[
F'(x) = f(x)
\]
The process of finding an antiderivative is called integration.

Common integration rules involve:

- Power rule for integration
- Substitution method
- Integration by parts
- Partial fractions

Analyzing the Given Problems

The expressions provided involve derivatives and integrals of algebraic functions. We will analyze each part separately for clarity.

(a) Find $\frac{d}{dx}(x^2 + 4x)$

This problem asks for the derivative of the polynomial $x^2 + 4x$.

Step-by-step solution:

1. Identify the terms: The function is $f(x) = x^2 + 4x$.
2. Apply the power rule: The derivative of x^n is $n x^{n-1}$.

$$
\frac{d}{dx} x^2 = 2x
$$
$$
\frac{d}{dx} 4x = 4
$$

3. Combine results:

$$
\frac{d}{dx} (x^2 + 4x) = 2x + 4
$$

Final answer:

$$
\boxed{\frac{d}{dx}(x^2 + 4x) = 2x + 4}
$$

(b) Find $\int (x^2 + 4x) \, dx$

This problem involves indefinite integration of the same polynomial.

Step-by-step solution:

1. Split the integral:

$$
\int (x^2 + 4x) dx = \int x^2 dx + \int 4x dx
$$

2. Apply the power rule for integration:

$$
\int x^n dx = \frac{x^{n+1}}{n+1} + C
$$

- For x^2:

$$
\int x^2 dx = \frac{x^{3}}{3}
$$

- For $4x$:

$$
\int 4x dx = 4 \times \frac{x^{2}}{2} = 2x^{2}
$$

3. Combine and add the constant of integration C:

$$
\int (x^2 + 4x) dx = \frac{x^3}{3} + 2x^2 + C
$$

Final answer:

$$
\boxed{\int (x^2 + 4x) dx = \frac{x^3}{3} + 2x^2 + C}
$$

(c) Find $\frac{d}{dx}[(x+5)(x^2)]$

This involves differentiating the product of two functions: $(x+5)$ and x^2. We will use the product rule.

Step-by-step solution:

1. Identify functions:

Let $u = x + 5$, $v = x^2$.

2. Compute derivatives:

$$
u' = \frac{d}{dx}(x + 5) = 1
$$
$$
v' = \frac{d}{dx}(x^2) = 2x
$$

3. Apply the product rule:

$$
\frac{d}{dx} (u v) = u' v + u v'
$$

$$
= (1)(x^2) + (x + 5)(2x)
$$

4. Simplify:

$$
x^2 + 2x(x + 5) = x^2 + 2x^2 + 10x = (x^2 + 2x^2) + 10x = 3x^2 + 10x
$$

Final answer:

$$
\boxed{\frac{d}{dx} [(x+5)(x^2)] = 3x^2 + 10x}
$$

(d) Find $\int (x+5)(x^2) \, dx$

This involves integrating the product $(x+5)(x^2)$. It might be simpler to expand the expression before integrating.

Step-by-step solution:

1. Expand the integrand:

$$
(x + 5)(x^2) = x \cdot x^2 + 5 \cdot x^2 = x^3 + 5x^2
$$

2. Set up the integral:

\[

$$
\int (x^3 + 5x^2) dx
$$

3. Integrate each term separately:

$$
\int x^3 dx = \frac{x^4}{4}
$$
$$
\int 5x^2 dx = 5 \times \frac{x^3}{3} = \frac{5x^3}{3}
$$

4. Combine results with the constant of integration C:

$$
\int (x^3 + 5x^2) dx = \frac{x^4}{4} + \frac{5x^3}{3} + C
$$

Final answer:

$$
\boxed{\int (x+5)(x^2) dx = \frac{x^4}{4} + \frac{5x^3}{3} + C}
$$

Additional Tips and Techniques for Derivatives and Integrals

Understanding how to efficiently compute derivatives and integrals is crucial in calculus. Here are some tips and techniques to help you master these processes:

1. Recognize Common Patterns

- Polynomial functions are straightforward to differentiate and integrate using the power rule.
- Products often require the product rule.
- Quotients necessitate the quotient rule.
- Compositions are handled with the chain rule.

2. Simplify Before Differentiating or Integrating

- Expand products to simplify expressions.
- Factor where possible to make derivatives and integrals more manageable.

3. Use Substitution for Complex Integrals

- Substitution simplifies integrals involving composite functions.
- Identify inner functions and set $u = g(x)$.

4. Practice the Fundamental Rules

- Power rule
- Sum and difference rule
- Constant multiple rule
- Chain rule
- Product and quotient rules

5. Check Your Work

- Differentiate your integral result to verify it matches the original function.
- Use derivative tests to verify the correctness of derivatives.

Conclusion

Calculus involves understanding and applying derivatives and integrals to analyze and compute properties of functions. The problems discussed—derivatives of polynomials and products, as well as integrals of polynomial expressions—are foundational. Mastery of these concepts provides a strong basis for tackling more complex calculus problems involving exponential, logarithmic, trigonometric, and other advanced functions.

By practicing the step-by-step methods such as the power rule, product rule, and substitution, learners can build confidence and proficiency. Remember, consistency and attention to detail are key to mastering calculus. Whether finding the slope of a curve at a point or calculating the area under a function, the skills developed here are invaluable in both academic and real-world applications.

Keywords: derivatives, antiderivatives, calculus, differentiation, integration, power rule, product rule, indefinite integral, calculus tips, polynomial functions

Frequently Asked Questions

What is the derivative of the function f(x) = x^2 +

4x?

The derivative of $f(x) = x^2 + 4x$ is $f'(x) = 2x + 4$.

How do you find the antiderivative of the function $f(x) = x^2 + 4x$?

The antiderivative of $f(x) = x^2 + 4x$ is $\int(x^2 + 4x)\,dx = (1/3)x^3 + 2x^2 + C$.

What is the derivative of the product $(x + 5)(x^2)$?

Using the product rule, $d/dx[(x + 5)(x^2)] = (x + 5)\cdot 2x + x^2\cdot 1 = 2x(x + 5) + x^2$.

How do you evaluate the integral $\int(x + 5)(x^2)\,dx$?

Expand the integrand: $(x + 5)(x^2) = x^3 + 5x^2$. Then, $\int(x^3 + 5x^2)\,dx = (1/4)x^4 + (5/3)x^3 + C$.

What is the derivative of the function $f(x) = (x + 5)(x^2)$?

Using the product rule, the derivative is $f'(x) = (x + 5)\cdot 2x + x^2\cdot 1 = 2x(x + 5) + x^2$.

Additional Resources

Find The Indicated Derivative Or Antiderivative (a)d/dx $x^2 + 4x\ x^1$ (b) $x^2 + 4x\ x^1$ dx (c) d/dx $(x + 5)(x^2)$ (d) $(x + 5)(x^2)$ dx

When working with calculus, one of the fundamental skills is to find derivatives and antiderivatives of various functions. These operations are crucial for understanding the behavior of functions, optimizing problems, and solving real-world applications. In this guide, we'll explore find the indicated derivative or antiderivative problems, focusing on the expressions provided. Whether you're a student brushing up on calculus concepts or a professional reviewing derivatives and integrals, this detailed breakdown will clarify each step involved.

Understanding the Core Concepts

Before diving into specific problems, let's briefly review the key ideas:

Derivative (d/dx)

- Derivative of a function measures the rate at which the function's value changes as its input changes.
- Denoted as $d/dx\ [f(x)]$, it provides the slope of the tangent line at any point on the function.
- Common rules include power rule, product rule, quotient rule, and chain rule.

Antiderivative (Indefinite Integral)

- The antiderivative of a function is the reverse process of differentiation.
- Denoted as $\int f(x)\, dx$, it finds a function whose derivative is $f(x)$.
- Includes constants of integration, usually represented as + C.

Problem Breakdown and Step-by-Step Solutions

Let's analyze each problem individually, understanding what is asked and how to approach it.

(a) Find the derivative $d/dx\ [x^2 + 4x]$

Expression:

$$ \frac{d}{dx} [x^2 + 4x] $$

Step 1: Recognize the sum rule

The derivative of a sum is the sum of derivatives:

$$ \frac{d}{dx} [f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x) $$

Step 2: Apply the power rule

- For x^2:

$$ \frac{d}{dx} x^2 = 2x $$

- For $4x$:

$$ \frac{d}{dx} 4x = 4 $$

Step 3: Combine results

$$ \boxed{2x + 4} $$

Final answer:

> The derivative of $x^2 + 4x$ with respect to x is 2x + 4.

(b) Find the antiderivative $\int (x^2 + 4x)\, dx$

Expression:

$$\int (x^2 + 4x) dx$$

Step 1: Use the sum rule for integrals

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

Step 2: Integrate term-by-term

- For x^2:

$$\int x^2 dx = \frac{x^3}{3} + C_1$$

- For $4x$:

$$\int 4x dx = 2x^2 + C_2$$

Note: Constants C_1 and C_2 consolidate into a single constant C.

Step 3: Write the combined antiderivative

$$\frac{x^3}{3} + 2x^2 + C$$

Final answer:

> The indefinite integral of $x^2 + 4x$ with respect to x is (1/3) x^3 + 2x^2 + C.

(c) Find d/dx $[(x + 5)(x^2)]$

Expression:

$$\frac{d}{dx} [(x + 5)(x^2)]$$

Step 1: Recognize the product

Since this is a product of two functions, apply the product rule:

$$\frac{d}{dx} [u \cdot v] = u' v + u v'$$

where:

- $u = x + 5$

- $v = x^2$

Step 2: Find derivatives of u and v

- $u' = \frac{d}{dx} (x + 5) = 1$
- $v' = \frac{d}{dx} x^2 = 2x$

Step 3: Apply the product rule

$$u' v + u v' = (1)(x^2) + (x + 5)(2x)$$

Step 4: Simplify

$$x^2 + 2x (x + 5) = x^2 + 2x^2 + 10x$$

$$= (x^2 + 2x^2) + 10x = 3x^2 + 10x$$

Final answer:

> The derivative of $(x + 5)(x^2)$ with respect to x is $3x^2 + 10x$.

(d) Find $\int (x + 5)(x^2)\, dx$

Expression:

$$\int (x + 5)(x^2)\, dx$$

Step 1: Expand the product

$$(x + 5)(x^2) = x \cdot x^2 + 5 \cdot x^2 = x^3 + 5x^2$$

Step 2: Integrate term-by-term

- $\int x^3 dx = \frac{x^4}{4} + C_1$
- $\int 5x^2 dx = 5 \cdot \frac{x^3}{3} = \frac{5}{3} x^3 + C_2$

Step 3: Combine results

$$\frac{x^4}{4} + \frac{5}{3} x^3 + C$$

Final answer:

> The indefinite integral of $(x + 5)(x^2)$ with respect to x is $(1/4) x^4 + (5/3) x^3 + C$.

Additional Tips and Insights

- Always identify whether you need a derivative or an antiderivative before beginning calculations.
- Use basic rules: power rule, product rule, quotient rule, chain rule, and sum rule.
- When expanding expressions, it often simplifies the integration or differentiation process.
- Remember constants of integration in indefinite integrals; they are essential for the general solution.
- Check your work by differentiating an antiderivative to see if it returns the original function.

Practical Applications

Understanding how to find derivatives and antiderivatives is essential across various fields:

- Physics: Calculating velocity and acceleration from position functions.
- Economics: Determining marginal cost or revenue.
- Engineering: Analyzing system responses and signals.
- Biology: Modeling growth and decay processes.

Summary

- The derivative of $x^2 + 4x$ is $2x + 4$.
- The antiderivative of $x^2 + 4x$ is $(1/3) x^3 + 2x^2 + C$.
- The derivative of $(x + 5)(x^2)$ is $3x^2 + 10x$.
- The antiderivative of $(x + 5)(x^2)$ is $(1/4) x^4 + (5/3) x^3 + C$.

Mastering these fundamental calculus techniques empowers you to solve a wide variety of mathematical problems with confidence. Practice applying these rules regularly to build intuition and efficiency in working with derivatives and integrals.

Find The Indicated Derivative Or Antiderivative A D Dx X2 4xx1 B X2 4xx1dx C D Dx X 5 X2 D X 5 X2 Dx

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