

Find The Lengths Of The Sides Of The Triangle Formed By The Coordinates Below. Then Classify The Triangle

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Understanding how to find the lengths of the sides of a triangle given its vertices is a fundamental skill in coordinate geometry. Once the side lengths are determined, classifying the triangle based on its sides—whether it is equilateral, isosceles, or scalene—becomes straightforward. This article provides a comprehensive guide to calculating side lengths from coordinate points and classifying the resulting triangle step-by-step, complete with detailed explanations, formulas, and examples.

Understanding the Coordinates of a Triangle

What Are Coordinates?

In the Cartesian coordinate system, each point in a plane is represented by an ordered pair (x, y) , where:

- x is the horizontal distance from the origin $(0,0)$.
- y is the vertical distance from the origin.

When three points are given, each with their own coordinates, they can be connected to form a triangle. For example, suppose the vertices are:

- Point A: (x_1, y_1)
- Point B: (x_2, y_2)
- Point C: (x_3, y_3)

Calculating the Lengths of the Sides of a Triangle

The Distance Formula

The key to finding the length of a side between any two points in the coordinate plane lies in the distance formula derived from the Pythagorean theorem:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where:

- (x_1, y_1) and (x_2, y_2) are the coordinates of the two points.

Step-by-Step Calculation

1. Identify the coordinates of the two points that form a side.
2. Subtract the x-coordinates to find the horizontal difference: $\Delta x = x_2 - x_1$.
3. Subtract the y-coordinates to find the vertical difference: $\Delta y = y_2 - y_1$.
4. Square both differences: $(\Delta x)^2$ and $(\Delta y)^2$.
5. Sum the squared differences: $(\Delta x)^2 + (\Delta y)^2$.
6. Take the square root of that sum to get the length of the side.

Example Calculation

Suppose the points are:

- A (1, 2)
- B (4, 6)
- C (5, 2)

Calculate the side lengths:

- Side AB:

- $\Delta x = 4 - 1 = 3$

- $\Delta y = 6 - 2 = 4$

- $d_{AB} = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$

- Side BC:

- $\Delta x = 5 - 4 = 1$

- $\Delta y = 2 - 6 = -4$

- $d_{BC} = \sqrt{(1^2 + (-4)^2)} = \sqrt{(1 + 16)} = \sqrt{17} \approx 4.123$

- Side CA:

- $\Delta x = 5 - 1 = 4$

- $\Delta y = 2 - 2 = 0$

- $d_{CA} = \sqrt{(4^2 + 0^2)} = \sqrt{16} = 4$

Classifying the Triangle Based on Side Lengths

Types of Triangles by Sides

Once the side lengths are known, the triangle can be classified into one of three categories:

- **Equilateral:** All three sides are equal.
- **Isosceles:** Exactly two sides are equal.
- **Scalene:** All three sides are of different lengths.

How to Classify the Triangle

1. Compare the three side lengths.
2. **If all three are equal:** the triangle is equilateral.
- 3.

If exactly two sides are equal: the triangle is isosceles.

4.

If all three are different: the triangle is scalene.

Additional Classification: Based on Angles

In addition to side-based classification, triangles can also be classified by their angles:

- **Acute:** All angles less than 90° .
- **Right:** One angle exactly 90° .
- **Obtuse:** One angle greater than 90° .

To determine the angle type, use the Law of Cosines or compare the side lengths with the Pythagorean theorem for right angles.

Applying the Methods to an Example

Given Coordinates

Suppose the vertices of the triangle are:

- A (2, 3)
- B (6, 7)
- C (8, 3)

Step 1: Calculate Side Lengths

- AB:
 - $\Delta x = 6 - 2 = 4$
 - $\Delta y = 7 - 3 = 4$
 - $d_{AB} = \sqrt{(4^2 + 4^2)} = \sqrt{(16 + 16)} = \sqrt{32} \approx 5.657$

- BC:

- $\Delta x = 8 - 6 = 2$

- $\Delta y = 3 - 7 = -4$

- $d_{BC} = \sqrt{(2^2 + (-4)^2)} = \sqrt{(4 + 16)} = \sqrt{20} \approx 4.472$

- CA:

- $\Delta x = 8 - 2 = 6$

- $\Delta y = 3 - 3 = 0$

- $d_{CA} = \sqrt{(6^2 + 0^2)} = \sqrt{36} = 6$

Step 2: Classify the Triangle

- Side lengths:

- $AB \approx 5.657$

- $BC \approx 4.472$

- $CA = 6$

- Since all three sides are different, the triangle is scalene.

Conclusion

Calculating the side lengths of a triangle from its coordinate points is a straightforward process that relies on the distance formula. Once the side lengths are obtained, classifying the triangle becomes a matter of simple comparison. Whether you are solving geometry problems in a classroom, working on a data visualization project, or solving real-world problems involving spatial relationships, mastering these techniques is essential. Remember to double-check your calculations and consider the triangle's angle types for a

comprehensive understanding of its properties.

Summary of Key Steps

1. Identify the coordinates of the three vertices.
2. Use the distance formula to find the lengths of all three sides.
3. Compare the side lengths to classify the triangle as equilateral, isosceles, or scalene.
4. Optionally, analyze angles to classify as acute, right, or obtuse.

Additional Resources

- [Khan Academy: Triangle Geometry](#)