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Understanding how magnetic fields are generated by current-carrying conductors is fundamental in electromagnetism. When two wire segments are positioned in close proximity, especially with opposite orientations, analyzing the magnetic field at a specific point becomes a fascinating problem. In this article, we delve into calculating the magnitude of the magnetic field at point P caused by two identical 1.50 mm segments of wire that are oriented in opposite directions. This problem exemplifies core principles such as the Biot–Savart Law, superposition of magnetic fields, and geometrical considerations in magnetic field calculations.

Background and Fundamental Concepts

Magnetic Fields and Current-Carrying Wires

When an electric current flows through a wire segment, it produces a magnetic field in the surrounding space. The direction of the magnetic field is given by the right-hand rule, and its magnitude depends on the current, the length of the wire segment, and the distance from the wire to the point where the field is being measured.

The Biot–Savart Law

The Biot–Savart Law provides a quantitative way to calculate the magnetic field generated by a small element of current:

- **Formula:** $\mathbf{dB} = \frac{\mu_0}{4\pi} \frac{I\,d\mathbf{l} \times \mathbf{\hat{r}}}{r^2}$
- **Variables:** where μ_0 is the permeability of free space ($4\pi \times 10^{-7}\,\mathrm{T\,m/A}$), I is the current, $d\mathbf{l}$ is a differential element of the wire, $\mathbf{\hat{r}}$ is the unit vector from the element to the point P, and r is the distance from the element to P.

Superposition Principle

The total magnetic field at a point due to multiple sources is the vector sum of the individual magnetic fields produced by each source. When dealing with two wire segments, this principle allows us to compute the field each segment produces at point P and then sum these vectorially.

Setup of the Problem

Geometry of the Wire Segments

Assuming two identical wire segments of length 1.50 mm each, positioned such that:

- They are located parallel to each other.
- One segment extends in one direction, and the other extends in the opposite direction.
- Point P is situated somewhere in the space between or near these segments, at a known position relative to them.

For simplicity, consider the following configuration:

- The two wire segments are aligned along the x-axis.
- Segment 1 runs from $x=0$ to $x=1.50\, \mathrm{mm}$.
- Segment 2 runs from $x=0$ to $x=-1.50\, \mathrm{mm}$.
- Point P is located above the midpoint between the two segments, at a height h along the y-axis.

This setup simplifies the calculations by exploiting symmetry and standard geometry.

Assumptions and Known Values

- Length of each wire segment: $L = 1.50\, \mathrm{mm} = 1.50 \times 10^{-3}\, \mathrm{m}$
- Current through each wire: I (assumed known or specified in the problem)
- Position of point P relative to the segments: specified by coordinates (x_P, y_P, z_P)
- Opposite directions of the currents in the two segments: one current flows in the positive x-direction, the other in the negative x-direction.

Understanding these parameters helps in setting up the integral for the magnetic field and facilitates the superposition process.

Calculating the Magnetic Field from a Single Wire Segment

Applying the Biot–Savart Law

To find the magnetic field at point P due to one segment, follow these steps:

1. Define the differential element $d\mathbf{l}$ along the wire. For a segment aligned along the x-axis, $d\mathbf{l} = dx\, \hat{\mathbf{x}}$.
2. Determine the position vector from the element to point P: $\mathbf{r} = \mathbf{r}_P - \mathbf{r}_\text{element}$.
3. Calculate the magnitude $r = |\mathbf{r}|$ and the unit vector $\mathbf{\hat{r}} = \mathbf{r} / r$.
4. Compute the cross product $d\mathbf{l} \times \mathbf{\hat{r}}$.
5. Express the differential magnetic field $d\mathbf{B}$ using the Biot–Savart Law.

Note: Due to the symmetry and the linear geometry, the integral over the segment simplifies, and standard formulas for the magnetic field due to a finite straight current-carrying wire can be used.

Magnetic Field Due to a Finite Straight Wire

The magnetic field at a point perpendicular to the center of a finite straight wire carrying current I is given by:

$$
B = \frac{\mu_0 I}{4\pi r} (\sin \theta_2 + \sin \theta_1)
$$

where:

- r is the perpendicular distance from the wire to point P.
- θ_1 and θ_2 are the angles between the wire's endpoints and point P relative to the perpendicular line from the wire to P.

This formula assumes the wire is straight and finite, making it ideal for the 1.50 mm segments in question.

Superposition of Magnetic Fields from Two Opposite Segments

Vector Addition of Magnetic Fields

Since the two segments carry currents in opposite directions, their magnetic fields at point P will have different directions. To find the total magnetic field:

- Calculate each segment's magnetic field vector at P separately.
- Determine the direction of each field using the right-hand rule.
- Sum the vectors to find the net magnetic field at P.

This process involves breaking down each magnetic field into components, especially if the fields are not aligned, and then performing vector addition.

Example Calculation Outline

Suppose:

- Current I flows in the positive x-direction in segment 1.
- Current $-I$ flows in the negative x-direction in segment 2.
- Point P is located directly above the midpoint between the segments at a height h.

The steps are:

1. Calculate B_1, the magnetic field at P due to segment 1.
2. Calculate B_2, the magnetic field at P due to segment 2.
3. Assign directions based on the right-hand rule:
- For segment 1: field points in a direction perpendicular to the plane determined by the current and the point P.
- For segment 2: similarly, but with opposite current direction.
4. Add the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$.

The superposition will reveal whether the fields reinforce or partially cancel each other, depending on their directions.

Final Calculation of the Magnetic Field Magnitude at Point P

Combining the Results

After computing the individual magnetic fields, the magnitude of the total magnetic field B_total at point P is:

$$
B_\text{total} = |\mathbf{B}_1 + \mathbf{B}_2|
$$

which involves calculating the vector sum, considering the directions and magnitudes of each component.

Special Cases and Simplifications

- If point P lies symmetrically between the two segments, some components may cancel or reinforce.
- For small h compared to L, approximations can simplify calculations.
- If currents are equal and opposite, certain symmetrical properties can be exploited.

Practical Applications and Importance

Understanding the magnetic field due to small wire segments in opposition has applications in:

- Designing magnetic shielding and field cancellation devices.
- Developing precision magnetic sensors and detectors.
- Analyzing electromagnetic interference in circuits.
- Educational demonstrations illustrating magnetic superposition.

Accurate calculations of these fields are critical for engineers and physicists working on magnetic field manipulation at small scales.

Conclusion