Find The Measures Of Two Angles, One Positive And One Negative, That Are Coterminal With The Given Angle.

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Understanding angles and their relationships is fundamental in geometry and trigonometry. Among these relationships, coterminal angles play a crucial role in simplifying problems, solving trigonometric equations, and understanding rotational movements. When dealing with coterminal angles, it's common to encounter angles that are positive, negative, or both, yet share the same terminal side when drawn in standard position. This article aims to provide a comprehensive guide on how to find two angles—one positive and one negative—that are coterminal with a given angle.

What Are Coterminal Angles?

Definition of Coterminal Angles

Coterminal angles are angles that share the same initial side and terminal side when drawn in standard position on the coordinate plane. In simpler terms, they are angles that point in the same direction but may differ by full rotations (multiples of 360° or 2π radians).

Example:
- 30°, 390°, and -330° are coterminal angles because they all end in the same position on the circle.

Why Are Coterminal Angles Important?

- They help simplify trigonometric calculations.
- They provide multiple angle measures for solving equations.
- They are essential in understanding periodic functions like sine and cosine.
- They aid in analyzing rotational movements and oscillations.

Understanding Angle Measures and Standard Position

Angles in Standard Position

An angle is said to be in standard position when its vertex is at the origin of the coordinate plane, and its initial side lies along the positive x-axis. The measure of the angle is then determined by the rotation from the initial side to the terminal side.

Positive and Negative Angles

- Positive angles: Measured counterclockwise from the positive x-axis.
- Negative angles: Measured clockwise from the positive x-axis.

Both positive and negative angles can be coterminal with each other if they differ by multiples of 360° (or 2π radians).

How to Find Coterminal Angles

General Approach

Given an angle, the process of finding coterminal angles involves adding or subtracting full rotations (360° or 2π radians):

- For positive coterminal angles, add multiples of 360°.
- For negative coterminal angles, subtract multiples of 360°.
- To find one positive and one negative coterminal angle with the same measure, apply the above steps accordingly.

Step-by-Step Method

1. Identify the given angle: Note its measure and whether it is positive or negative.

2. Add or subtract 360° (or 2π radians):
- To find a positive coterminal angle, add 360° repeatedly until the angle is positive.
- To find a negative coterminal angle, subtract 360° repeatedly until the angle is negative.

3. Ensure the angles are coterminal:
- Confirm that the difference between the angles is a multiple of 360° (or 2π radians).

4. Select one positive and one negative angle:
- Typically, choose the smallest positive coterminal angle greater than 0.
- And the smallest negative coterminal angle less than 0.

Examples of Finding One Positive and One Negative Coterminal Angle

Example 1: Given Angle is 150°

- Find a positive coterminal angle:

Since 150° is already positive, we can find another positive coterminal angle by adding 360°:

150° + 360° = 510°

Alternatively, subtracting 360° gives:

150° - 360° = -210° (which is negative)

- Find a negative coterminal angle:

Subtract 360° from 150°:

150° - 360° = -210°

- Results:

- Positive coterminal angle: 510°
- Negative coterminal angle: -210°

Example 2: Given Angle is -45°

- Find a positive coterminal angle:

Add 360°:

-45° + 360° = 315°

- Find a negative coterminal angle:

Subtract 360°:

-45° - 360° = -405°

- Results:

- Positive coterminal angle: 315°
- Negative coterminal angle: -405°

Using Radians to Find Coterminal Angles

Angles can also be measured in radians, where the full rotation is 2π radians.

Steps:

1. Convert the given angle to radians if necessary.

2. Add or subtract multiples of 2π:

- Positive coterminal angles: add 2π repeatedly.
- Negative coterminal angles: subtract 2π repeatedly.

3. Select the smallest positive and negative coterminal angles after conversion.

Practical Applications of Finding Coterminal Angles

- Trigonometry Problems: Simplify sine, cosine, and tangent calculations.
- Engineering and Physics: Analyze rotational motion and oscillations.
- Navigation and Robotics: Determine directions and rotations.
- Computer Graphics: Calculate rotations and transformations.

Tips for Finding Coterminal Angles Effectively

- Always keep track of whether the angle is positive or negative.
- Remember that coterminal angles differ by multiples of 360° or 2π radians.
- Use modular arithmetic for quick calculations:
- For degrees: angle mod 360°
- For radians: angle mod 2π
- Convert between degrees and radians if necessary to avoid confusion.

Conclusion

Finding the measures of two angles—one positive and one negative—that are coterminal with a given angle is a fundamental skill in trigonometry and geometry. By understanding the concept of coterminal angles, using the periodic nature of angles, and applying simple addition or subtraction of full rotations, you can easily determine angles that share the same terminal side. Whether working in degrees or radians, these techniques help simplify complex problems and deepen your understanding of rotational symmetries and periodic functions.

Summary Points

- Coterminal angles share the same terminal side.
- They differ by multiples of 360° or 2π radians.
- To find positive coterminal angles, add 360° (or 2π radians).
- To find negative coterminal angles, subtract 360° (or 2π radians).
- Always confirm the angles are coterminal by checking their difference is a multiple of the full rotation measure.

By mastering these methods, you will confidently find both positive and negative coterminal angles, enhancing your understanding of angular relationships and their applications across various fields.

Frequently Asked Questions

What does it mean for two angles to be coterminal?

Two angles are coterminal if they share the same initial and terminal sides, meaning they differ by a full rotation (360° or 2π radians).

How can I find a positive and a negative angle coterminal with a given angle?

You can add or subtract multiples of 360° (or 2π radians) to the given angle. For a positive coterminal angle, add 360°; for a negative one, subtract 360°.

What is the general formula for coterminal angles?

The general formula is: angle + 360°·k, where k is any integer (positive, negative, or zero).

If the given angle is 45°, what are the measures of a positive

and negative coterminal angle?

A positive coterminal angle is 45° + 360° = 405°, and a negative coterminal angle is 45° - 360° = -315°.

Can the measures of coterminal angles be negative or greater than 360°?

Yes, coterminal angles can be negative or greater than 360°, as long as they differ from the original by full rotations of 360°.

Why is it important to find angles coterminal with a given angle?

Finding coterminal angles helps in simplifying problems involving angles, especially in trigonometry, and understanding angle rotations beyond standard ranges.

How do radians relate to finding coterminal angles?

In radians, coterminal angles can be found by adding or subtracting 2π radians. For example, given an angle θ, coterminal angles are θ + 2π·k.

Can you give an example of finding both a positive and negative coterminal angle for 120°?

Yes. A positive coterminal angle is 120° + 360° = 480°, and a negative coterminal angle is 120° - 360° = -240°.

Additional Resources

Find The Measures Of Two Angles, One Positive And One Negative, That Are Coterminal With The Given Angle

Understanding the relationship between angles is fundamental in geometry and trigonometry, particularly when dealing with coterminal angles. These angles, which share the same terminal side when drawn in standard position, provide insight into the periodic nature of angle measures and their applications across various fields such as physics, engineering, and computer graphics. Specifically, exploring how to find two coterminal angles—one positive and one negative—that correspond to a given angle enhances our comprehension of angle equivalence and their practical utility. This article delves into the concepts, methods, and significance of identifying such angles, offering a comprehensive guide that balances theoretical explanations with practical examples.

Understanding Coterminal Angles

Definition and Conceptual Foundation

In the realm of geometry, coterminal angles are angles that share the same terminal side when placed in standard position—that is, when the initial side lies along the positive x-axis. Despite potentially differing in measure, these angles end at the same position on the coordinate plane, making them equivalent in terms of their directional orientation.

Mathematically, two angles, θ and ϕ, are coterminal if:

$$
\theta = \phi + 360^\circ \times k
$$

where k is any integer (positive, negative, or zero), and angles are measured in degrees. For radian measure, the equivalent formula is:

$$
\theta = \phi + 2\pi \times k
$$

This periodicity stems from the nature of circles, where rotating by full turns (360° or 2π radians) brings you back to the same position.

Importance of Coterminal Angles

Coterminal angles are crucial because they allow us to analyze angles beyond the typical 0°–360° range, facilitating problem-solving in periodic functions like sine and cosine. They also help in simplifying complex angle calculations, understanding wave phenomena, and designing rotational systems.

The Challenge: One Positive and One Negative Coterminal Angle

While many exercises focus on finding coterminal

angles within a standard range (such as 0°–360°), the task of identifying one positive and one negative coterminal angle for a given angle adds an extra layer of depth. This approach emphasizes understanding the full spectrum of angle equivalence, including negative rotations, which often appear in real-world scenarios such as counter-clockwise and clockwise rotations.

Why is this important?

- It provides a more complete understanding of angle measures.
- It reinforces the concept of periodicity, including negative periods.
- It has practical applications in navigation, robotics, and physics, where directions and rotations can be positive or negative depending on orientation.

Step-by-Step Procedure for Finding Coterminal Angles

To find the two angles—one positive and one negative—that are coterminal with a given angle, follow these systematic steps:

Step 1: Identify the Given Angle

The initial step involves noting the measure of the given angle, whether in degrees or radians. For illustration, suppose the angle is $\theta_0 = 125^\circ$.

Step 2: Use Coterminal Angle Formulas

Recall the general formulas:

- Positive coterminal angle:

$$
\theta_{positive} = \theta_0 + 360^\circ \times k, \quad k \in \mathbb{Z}
$$

- Negative coterminal angle:

$$
\theta_{negative} = \theta_0 - 360^\circ \times m, \quad m \in \mathbb{Z}
$$

To find one positive and one negative coterminal angle, choose appropriate integer values for k and m.

Step 3: Find the Positive Coterminal Angle

- Start with $k=1$:

$$
\theta_{positive} = 125^\circ + 360^\circ \times 1 = 125^\circ + 360^\circ = 485^\circ
$$

- Alternatively, if you want the smallest positive coterminal angle, you could set $k=0$:

$$
\theta_{positive} = 125^\circ + 360^\circ \times 0 = 125^\circ
$$

But the typical approach is to find a positive coterminal angle different from the original, so 485° is a suitable choice.

Step 4: Find the Negative Coterminal Angle

- Set $m=1$:

$$
\theta_{negative} = 125^\circ - 360^\circ \times 1 = 125^\circ - 360^\circ = -235^\circ
$$

Alternatively, for a more negative angle, choose $m=2$):

$$
\theta_{negative} = 125^\circ - 360^\circ \times 2 = 125^\circ - 720^\circ = -595^\circ
$$

The standard choice for the "nearest" negative coterminal angle is often -235°.

General Formula and Examples

The process can be summarized as follows:

- Positive coterminal angle:

$$
\theta_{positive} = \theta_0 + 360^\circ \times k, \quad k \in \mathbb{Z}^+
$$

- Negative coterminal angle:

$$
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$$
\theta_{negative} = \theta_0 - 360^\circ \times m, \quad m \in \mathbb{Z}^+
$$

Example 1:

Given $\theta_0 = 150^\circ$, find one positive and one negative coterminal angle.

- Positive:

$$
\theta_{positive} = 150^\circ + 360^\circ = 510^\circ
$$

- Negative:

$$
\theta_{negative} = 150^\circ - 360^\circ = -210^\circ
$$

Example 2:

Given $\theta_0 = -45^\circ$:

- Positive:

$$
\theta_{positive} = -45^\circ + 360^\circ = 315^\circ
$$

\]

- Negative:

\[
\theta_{negative} = -45^\circ - 360^\circ = -405^\circ
\]

This illustrates that negative angles can be more than just the initial measure, extending infinitely in both directions.

Extension to Radians

In many scientific and engineering contexts, angles are measured in radians. The formulas adapt accordingly:

- Positive coterminal angle:

\[
\theta_{positive} = \theta_0 + 2\pi \times k
\]

- Negative coterminal angle:

$$
\theta_{negative} = \theta_0 - 2\pi \times m
$$

where $k, m \in \mathbb{Z}$.

Example:

Given $\theta_0 = \frac{\pi}{4}$:

- Positive:

$$
\theta_{positive} = \frac{\pi}{4} + 2\pi = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}
$$

- Negative:

$$
\theta_{negative} = \frac{\pi}{4} - 2\pi = \frac{\pi}{4} - \frac{8\pi}{4} = -\frac{7\pi}{4}
$$

Significance and Applications

The ability to find coterminal angles, especially one positive and one negative, has numerous practical applications:

1. Navigation and Orientation: Understanding rotations in clockwise and counter-clockwise directions involves negative and positive angles, respectively. Coterminal angles help in standardizing directions.

2. Wave and Signal Analysis: Periodic functions like sine and cosine repeat their values every 360° or 2π, making coterminal angles essential in analyzing phase shifts and waveforms.

3. Robotics and Engineering: Precise control of rotational components requires knowledge of angles beyond the standard 0°–360°, including negative rotations.

4. Mathematical Problem Solving: Simplification of trigonometric expressions often involves converting angles into coterminal ones within a preferred range, which may include negative measures.

Conclusion: The Broader Perspective

Understanding how to find two coterminal angles—one positive and one negative—for a given angle is a foundational skill that deepens grasp of rotational symmetry and periodicity in mathematics. It underscores the cyclical nature of angles and how they can extend infinitely in both directions on the coordinate plane. This knowledge is not only theoretically enriching but also practically vital across numerous scientific and technological disciplines.

By mastering the formulas and conceptual framework outlined here, students and professionals alike can approach problems involving angles with greater confidence and clarity. Whether dealing with degrees or radians, the principles remain consistent, highlighting the universality of these geometric concepts. Ultimately, recognizing the interconnectedness of angles through coterminality enhances our ability to analyze, interpret, and manipulate rotational systems in diverse contexts.

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