

Find The Minimum And Maximum Values For The Function With The Given Domain Interval. $L(x)=e^x$, Given $[4,6]$

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Determining the minimum and maximum values of a function within a specific domain interval is a fundamental concept in calculus, essential for understanding the behavior of functions and solving real-world problems. In this article, we focus on the function $L(x) = e^x$ over the domain interval $[4, 6]$. We will explore how to find the minimum and maximum values of this exponential function on the given interval, including the application of key calculus principles such as critical points and endpoint analysis.

Understanding the Function $L(x) = e^x$

Before diving into the process of finding the extrema, it's important to understand the characteristics of the exponential function e^x .

Properties of e^x

- **Monotonically Increasing:** The function e^x increases continuously as x increases.
- **Always Positive:** For all real numbers x , $e^x > 0$.
- **Smooth and Differentiable:** The derivative of e^x is e^x , which is always positive, reinforcing its increasing nature.

Given these properties, it is intuitive that on any interval, the minimum value of e^x occurs at the smallest x , and the maximum at the largest x , but we will verify this through formal calculus methods.

Step-by-Step Process to Find the Min and Max of $L(x) = e^x$ on $[4,6]$

The process involves evaluating the function at critical points and endpoints

within the domain interval.

1. Identify Critical Points

Critical points occur where the first derivative of the function is zero or undefined. For $L(x) = e^x$:

- Compute the derivative $L'(x)$:

$$L'(x) = \frac{d}{dx} e^x = e^x$$

- Find where $L'(x) = 0$:

$$e^x = 0$$

Since e^x is never zero for any real x , there are no critical points within the interval.

2. Evaluate the Endpoints

Because there are no critical points, the extrema must occur at the endpoints of the interval $[4, 6]$.

- Evaluate $L(4)$:

$$L(4) = e^4$$

- Evaluate $L(6)$:

$$L(6) = e^6$$

Using known values:

$$e^4 \approx 54.5982$$

$$e^6 \approx 403.429$$

3. Determine the Minimum and Maximum Values

Since e^x is increasing over the entire real line, the function's minimum value on $[4, 6]$ is at the lower endpoint $x=4$, and the maximum is at the upper endpoint $x=6$.

Minimum value:

$$\boxed{L_{\min}} = e^4 \approx 54.5982$$

Maximum value:

$$\boxed{L_{\max}} = e^6 \approx 403.429$$

Implications and Applications

Understanding how to find the minimum and maximum of a function like e^x over a specific interval has significant practical implications across

many fields.

Applications in Real-World Scenarios

- **Population Growth Models:** Exponential functions model populations over time, helping identify the lowest and highest populations within a time frame.
- **Financial Forecasting:** Compound interest calculations often involve exponential functions, allowing analysts to determine minimum and maximum investment returns over specific periods.
- **Physics and Engineering:** Exponential decay or growth processes can be analyzed within particular domains to optimize systems or predict behaviors.

Why It's Important to Find Min and Max Values

- These values provide insights into the range or extent of a function's output within a domain.
- They are essential for optimization problems, where the goal is to maximize profit or minimize cost.
- They aid in understanding the behavior of natural phenomena modeled by exponential functions.

Summary and Key Takeaways

- The function $L(x) = e^x$ is strictly increasing over its entire domain.
- In the interval $[4, 6]$, the minimum value of e^x is at $x=4$, which is approximately 54.5982.
- The maximum value occurs at $x=6$, which is approximately 403.429.
- No critical points exist within the interval because the derivative e^x is never zero.
- To find the extrema of continuous functions on closed intervals, evaluate the function at critical points and endpoints; here, only endpoints are relevant.

Conclusion

Finding the minimum and maximum values of a function within a specific domain interval is a foundational skill in calculus, applicable across various

scientific and engineering disciplines. For the function $L(x) = e^x$ over $[4,6]$, the process is straightforward due to the function's monotonic increasing nature. By evaluating the function at the endpoints, we confirm that e^4 is the minimum and e^6 is the maximum within this interval. Mastery of these techniques enables more complex analysis, including optimization and modeling tasks essential in academic and professional contexts.

If you want to explore further, consider how these principles apply to other types of functions, such as quadratic or trigonometric functions, or how to handle functions with multiple critical points within a domain.

Frequently Asked Questions

What is the function we are analyzing for the given domain interval?

The function is $L(x) = e^x$.

What is the domain interval provided for this function?

The domain interval given is $[4, 6]$.

How do we find the minimum and maximum values of $L(x) = e^x$ on $[4, 6]$?

Since e^x is continuous and increasing, its minimum on $[4,6]$ occurs at $x=4$, and maximum at $x=6$.

What are the minimum and maximum values of $L(x) = e^x$ on the interval $[4, 6]$?

The minimum value is e^4 , and the maximum value is e^6 .

Why does the increasing nature of e^x matter in finding the extrema on $[4,6]$?

Because e^x is strictly increasing, its minimum on the interval is at the left endpoint, and maximum at the right endpoint.

Can you compute the approximate numerical values of

the minimum and maximum?

Yes, $e^4 \approx 54.60$ and $e^6 \approx 403.43$, so the minimum is approximately 54.60 and the maximum approximately 403.43.

Additional Resources

Maximum and Minimum Values of the Function $(L(x) = e^x)$ on the Interval $[4, 6]$

Introduction

In the world of calculus and mathematical analysis, determining the minimum and maximum values of a function over a specific interval is a fundamental task. These values, known as the extrema of the function, provide critical insights into the behavior of the function within the domain. When analyzing exponential functions such as $(L(x) = e^x)$, the process involves understanding the function's inherent properties, especially its monotonicity, and applying the Extreme Value Theorem.

This article delves into a comprehensive exploration of how to find the minimum and maximum values of $(L(x) = e^x)$ over the domain interval $([4, 6])$. Whether you're a student grasping the basics or a professional seeking clarity, this detailed examination aims to clarify the methodology, reasoning, and mathematical principles involved in this classical problem.

Understanding the Function $(L(x) = e^x)$

Before calculating the extrema, it's crucial to understand the nature of the function:

- Exponential Function: $(L(x) = e^x)$ is a classic exponential function, where (e) is Euler's number, approximately 2.71828.
- Monotonicity: The function is strictly increasing over the entire real line, meaning that as (x) increases, (e^x) increases without exception.
- Differentiability: (e^x) is smooth and differentiable everywhere, with the derivative $(L'(x) = e^x)$.

These properties suggest that within any interval, the maximum value will be at the right endpoint, and the minimum at the left endpoint, since the function increases monotonically.

Step-by-Step Approach to Finding Extrema

To systematically find the minimum and maximum of $L(x) = e^x$ on $[4, 6]$, we follow these steps:

1. Verify the Function's Continuity and Differentiability

According to the Extreme Value Theorem, a continuous function on a closed interval attains both its minimum and maximum values somewhere within that interval. Since e^x is continuous and differentiable everywhere, the theorem applies directly.

2. Determine Critical Points

Critical points are where the derivative is zero or undefined. For e^x :

- $L'(x) = e^x$
- Since e^x is never zero and never undefined, there are no critical points within the open interval $(4, 6)$.

This is a key insight: the function does not have any internal maxima or minima within the interval because it's strictly increasing.

3. Evaluate Endpoints

Because there are no critical points inside $[4, 6]$, the extrema must occur at the endpoints:

- At $x = 4$
- At $x = 6$

Calculating the function's value at these points:

- $L(4) = e^4$
- $L(6) = e^6$

Using known approximations:

- $e^4 \approx 54.5982$
- $e^6 \approx 403.429$

Final Results: Minimum and Maximum Values

Based on the above analysis:

- Minimum value: $L(4) = e^4 \approx 54.5982$
- Maximum value: $L(6) = e^6 \approx 403.429$

Since the function is monotonically increasing, the minimum on $[4, 6]$ is at the starting point $x=4$, and the maximum is at the endpoint $x=6$.

Additional Insights and Practical Considerations

While the theoretical approach is straightforward for monotonic functions like (e^x) , more complex functions require a thorough examination of critical points, potential points of discontinuity, and the behavior at infinity.

Key aspects to consider in general cases include:

- Existence of Critical Points: Solving $(f'(x) = 0)$ to find potential internal extrema.
- Boundary Values: Always evaluate at interval endpoints, especially when the function is monotonic or when critical points are absent.
- Function Behavior: For non-monotonic functions, multiple critical points may exist; in such cases, compare all critical points and endpoints.

In the specific case of (e^x) :

- The simplicity of the exponential function's derivative eliminates the need for complex calculations.
- Its strict monotonicity guarantees that the extrema are at the endpoints.
- This insight simplifies analysis, making exponential functions a favorite for illustrating calculus principles.

Summary

Aspect	Details
Function	$(L(x) = e^x)$
Domain	$([4, 6])$
Critical Points	None (since (e^x) is always increasing)
Min Value	$(e^4 \approx 54.5982)$ at $(x=4)$
Max Value	$(e^6 \approx 403.429)$ at $(x=6)$

In conclusion, the extremal values of $(L(x) = e^x)$ over $([4, 6])$ are straightforward to determine due to the function's properties. The minimum occurs at the lower boundary, $(x=4)$, and the maximum at the upper boundary, $(x=6)$. This example exemplifies the elegance of calculus in analyzing exponential functions and highlights the importance of understanding the function's behavior in simplifying extremum calculations.

Final Thoughts

This exploration underscores a fundamental principle: the nature of the function dictates the approach. For strictly increasing or decreasing

functions, evaluating endpoints suffices. For more complex functions, critical point analysis becomes essential. Mastering these techniques enhances problem-solving skills and deepens understanding of calculus, laying a solid foundation for tackling advanced mathematical challenges.

Whether you're preparing for exams, working on real-world modeling, or simply sharpening your mathematical intuition, recognizing the properties of functions like e^x streamlines the process of finding extrema, turning what might seem like complex calculations into straightforward exercises.

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