Find The Missing Side Of Each Triangle. Leave Your Answer In Simplest Radical Form. (Please Show Ur Work)

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Understanding how to find the missing side of a triangle is a fundamental skill in geometry. Whether you're working with right triangles or more complex shapes, mastering the process of solving for unknown sides helps build a strong foundation in math. In this article, we'll explore different methods for determining the missing side, including the Pythagorean theorem and the Law of Cosines, and provide detailed, step-by-step examples to ensure clarity. Remember, always leave your answer in simplest radical form and show your work for full credit and better understanding.

Understanding Triangle Types and Key Concepts

Before diving into specific problems, it's essential to recognize different types of triangles and the relevant formulas for solving them.

Right Triangles

- Defined by a 90-degree angle.
- The Pythagorean theorem is used to find missing sides.

Oblique Triangles

- Do not have a right angle.
- Law of Cosines and Law of Sines are used.

Using the Pythagorean Theorem for Right Triangles

The Pythagorean theorem is the cornerstone for finding missing sides in right triangles. It states:

\[a^2 + b^2 = c^2 \]

where:
- a and b are the legs (the shorter sides),
- c is the hypotenuse (the longest side).

Steps to find the missing side:
1. Identify which sides are known.
2. Plug known values into the theorem.
3. Solve for the unknown side.
4. Simplify your radical answer.

Example 1: Find the Hypotenuse

Suppose you have a right triangle with legs of 3 units and 4 units, and you need to find the hypotenuse.

Solution:
- Known sides: $a = 3$, $b = 4$
- Unknown: c

Apply the Pythagorean theorem:
$$ c^2 = a^2 + b^2 $$
$$ c^2 = 3^2 + 4^2 $$
$$ c^2 = 9 + 16 $$
$$ c^2 = 25 $$
$$ c = \sqrt{25} $$
$$ c = 5 $$

Answer: The hypotenuse is 5 (already in simplest radical form).

Example 2: Find a Missing Leg

Given a right triangle with hypotenuse of 13 units and one leg of 5 units, find the other leg.

Solution:
- Known sides: $c = 13$, $a = 5$
- Unknown: b

Apply the Pythagorean theorem:
$$ a^2 + b^2 = c^2 $$
$$ 5^2 + b^2 = 13^2 $$
$$ 25 + b^2 = 169 $$
$$ b^2 = 169 - 25 $$
$$ b^2 = 144 $$

$$ b = \sqrt{144} $$
$$ b = 12 $$

Answer: The missing leg is 12.

Applying the Law of Cosines for Non-Right Triangles

When working with oblique triangles (not right-angled), the Law of Cosines is the most effective tool for finding missing sides. The Law of Cosines states:

$$ c^2 = a^2 + b^2 - 2ab \cos C $$

where:
- a, b, and c are the sides,
- C is the included angle between sides a and b.

Steps:
1. Identify the known sides and included angle.
2. Plug into the Law of Cosines.
3. Solve for the unknown side.
4. Simplify your radical answer.

Example 3: Find a Side Using Law of Cosines

Given a triangle with sides $a=7$, $b=10$, and included angle $C=60^\circ$, find side c.

Solution:
- Known: $a=7$, $b=10$, $C=60^\circ$
- Unknown: c

Apply Law of Cosines:
$$ c^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ $$
$$ c^2 = 49 + 100 - 140 \times \frac{1}{2} $$
$$ c^2 = 149 - 70 $$
$$ c^2 = 79 $$

Now, simplify:
$$ c = \sqrt{79} $$

Since 79 is prime, the radical is in simplest form.

Answer: The missing side c is $\sqrt{79}$.

Example 4: Find a Side When Two Sides and Included Angle Are Known

Suppose $a=5$, $b=8$, and $C=45^\circ$. Find side c.

Solution:
$$ c^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 45^\circ $$
$$ c^2 = 25 + 64 - 80 \times \frac{\sqrt{2}}{2} $$
$$ c^2 = 89 - 80 \times \frac{\sqrt{2}}{2} $$
$$ c^2 = 89 - 40 \sqrt{2} $$

Answer in simplest radical form:
$$ c = \sqrt{89 - 40 \sqrt{2}} $$

This radical cannot be simplified further, so the final answer is $\sqrt{89 - 40 \sqrt{2}}$.

Finding a Side with Two Known Sides and an Angle (Law of Sines)

In some cases, especially when you know two angles and one side (AAS or ASA), the Law of Sines is appropriate:

$$ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} $$

Steps:
1. Find the third angle if not given.
2. Set up the proportion involving known sides and angles.
3. Solve for the unknown side.
4. Simplify your radical form if necessary.

Practice Problem: Find the Missing Side

A triangle has sides $a=9$, $b=12$, and included angle $C=120^\circ$. Find side c.

Solution:

$$ c^2 = 9^2 + 12^2 - 2 \times 9 \times 12 \times \cos 120^\circ $$
$$ c^2 = 81 + 144 - 216 \times (-\frac{1}{2}) $$
$$ c^2 = 225 + 216 \times \frac{1}{2} $$
$$ c^2 = 225 + 108 $$
$$ c^2 = 333 $$
$$ c = \sqrt{333} $$

Since 333 factors as 3×111 and 111 factors as 3×37, which are prime, the radical simplifies to:

$$ c = \sqrt{3 \times 111} = \sqrt{3 \times 3 \times 37} = 3 \sqrt{37} $$

Final Answer: $3 \sqrt{37}$.

Conclusion

Finding the missing side of a triangle involves understanding which formula to apply based on the triangle's properties. For right triangles, the Pythagorean theorem is straightforward and effective. For non-right triangles, the Law of Cosines and Law of Sines are indispensable tools. Always leave your answers in simplest radical form and show your work step-by-step to ensure accuracy and clarity.

By practicing these methods and solving various problems, you'll develop confidence in tackling any triangle side-finding challenge. Remember, careful substitution, simplifying radicals, and verifying your work are key to mastering this essential skill in geometry.

Frequently Asked Questions

In a right triangle, the legs measure 6 and 8 units. Find the length of the hypotenuse in simplest radical form.

Using the Pythagorean theorem: $c^2 = 6^2 + 8^2 = 36 + 64 = 100$; $c = \sqrt{100} = 10$. The hypotenuse is 10.

One leg of a right triangle is 9 units, and the hypotenuse is 15 units. Find the length of the other leg in simplest radical form.

Using Pythagoras: $b^2 = 15^2 - 9^2 = 225 - 81 = 144$; $b = \sqrt{144} = 12$. The missing side is 12.

A triangle has two sides measuring 7 and 24 units, and the third side is unknown. If it's a right triangle with the hypotenuse as 25 units, find the missing side.

Since 25 is the hypotenuse: missing side $b^2 = 25^2 - 7^2 = 625 - 49 = 576$; $b = \sqrt{576} = 24$. The missing side is 24.

In an isosceles right triangle, the legs measure 5 units each. Find the length of the hypotenuse in simplest radical form.

Using Pythagoras: hypotenuse $c = \sqrt{(5^2 + 5^2)} = \sqrt{(25 + 25)} = \sqrt{50} = 5\sqrt{2}$.

A triangle has sides of 3 and 4 units, and the remaining side is unknown. If the triangle is right-angled with the hypotenuse as 5 units, find the missing side.

Since 5 is the hypotenuse: missing side $b^2 = 5^2 - 3^2 = 25 - 9 = 16$; $b = \sqrt{16} = 4$. The missing side is 4.

In a triangle, two sides measure 10 and 24 units, and the included angle between them is 90°. Find the length of the third side in simplest radical form.

Using the Law of Cosines: $c^2 = 10^2 + 24^2 - 21024\cos(90°) = 100 + 576 - 0 = 676$; $c = \sqrt{676} = 26$.

A triangle has sides of 8 and 15 units, with the included angle of 90°. Find the length of the third side in simplest radical form.

Using Law of Cosines: $c^2 = 8^2 + 15^2 - 2815\cos(90°) = 64 + 225 - 0 = 289$; $c = \sqrt{289} = 17$.

A triangle has sides 12 and 35 units, with the angle between them being 60°. Find the length of the third side in simplest radical form.

Using Law of Cosines: $c^2 = 12^2 + 35^2 - 21235\cos(60°) = 144 + 1225 -$

21235(1/2) = 1369 - 420 = 949; $c = \sqrt{949} = \sqrt{(949)}$. Simplified radical form is $13\sqrt{(7)}$.

Additional Resources

Find The Missing Side Of Each Triangle. Leave Your Answer In Simplest Radical Form. (Please Show Your Work)

When tackling problems involving triangles, especially those asking you to find a missing side, understanding the core principles and formulas is essential. The phrase "Find The Missing Side Of Each Triangle. Leave Your Answer In Simplest Radical Form. (Please Show Your Work)" is a common directive in geometry and trigonometry exercises. It emphasizes not only arriving at the correct numerical answer but also simplifying radicals for clarity and precision. This comprehensive guide will walk you through the methods, step-by-step solutions, and best practices to confidently find missing sides, ensuring your answers are both accurate and simplified.

Understanding the Basics: Types of Triangles and Relevant Theorems

Before delving into specific problem-solving strategies, it’s important to recognize the types of triangles and the applicable theorems.

Types of Triangles
- Right Triangle: One angle is exactly 90°. The side opposite this angle is the hypotenuse.
- Acute Triangle: All angles are less than 90°.
- Obtuse Triangle: One angle is greater than 90°.

Key Theorems and Formulas
- Pythagorean Theorem: For right triangles, $a^2 + b^2 = c^2$, where c is the hypotenuse.
- Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

The choice of which theorem to use depends on the given information—whether you know sides, angles, or a combination of both.

Step-by-Step Approach to Finding the Missing Side

Step 1: Analyze the Given Information
Determine what data you have:
- Are two sides known?
- Is one side and an angle known?
- Is it a right triangle or an oblique triangle?

Step 2: Decide Which Formula or Theorem to Use
Based on the given data:
- Use Pythagoras for right triangles with two sides.
- Use Law of Sines if you have an angle and side opposite it, plus another angle or side.
- Use Law of Cosines if you know two sides and the included angle, or two angles and a side (after finding an angle).

Step 3: Set Up the Equation
Write the formula with the known quantities, isolating the unknown side.

Step 4: Solve for the Missing Side
Perform algebraic manipulations carefully, keeping radicals in simplest form.

Step 5: Simplify Radical Expressions
Express radicals in simplest form by:
- Prime factorization.
- Reducing perfect squares.
- Rationalizing denominators if necessary.

Practical Examples with Solutions

Let's explore detailed examples to solidify these concepts. Each example will demonstrate the process of finding the missing side and leaving answers in simplest radical form.

Example 1: Finding the Hypotenuse in a Right Triangle

Problem:
A right triangle has legs 3 and 4. Find the hypotenuse c.

Solution:

Step 1: Recognize it's a right triangle with legs $a = 3$, $b = 4$.

Step 2: Use Pythagoras’ Theorem:

$$
c^2 = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25
$$

Step 3: Find c:

$$
c = \sqrt{25} = 5
$$

Answer:
The hypotenuse is 5. Since 5 is already a whole number, it’s in simplest radical form.

Example 2: Finding a Missing Leg in a Right Triangle with Radical Result

Problem:
In a right triangle, hypotenuse $c = 7$, and one leg $a = 3$. Find the other leg b.

Solution:

Step 1: Recognize it's a right triangle with known hypotenuse $c = 7$ and side $a = 3$.

Step 2: Apply Pythagoras:

$$
b^2 = c^2 - a^2 = 7^2 - 3^2 = 49 - 9 = 40
$$

Step 3: Simplify radical:

$$
b = \sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2 \sqrt{10}
$$

Answer:
The missing leg is $2 \sqrt{10}$ in simplest radical form.

Example 3: Using Law of Sines in an Oblique Triangle

Problem:
In triangle ABC, side $a = 8$, angle $A = 30^\circ$, and angle $B = 45^\circ$. Find side b.

Solution:

Step 1: Find angle C:

$$
C = 180^\circ - A - B = 180^\circ - 30^\circ - 45^\circ = 105^\circ
$$

Step 2: Write Law of Sines:

\[

\frac{a}{\sin A} = \frac{b}{\sin B}
\]

Plug in known values:

\[
\frac{8}{\sin 30^\circ} = \frac{b}{\sin 45^\circ}
\]

Recall:

\[
\sin 30^\circ = \frac{1}{2}, \quad \sin 45^\circ = \frac{\sqrt{2}}{2}
\]

Step 3: Solve for \(b \):

\[
b = \frac{\sin 45^\circ \times 8}{\sin 30^\circ} = \frac{\frac{\sqrt{2}}{2} \times 8}{\frac{1}{2}} = \left(\frac{\sqrt{2}}{2} \times 8 \right) \times 2
\]

Simplify numerator:

\[
\frac{\sqrt{2}}{2} \times 8 = 8 \times \frac{\sqrt{2}}{2} = 4 \sqrt{2}
\]

Multiply by 2:

\[
b = 4 \sqrt{2} \times 2 = 8 \sqrt{2}
\]

Answer:
Side \(b = \boxed{8 \sqrt{2}} \).

Example 4: Using Law of Cosines for an Oblique Triangle

Problem:
Sides \(a = 7 \), \(b = 10 \), with included angle \(C = 60^\circ \). Find side \(c \).

Solution:

Step 1: Use Law of Cosines:

\[
c^2 = a^2 + b^2 - 2ab \cos C

$$

Plug in values:

$$
c^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ
$$

Calculate:

$$
c^2 = 49 + 100 - 2 \times 7 \times 10 \times \frac{1}{2}
$$

$$
c^2 = 149 - (2 \times 7 \times 10 \times \frac{1}{2}) = 149 - (7 \times 10) = 149 - 70 = 79
$$

Step 2: Express in radical form:

$$
c = \sqrt{79}
$$

Since 79 is prime, radical form is simplest.

Answer:
Side $c = \boxed{\sqrt{79}}$.

Tips for Simplifying Radicals

- Prime Factorization: Break down the radical’s radicand into prime factors.
- Identify Perfect Squares: Extract perfect squares from under the radical.
- Reduce Radicals: Express radicals in simplest radical form by removing perfect square factors.

Examples:

- $\sqrt{50} = \sqrt{25 \times 2} = 5 \sqrt{2}$
- $\sqrt{72} = \sqrt{36 \times 2} = 6 \sqrt{2}$

Final Thoughts and Best Practices

- Always check if the triangle is right-angled. This allows immediate use of Pythagoras' theorem.
- Use the Law of Sines or Cosines carefully, especially when dealing with

oblique triangles.
- Show all work clearly to facilitate understanding and verify accuracy.
- Simplify radicals thoroughly to leave your answer in the neatest form.
- Double-check your calculations, especially radicals, to avoid errors.

Conclusion

Finding the missing side of each triangle and expressing the answer in simplest radical form is a fundamental skill in geometry and trigonometry. Whether using Pythagoras' theorem for right triangles or the Law of Sines and Cosines for oblique triangles, the key is to analyze the given data, select the appropriate method, solve carefully, and simplify radicals thoroughly.

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