Find The Points Of Intersection Of The Graphs Of The Functions. F(x) = X^2 3x + 9; G(x) = 9/2x + 5/2(x,y)

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Understanding how to find the points of intersection of two functions is a fundamental skill in algebra and calculus. When analyzing the functions $F(x) = x^2 + 3x + 9$ and $G(x) = (9/2)x + (5/2)$, determining where their graphs intersect helps us understand where the two equations share common solutions. This article provides a comprehensive guide to finding these intersection points, exploring the underlying concepts, step-by-step methods, and practical applications.

Understanding the Functions Involved

Before diving into the process of finding intersection points, it is essential to understand the functions involved.

Function F(x): Quadratic Function

The function $F(x) = x^2 + 3x + 9$ is a quadratic function, which graphs as a parabola opening upwards since the coefficient of x^2 is positive. Key features of this parabola include:

- Vertex: The highest or lowest point of the parabola.
- Axis of symmetry: The vertical line that passes through the vertex.
- Y-intercept: The point where the graph crosses the y-axis.

Function G(x): Linear Function

The function $G(x) = (9/2)x + (5/2)$ is a linear function graphing as a straight line. Its characteristics include:

- Slope: 9/2, indicating the steepness of the line.
- Y-intercept: 5/2, indicating where the line crosses the y-axis.
- Direction: The line rises as x increases because the slope is positive.

Methods to Find the Intersection Points

To find the points where the graphs of $F(x)$ and $G(x)$ intersect, we need to find the solutions to the equation where $F(x) = G(x)$. This involves setting the two functions equal to each other and solving for x.

Step 1: Set the Functions Equal

Start with the equation:

```
x^2 + 3x + 9 = (9/2)x + (5/2)
```

This equation represents the x-values where the two graphs intersect.

Step 2: Rearrange the Equation

Bring all terms to one side to form a quadratic equation:

```
x^2 + 3x + 9 - (9/2)x - (5/2) = 0
```

Combine like terms:

```
x^2 + (3x - (9/2)x) + (9 - 5/2) = 0
```

Calculate the coefficients:

- $3x - (9/2)x = (6/2)x - (9/2)x = (-3/2)x$
- $9 - 5/2 = (18/2) - (5/2) = (13/2)$

So, the quadratic equation simplifies to:

```
x^2 - (3/2)x + (13/2) = 0
```

Step 3: Solve the Quadratic Equation

Use the quadratic formula:

```
x = [-b ± √(b^2 - 4ac)] / 2a
```

Identify coefficients:

- $a = 1$
- $b = -3/2$

- c = 13/2

Calculate the discriminant:

$$\Delta = b^2 - 4ac = (-3/2)^2 - 4 \cdot 1 \cdot (13/2) = (9/4) - (52/2) = (9/4) - (26) = (9/4) - (104/4) = -95/4$$

Since the discriminant is negative (*Δ* = -95/4), there are no real solutions. This indicates the graphs do not intersect at any real point. However, in the context of complex solutions, we can find the complex intersection points.

Interpreting the Results

Given that the discriminant is negative, the graphs of F(x) and G(x) do not cross each other in the real coordinate plane. They are either entirely separate or tangent at complex points.

Real vs. Complex Intersection Points

- **Real Intersection Points:** Solutions with real x-values where the graphs meet.
- **Complex Intersection Points:** Solutions with imaginary components when the discriminant is negative.

Finding Complex Intersection Points

When the discriminant is negative, solutions involve complex numbers:

$$x = [-b \pm i\sqrt{|\Delta|}] / 2a$$

Calculating:

$$x = [3/2 \pm i\sqrt{(95/4)}] / 2 = [3/2 \pm i \cdot (\sqrt{95})/2] / 2$$

Simplify:

$$x = (3/2) / 2 \pm i \cdot (\sqrt{95})/2 / 2 = (3/4) \pm i \cdot (\sqrt{95})/4$$

Corresponding y-values can be found by substituting these complex x-values into either F(x) or G(x), typically G(x) for simplicity.

Practical Applications of Finding Intersection Points

Understanding how to find the points of intersection between functions has numerous practical applications:

1. Engineering and Physics

- Determining where two forces intersect or balance.
- Designing structures where stress or load functions intersect.

2. Economics and Business

- Identifying equilibrium points where supply equals demand.
- Analyzing cost and revenue functions to maximize profit.

3. Computer Graphics and Data Visualization

- Rendering graphs accurately by finding intersection points.
- Collision detection in game development.

Summary and Key Takeaways

- To find the intersection points of the functions $F(x) = x^2 + 3x + 9$ and $G(x) = (9/2)x + (5/2)$, set them equal and solve for x.
- The resulting quadratic may have real or complex solutions, depending on the discriminant.
- In our example, the discriminant is negative, indicating no real intersections but complex solutions.
- Complex solutions involve imaginary numbers and are essential in advanced mathematics and physics.
- The process involves algebraic manipulation, quadratic formula application, and interpretation of results.

Conclusion

Mastering how to find the points of intersection between different types of functions is a vital algebraic skill that extends to many real-world scenarios. Whether dealing with quadratic and linear functions or more complex relationships, understanding the methods to identify where their graphs meet allows for better analysis, prediction, and problem-solving across various disciplines. Remember, the key steps involve setting the functions equal, simplifying the resulting equation, and applying the quadratic formula or other solution methods. With practice, identifying intersection points becomes an intuitive part of analyzing mathematical models and their applications.

For further practice, consider exploring functions with different shapes and intersection behaviors, and utilize graphing calculators or software to visualize the solutions!

Frequently Asked Questions

How do I find the points of intersection between the functions $F(x) = x^2 - 3x + 9$ and $G(x) = (9/2)x + (5/2)$?

Set F(x) equal to G(x), which gives $x^2 - 3x + 9 = (9/2)x + (5/2)$. Then, solve the resulting quadratic equation for x, and substitute those x-values into either function to find the corresponding y-values.

What steps are involved in solving for intersection points of quadratic and linear functions?

First, set the two functions equal to each other. Next, clear fractions if necessary, then rearrange into standard quadratic form. Solve for x using factoring, completing the square, or the quadratic formula. Finally, find y by plugging x-values back into either original function.

Can I graph both functions to visually find their intersection points?

Yes, graphing both functions on the same coordinate plane can help visually identify the intersection points. The points where the graphs cross are the solutions, which can then be verified algebraically.

What is the quadratic equation I get when solving for the intersection points of $F(x) = x^2 - 3x + 9$ and $G(x) = (9/2)x + (5/2)$?

Subtract G(x) from F(x): $x^2 - 3x + 9 - [(9/2)x + (5/2)] = 0$. Simplifying, you get $x^2 - (3 + 9/2)x + (9 - 5/2) = 0$, which simplifies further to $x^2 - (15/2)x + (13/2) = 0$.

How do I solve the quadratic equation $x^2 - (15/2)x + (13/2) =$

0?

Use the quadratic formula: $x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$, where $a=1$, $b= -15/2$, and $c= 13/2$. Calculate the discriminant and then find the roots accordingly.

What are the approximate intersection points of the functions F(x) and G(x)?

By solving the quadratic, you get the x-values. Substituting these back into either function gives the y-values. The exact points depend on the roots found from the quadratic formula.

Are there cases where quadratic and linear functions do not intersect?

Yes. If the quadratic and linear functions are tangent or do not cross at any point (discriminant less than zero), then there are no real intersection points. In this case, the discriminant will determine the number of real solutions.

Additional Resources

Find The Points Of Intersection Of The Graphs Of The Functions. $F(x) = x^2 - 3x + 9$; $G(x) = (9/2)x + (5/2)$

Understanding how to find the points of intersection of the graphs of functions is a fundamental skill in algebra and calculus. These points reveal where two functions meet on the coordinate plane, offering insights into their relationships, solutions to equations, and real-world applications such as optimization and modeling. In this guide, we'll explore a detailed, step-by-step approach to determine the points of intersection between the quadratic function $F(x) = x^2 - 3x + 9$ and the linear function $G(x) = \frac{9}{2}x + \frac{5}{2}$.

Introduction: Why Find Points of Intersection?

Before diving into the calculations, it’s essential to understand the significance of points of intersection:

- Solve systems of equations: Finding where two functions meet is equivalent to solving a system of equations.
- Graphical interpretation: The intersection points are where the graphs of the functions cross.
- Applications: In real-world contexts, intersection points can represent solutions like equilibrium points in economics, collision points in physics, or optimal solutions in engineering.

Step 1: Understand the Given Functions

Let's analyze the functions we're working with:

Function $F(x) = x^2 - 3x + 9$

- This is a quadratic function, which graphs as a parabola.
- The parabola opens upwards since the coefficient of x^2 is positive.
- The vertex form can be found to understand its shape better, but for intersection purposes, standard form suffices.

Function $G(x) = \frac{9}{2}x + \frac{5}{2}$

- This is a linear function.
- Its graph is a straight line with slope $\frac{9}{2}$ and y-intercept $\frac{5}{2}$.

Step 2: Set the Functions Equal to Find Intersection Points

To find the points where their graphs intersect, set $F(x)$ equal to $G(x)$:

$$
x^2 - 3x + 9 = \frac{9}{2}x + \frac{5}{2}
$$

This equation represents the x-coordinates where the graphs meet. Solving for x involves rearranging and simplifying.

Step 3: Solve the Equation for x

Step 3.1: Move all terms to one side

$$
x^2 - 3x + 9 - \frac{9}{2}x - \frac{5}{2} = 0
$$

Step 3.2: Combine like terms

First, combine the x terms:

$$
-3x - \frac{9}{2}x
$$

Express $-3x$ as $-\frac{6}{2}x$ to combine with $-\frac{9}{2}x$:

$$
-\frac{6}{2}x - \frac{9}{2}x = -\frac{15}{2}x
$$

Next, combine the constant terms:

$$
9 - \frac{5}{2}
$$

Express 9 as $\frac{18}{2}$:

$$
\frac{18}{2} - \frac{5}{2} = \frac{13}{2}
$$

Now, the quadratic equation simplifies to:

$$
x^2 - \frac{15}{2}x + \frac{13}{2} = 0
$$

Step 3.3: Eliminate fractions by multiplying through by 2

Multiply every term by 2 to clear denominators:

$$
2x^2 - 15x + 13 = 0
$$

This is a standard quadratic equation.

Step 4: Use the Quadratic Formula

The quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=2$, $b=-15$, and $c=13$.

Step 4.1: Calculate discriminant

$$
D = b^2 - 4ac = (-15)^2 - 4 \times 2 \times 13 = 225 - 104 = 121
$$

Since $D=121 > 0$, there are two real solutions, meaning the graphs intersect at two points.

Step 4.2: Compute the roots

$$
x = \frac{-(-15) \pm \sqrt{121}}{2 \times 2} = \frac{15 \pm 11}{4}
$$

Calculate both solutions:

- First root:

$$
x = \frac{15 + 11}{4} = \frac{26}{4} = \frac{13}{2} = 6.5
$$

- Second root:

$$
x = \frac{15 - 11}{4} = \frac{4}{4} = 1
$$

Step 5: Find Corresponding y-Values

Now, substitute each x-value back into either original function to find the corresponding y-coordinate.

For $x=6.5$:

Using $G(x) = \frac{9}{2}x + \frac{5}{2}$:

$$
G(6.5) = \frac{9}{2} \times 6.5 + \frac{5}{2}
$$

Calculate:

$$
\frac{9}{2} \times 6.5 = \frac{9}{2} \times \frac{13}{2} = \frac{9 \times 13}{4} = \frac{117}{4}
$$

Add $\frac{5}{2}$:

$$
\frac{117}{4} + \frac{5}{2} = \frac{117}{4} + \frac{10}{4} = \frac{127}{4}
$$

Express as a decimal:

$$
\frac{127}{4} = 31.75
$$

So, the point is:

$$
\boxed{\left(6.5, 31.75\right)}
$$

For $x=1$:

Using the same $G(x)$:

$$
G(1) = \frac{9}{2} \times 1 + \frac{5}{2} = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7
$$

Thus, the second intersection point is:

$$
\boxed{\left(1, 7\right)}
$$

Final Result:

The points of intersection of the graphs of $F(x) = x^2 - 3x + 9$ and $G(x) = \frac{9}{2}x + \frac{5}{2}$ are:

- $\boxed{\left(1, 7\right)}$
- $\boxed{\left(6.5, 31.75\right)}$

Additional Insights and Applications

Visual Interpretation

Plotting these functions confirms that they intersect at the calculated points. The parabola $F(x)$ opens upward, with the line $G(x)$ crossing it at two points. Understanding where this occurs helps in grasping the behavior of quadratic and linear functions together.

Real-life Contexts

- Physics: Intersection points may represent moments where two moving objects occupy the same position.
- Economics: Equilibrium points where supply and demand curves intersect.
- Engineering: Points where different system responses meet on a graph.

Extending the Analysis

- Vertex of the parabola: To understand the shape better, find the vertex of $F(x)$. The vertex x-coordinate is $-\frac{b}{2a} = -\frac{-3}{2} = 1.5$. The vertex point is at:

$$
F(1.5) = (1.5)^2 - 3(1.5) + 9 = 2.25 - 4.5 + 9 = 6.75
$$

- Comparing the parabola and line: Since the line intersects the parabola at two points, the parabola is above the line between those points or vice versa, depending on the parabola's vertex.

Conclusion

Finding the points of intersection of two functions is an essential process in algebra, providing valuable insights into their relationship and behavior. By setting the functions equal, simplifying the resulting equation, applying the quadratic formula, and substituting back to find corresponding y-values, you can precisely determine where and how two graphs cross. The specific example with $F(x) = x^2 - 3x + 9$ and $G(x) = \frac{9}{2}x + \frac{5}{2}$ illustrates this process clearly and demonstrates the importance of algebraic manipulation and understanding quadratic and linear functions in tandem.

Whether you're tackling academic problems or applying these concepts in real-world scenarios, mastering the method of finding points of intersection empowers you to analyze complex relationships between functions confidently.

Find The Points Of Intersection Of The Graphs Of The Functions F X X 2 3x 9 G X 9 2x 5 2 X Y

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