Find The Volume Of The Solid Generated By Revolving The Region Bounded By The Graphs Of Y=x +8 And Y=x+14

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Understanding how to find the volume of a solid generated by revolving a region around an axis is a fundamental concept in calculus. In this article, we will explore the process step-by-step to determine the volume of the solid formed when the region bounded by the lines Y = x + 8 and Y = x + 14 is revolved about a specified axis. This problem provides a great opportunity to review the methods of disks and washers, as well as the application of definite integrals in volume calculations.

Understanding the Region Bounded by the Graphs

Graphs of the Lines Y = x + 8 and Y = x + 14

The two lines in question are:

- Y = x + 8
- Y = x + 14

These are straight lines with the same slope (1), but different y-intercepts (8 and 14). Graphically, they are parallel lines that run diagonally across the coordinate plane.

Identifying the Bounded Region

Since both lines are parallel, the region bounded between them at any x-value is a strip of the plane with height:

$$ \text{Height} = (x + 14) - (x + 8) = 6 $$

This indicates that at every x along the region, the vertical distance between the two lines is constant at 6 units.

Determining the Limits of Integration

To find the volume, we need to establish the interval over which the region extends. Since these lines are unbounded in the x-direction, we need to define the interval based on the problem context or choose specific bounds for demonstration.

For simplicity, let’s assume the region is bounded between x = a and x = b, where a and b

are finite. For example, consider the region between x = 0 and x = 10.

Choosing the Axis of Revolution

The problem specifies the region bounded by the lines, but the axis of revolution is not explicitly given. Common axes include:

- The x-axis (horizontal axis)
- The y-axis (vertical axis)
- Any vertical or horizontal line

For this example, assume the region is revolved around the x-axis, which is a common scenario in volume problems.

Applying the Disk and Washer Method

Understanding the Method

When a region is revolved about an axis, the volume can be computed using the disk or washer method:

- Disk Method: Used when the region is revolved around an axis that is perpendicular to the cross-sectional slices, and the slices are disks.
- Washer Method: Used when the region has a hole in the middle (like a washer), often when revolving around lines not coinciding with the boundary of the region.

Since our region is between two lines, and we're revolving around the x-axis, the washer method is appropriate because the region is between two curves, creating a hollow shape (a "washer") in the solid.

Formulating the Volume Integral

For revolution around the x-axis, the volume V can be found using:

$$
V = \pi \int_{a}^{b} \left[R_{\text{outer}}^2 - R_{\text{inner}}^2 \right] dx
$$

where:
- R_{outer} is the distance from the x-axis to the outer boundary of the region.
- R_{inner} is the distance from the x-axis to the inner boundary of the region.

In our case:
- The upper boundary (top line): $y = x + 14$
- The lower boundary (bottom line): $y = x + 8$

When revolving around the x-axis, the outer radius corresponds to the maximum y-value at each x, which is $y = x + 14$, and the inner radius corresponds to the minimum y-value, $y = x + 8$.

Thus:

$$
V = \pi \int_{a}^{b} \left[(x + 14)^2 - (x + 8)^2 \right] dx
$$

Calculating the Volume Step-by-Step

Step 1: Set the Limits of Integration

Assuming the region is between $x = 0$ and $x = 10$, we have:

$$
a = 0, \quad b = 10
$$

Step 2: Write the Integral Expression

$$
V = \pi \int_{0}^{10} \left[(x + 14)^2 - (x + 8)^2 \right] dx
$$

Step 3: Expand the Squares

$$
(x + 14)^2 = x^2 + 28x + 196
$$

$$
(x + 8)^2 = x^2 + 16x + 64
$$

Subtracting:

$$
(x + 14)^2 - (x + 8)^2 = (x^2 + 28x + 196) - (x^2 + 16x + 64) = (28x - 16x) + (196 - 64) = 12x + 132
$$

Step 4: Simplify the Integral

$$
V = \pi \int_{0}^{10} (12x + 132) dx

\]

Step 5: Integrate

\[
V = \pi \left[6x^2 + 132x \right]_0^{10}
\]

Calculating at the bounds:

At \(x = 10 \):

\[
6 \times 10^2 + 132 \times 10 = 6 \times 100 + 1320 = 600 + 1320 = 1920
\]

At \(x = 0 \):

\[
0 + 0 = 0
\]

Therefore:

\[
V = \pi (1920 - 0) = 1920 \pi
\]

Final Result:

\[
\boxed{
\text{The volume of the solid generated by revolving the region between } y = x + 8 \text{ and } y = x + 14 \text{ from } x=0 \text{ to } x=10 \text{ about the x-axis is } 1920 \pi
}
\]

Extensions and Generalizations

While this example considers specific bounds, the method generalizes to any interval \([a, b]\). To find the volume for a different interval, simply replace the limits in the integral.

Additionally, if the axis of revolution changes—such as revolving around the y-axis or a vertical line—the integral setup adjusts accordingly, often involving horizontal slices or different radii.

Key Takeaways for Calculus Students

- Recognize the shape of the boundary region using the equations of the curves.
- Choose the appropriate method (disk, washer, shell) based on the axis of revolution.
- Set correct limits of integration based on the bounded region.
- Expand and simplify integrands carefully to facilitate integration.
- Remember that the volume formula involves the difference of squared radii, especially when dealing with washers.

Conclusion

Finding the volume of a solid generated by revolving a region bounded by two lines is a fundamental skill in integral calculus. By understanding the geometry of the region, selecting the correct method, and carefully setting up the integral, you can accurately compute the volume. In our example, revolving the region between $y = x + 8$ and $y = x + 14$ around the x-axis yields a straightforward integral that results in a volume of 1920π cubic units over the specified bounds. Mastery of these techniques enhances your problem-solving toolkit for a wide range of volume and surface area calculations in calculus.

Frequently Asked Questions

How do I set up the integral to find the volume of the solid generated by revolving the region between y = x + 8 and y = x + 14 around the x-axis?

To find the volume, identify the region bounded by the two lines, then use the washer method. The volume V is given by $V = \pi \int_{a}^{b} [R(x)^2 - r(x)^2]\, dx$, where $R(x)$ and $r(x)$ are the outer and inner radii respectively. Since the region is between $y = x + 14$ (upper) and $y = x + 8$ (lower), and revolving around the x-axis, the radii are $R(x) = x + 14$ and $r(x) = x + 8$.

What are the limits of integration for calculating the volume of the solid formed by revolving the region between y = x + 8 and y = x + 14?

The limits of integration are determined by the intersection points of the two lines. Setting $y = x + 8$ and $y = x + 14$ equal to each other yields no solution, as they are parallel lines. Therefore, the region extends infinitely in x unless specified. Typically, you choose a finite interval based on the problem context or limits provided. If no bounds are given, the problem may involve an indefinite integral, or a specific interval needs to be provided.

What is the formula for calculating the volume of the solid generated by revolving the region between y = x + 8 and y =

x + 14 around the x-axis?

The volume V is given by the integral V = π ∫ [a to b] [(x + 14)^2 - (x + 8)^2] dx, where a and b are the x-values defining the region's bounds. Simplifying the integrand, (x + 14)^2 - (x + 8)^2 = (x^2 + 28x + 196) - (x^2 + 16x + 64) = 12x + 132.

How do I compute the volume once I have set up the integral for the region between y = x + 8 and y = x + 14?

First, simplify the integrand, then evaluate the definite integral over the chosen bounds. The integral becomes V = π ∫ [a to b] (12x + 132) dx. Integrate term-by-term: ∫ 12x dx = 6x^2 and ∫ 132 dx = 132x. Plug in the bounds a and b to find the volume: V = π [6b^2 + 132b - (6a^2 + 132a)].

Additional Resources

Find The Volume Of The Solid Generated By Revolving The Region Bounded By The Graphs Of Y = x + 8 And Y = x + 14

Introduction

The problem of determining the volume of a solid generated by revolving a region bounded by specific curves around an axis is a classical exercise in integral calculus with widespread applications in engineering, physics, and mathematics. Specifically, this article aims to provide a comprehensive analysis of the problem: Find the volume of the solid generated by revolving the region bounded by the graphs of y = x + 8 and y = x + 14.

This problem, while seemingly straightforward, involves a careful understanding of the geometric region, the choice of the axis of revolution, and the application of integral calculus techniques such as the disk/washer method or the cylindrical shells method. We will dissect each component, explore the underlying principles, and finally, derive the explicit formula for the volume.

Understanding the Geometric Region

Graphical Interpretation

The two given equations are:

- y = x + 8
- y = x + 14

These are straight lines with identical slopes (slope = 1) but different y-intercepts (8 and 14). Graphically, they are parallel lines that extend infinitely in both directions.

Boundaries and Region

The bounded region between these two lines, in the xy-plane, is the "strip" between them:

- The lower boundary: $y = x + 8$
- The upper boundary: $y = x + 14$

Since the lines are parallel, the region is a "slab" extending indefinitely along the x-axis unless bounded by specific x-values. For the purpose of volume calculation, it's essential to define the limits of integration. Typically, the problem implies a finite segment, which can be specified by choosing x-values or considering the lines over a specific interval.

For a comprehensive analysis, assume the region is bounded between $x = a$ and $x = b$, with $a < b$. This interval can be arbitrary or specified; for simplicity, let's consider the region between $x = 0$ and $x = c > 0$.

Choosing the Axis of Revolution

The problem's wording does not specify the axis about which the region is revolved. Common axes include:

- The x-axis
- The y-axis
- A line parallel to the axes

Given the symmetry and the typical context, we will analyze the volume generated by revolving the region around the x-axis. This choice simplifies the computation and is common in such problems.

Mathematical Approach

Visualizing the Revolution

Revolving the region bounded by $y = x + 8$ and $y = x + 14$ around the x-axis results in a three-dimensional solid with a characteristic "washer" shape at each x-value.

At each x in [a, b], the cross-section perpendicular to the x-axis is a washer with:

- Outer radius: $|y = x + 14|$ (distance from x-axis to the upper line)
- Inner radius: $|y = x + 8|$ (distance from x-axis to the lower line)

Because y is always positive for $x \geq 0$ in this scenario, the radii are simply:

- Outer radius: $R_{outer} = x + 14$
- Inner radius: $R_{inner} = x + 8$

Applying the Washer Method

The volume of the solid can be expressed as:

$$ V = \int_{a}^{b} \pi \left(R_{\text{outer}}^2 - R_{\text{inner}}^2 \right) dx $$

Where:

- $R_{\text{outer}} = x + 14$
- $R_{\text{inner}} = x + 8$

The integral becomes:

$$ V = \pi \int_{a}^{b} \left[(x + 14)^2 - (x + 8)^2 \right] dx $$

Derivation of the Volume Formula

Step 1: Expand the Squares

$$
(x + 14)^2 = x^2 + 28x + 196
$$
$$
(x + 8)^2 = x^2 + 16x + 64
$$

Subtracting:

$$
(x + 14)^2 - (x + 8)^2 = (x^2 + 28x + 196) - (x^2 + 16x + 64) = (28x - 16x) + (196 - 64) = 12x + 132
$$

Step 2: Set up the Integral

$$
V = \pi \int_{a}^{b} (12x + 132) dx
$$

This simplifies the problem to integrating a linear function.

Step 3: Integrate

$$
V = \pi \left[6x^2 + 132x \right]_a^b
$$

So, the volume is:

$$
V = \pi \left(6b^2 + 132b - (6a^2 + 132a) \right)
$$

Selecting the Limits of Integration

To finalize the volume, specific limits are necessary. For instance, if the region is bounded between $x = 0$ and $x = c$, then:

$$
V = \pi \left(6c^2 + 132c - (0 + 0) \right) = \pi (6c^2 + 132c)
$$

Alternatively, if the bounded region is from $x = a$ to $x = b$, replace these accordingly.

Special Cases and Further Considerations

Infinite Extent

If the region extends infinitely, the volume becomes unbounded, and the integral diverges. Therefore, practical problems specify finite bounds.

Symmetry and Axis of Revolution

If the axis of rotation is the y-axis or a different line, the method would change—possibly requiring the shell method or a different approach.

Practical Application

Such calculations are fundamental in engineering design, for example, in designing cylindrical or toroidal components where precise volume calculations are necessary.

Final Expression for the Volume

Assuming the region between $x = 0$ and $x = c$, revolved about the x-axis, the volume is:

$$
\boxed{
V = \pi \left(6c^2 + 132c \right)
}
$$

This formula provides a direct means to compute the volume once the bounds are specified.

Conclusion

The problem of Find The Volume Of The Solid Generated By Revolving The Region Bounded By The

Graphs Of y = x + 8 And y = x + 14 exemplifies the elegance and power of integral calculus in geometric applications. By carefully analyzing the region's boundaries, selecting an appropriate axis of rotation, and applying the washer method, we derive a straightforward integral expression that captures the essence of the three-dimensional shape.

This analysis underscores the importance of understanding the geometry of the problem, the assumptions involved, and the flexibility of integral techniques in solving complex volume problems. Whether applied in engineering, physics, or pure mathematics, such calculations form the backbone of volumetric analysis in the real world.

References

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About the Author

[Insert author bio here, emphasizing expertise in calculus, mathematics education, or related fields.]

Note: For specific numerical results, define the bounds of the region explicitly.

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