Find The Volume When The Region Between = 2sin() 1 And The X-axis For 6 5 6 Is Revolved About The Y-axis.

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This problem involves calculating the volume of a solid of revolution generated by revolving a specific region around the y-axis. The region is bounded by the curve $y = 2\sin(x)$, the x-axis, and the vertical lines $x = 5$ and $x = 6$. Understanding how to approach such problems is fundamental in integral calculus, especially when working with volumes generated by revolution. In this article, we will explore the step-by-step process to determine this volume, covering necessary concepts, methods, and calculations.

Understanding the Problem and Setting Up the Integral

The Region of Interest

The problem defines a region bounded by:
- The curve $y = 2\sin(x)$,
- The x-axis $y=0$,
- Vertical lines at $x=5$ and $x=6$.

Since $y=2\sin(x)$, the function oscillates between -2 and 2. However, because we're bounded below by the x-axis, only the parts where $2\sin(x) \geq 0$ contribute to the region.

For $\sin(x)$:
- $\sin(x) \geq 0$ in intervals $[2\pi n, \pi + 2\pi n]$,
- Since 5 and 6 are within the interval $[0, 2\pi]$, and $2\pi \approx 6.283$,
- The segment from $x=5$ to $x=6$ lies within the first period, where $\sin(x)$ is positive between approximately 0 and π, specifically for $\sin(x) \geq 0$.

Hence, the region between $x=5$ and $x=6$ is above the x-axis, and the curve $y=2\sin(x)$ is positive within this interval.

Methodology for Calculating the Volume

The key to solving this volume problem is choosing the appropriate method of integration. Since the problem involves revolving around the y-axis, the cylindrical shells method is most suitable.

Cylindrical Shells Method

This method involves imagining the solid as composed of cylindrical shells:

- Each shell corresponds to a vertical strip at position x,
- The shell's height is $y = 2\sin(x)$,
- The radius of each shell is x,
- The circumference of each shell is $2\pi x$,
- The thickness is dx.

The volume of each shell is approximately:

$$
dV = \text{circumference} \times \text{height} \times \text{thickness} = 2\pi x \times 2\sin(x) \times dx
$$

Total volume:

$$
V = \int_{x=5}^{x=6} 2\pi x \times 2\sin(x) \, dx
$$

Simplify:

$$
V = 4\pi \int_{5}^{6} x \sin(x) \, dx
$$

Calculating the Integral

Integral to Solve

The main integral:

$$
V = 4\pi \int_{5}^{6} x \sin(x) \, dx
$$

This involves integrating $x \sin(x)$, which requires integration by parts.

Integration by Parts

Recall the formula:

$$
\int u \, dv = uv - \int v \, du
$$

Choose:

- $u = x \Rightarrow du = dx$,
- $dv = \sin(x) dx \Rightarrow v = -\cos(x)$.

Applying integration by parts:

$$
\int x \sin(x) dx = -x \cos(x) + \int \cos(x) dx
$$

Further integrating:

$$
\int \cos(x) dx = \sin(x) + C
$$

Thus,

$$
\int x \sin(x) dx = -x \cos(x) + \sin(x) + C
$$

Now, evaluate this from $x=5$ to $x=6$:

$$
V = 4\pi \left[-x \cos(x) + \sin(x) \right]_5^6
$$

Computing the Definite Integral

Evaluate at the bounds:

$$
V = 4\pi \left(\left[-6 \cos(6) + \sin(6)\right] - \left[-5 \cos(5) + \sin(5)\right] \right)
$$

Expressed explicitly:

$$
V = 4\pi \left(-6 \cos(6) + \sin(6) + 5 \cos(5) - \sin(5) \right)
$$

Using approximate values of the trigonometric functions (in radians):

- $\cos(5) \approx 0.28366$,
- $\sin(5) \approx -0.95892$,
- $\cos(6) \approx 0.96017$,
- $\sin(6) \approx -0.27942$.

Calculate each term:

$$
-6 \times 0.96017 \approx -5.76099
$$
$$
\sin(6) \approx -0.27942
$$
$$
5 \times 0.28366 \approx 1.4183
$$
$$
-\sin(5) \approx 0.95892
$$

Putting it all together:

$$
V \approx 4\pi \left(-5.76099 - 0.27942 + 1.4183 + 0.95892 \right)
$$

Combine the terms inside the parentheses:

$$
(-5.76099 - 0.27942) = -6.04041
$$
$$
(1.4183 + 0.95892) = 2.37722
$$
$$
-6.04041 + 2.37722 = -3.66319
$$

Finally, multiply by 4π:

$$
V \approx 4 \times 3.1416 \times (-3.66319) \approx 12.5664 \times (-3.66319) \approx -46.064
$$

Since volume cannot be negative, take the absolute value:

$$
V \approx 46.064
$$

Result:

$$
\boxed{
V \approx 46.06 \text{ cubic units}
}
$$

Summary and Final Remarks

- The problem involves calculating the volume of a solid generated by revolving the specified region about the y-axis.
- The cylindrical shells method simplifies the process for revolution about the y-axis.
- Setting up the integral involves understanding the bounds and the shape of the region.
- Integration by parts is necessary to evaluate the integral \(\int x \sin(x) dx \).
- Approximate numerical calculations provide an estimate of the volume, approximately 46.06 cubic units.

Important tips for similar problems:

- Always analyze the region carefully to determine bounds and whether the function stays positive.
- Choose the most suitable method—cylindrical shells or washers/disks—based on the axis of revolution.
- Simplify the integral before evaluation and verify bounds and function positivity.
- Use appropriate numerical approximations for trigonometric functions, being mindful of radians.

By mastering these steps, you can confidently compute volumes of complex regions revolved around different axes, enhancing your understanding of integral calculus applications.

Frequently Asked Questions

What is the main problem involving the region between y = 2sin(x) + 1 and the x-axis when revolved around the y-axis?

The problem is to find the volume of the solid formed when the region between y = 2sin(x) + 1 and the x-axis, over the interval from x = 6 to x = 5 (assuming a typo and the interval is from x = 5 to x = 6), is revolved about the y-axis.

How do you set up the integral for finding the volume of the solid formed by revolving the region around the y-axis?

You use the shell method, setting up the integral as V = 2π ∫_{a}^{b} x · f(x) dx, where x is the radius and f(x) is the height of the shell, integrating over the interval from x = 5 to x = 6.

What is the function representing the height of the region in this problem?

The height of the region at a given x is y = 2sin(x) + 1, as long as this is above the x-axis.

What are the limits of integration for this problem?

The limits of integration are from $x = 5$ to $x = 6$, corresponding to the interval over which the region exists and is revolved.

How do you verify that the region is above the x-axis over the interval from 5 to 6?

Evaluate $y = 2\sin(x) + 1$ at $x = 5$ and $x = 6$. Since $\sin(x)$ ranges between -1 and 1, $2\sin(x)$ varies between -2 and 2, so y varies between -1 and 3. For the region to be above the x-axis, y must be positive; check if $2\sin(x) + 1 > 0$ over the interval.

What is the formula for the volume of the solid when revolving the region about the y-axis using the shell method?

$V = 2\pi \int_{x=5}^{6} x \cdot (2\sin(x) + 1)\, dx$.

How do you compute the integral for the volume in this problem?

Compute $V = 2\pi \int_{5}^{6} x \cdot (2\sin(x) + 1)\, dx$ by splitting the integral into two parts: $2\pi \left[\int_{5}^{6} 2x \sin(x)\, dx + \int_{5}^{6} x\, dx\right]$, and then evaluate each integral separately.

What are common challenges in solving this volume problem, and how can they be addressed?

Challenges include accurately setting up the integral, verifying the region is above the x-axis, and computing the integrals involving x and $\sin(x)$. These can be addressed by carefully analyzing the functions, confirming the bounds, and applying integration by parts where necessary.

Additional Resources

Find The Volume When The Region Between $y = 2\sin(x) + 1$ And The X-Axis For $6 \le x \le 6$ Is Revolved About The Y-Axis

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An In-Depth Investigation into the Volume of a Revolved Region: The Case of $y = 2\sin(x) + 1$

Understanding the volume of a solid generated by revolving a plane region around an axis is a fundamental problem in calculus, often revealing deep insights into the interplay between functions, geometry, and integration. In this comprehensive analysis, we explore the problem of finding the volume when the region between the curve $y = 2\sin(x) + 1$ and the x-axis, over a specified interval, is revolved about the y-axis. This case presents intriguing challenges due to the nature of the trigonometric function involved and the choice of the axis of revolution.

Context and Significance of the Problem

Revolution of area segments around axes is a classical topic in integral calculus, with applications spanning engineering, physics, and computer-aided design. Calculating the volume of such solids not only reinforces key techniques like the Disk/Washer and Shell methods but also exemplifies the importance of understanding the geometric properties of functions.

Specifically, the region under $y = 2\sin(x) + 1$ between certain bounds and the x-axis forms a non-trivial shape bounded by oscillatory behavior of the sine function and a constant offset. When this region is revolved about the y-axis, the resulting solid's volume encapsulates the intricate relationship between trigonometric oscillations and rotational symmetry.

Clarifying the Problem Statement

The problem as initially stated contains some ambiguities, likely due to transcription errors. For clarity, we interpret the problem as follows:

- Function: $y = 2\sin(x) + 1$
- Interval: $x \in [a, b]$, where the specific bounds are from $x=6$ to $x=6$, which appears to be a typo or incomplete.

Given the context, a plausible interpretation is:

- The interval of interest is from $x = 0$ to $x = \pi$ (or another interval), where the function is positive and the region is well-defined.

Alternatively, if the original problem intended x from 6 to 6, that would be a degenerate interval with zero width, resulting in zero volume. Therefore, assuming the problem is to find the volume generated by revolving the region between $y = 2\sin(x) + 1$ and the x-axis over an interval $[a, b]$, where:

- $a = 0$
- $b = \pi$

This interval captures one full oscillation of the sine function, ensuring the region is meaningful and the volume finite.

Mathematical Framework

The key to solving this problem lies in selecting the appropriate method—either the shell method or the disk/washer method. For rotation around the y-axis, the shell method often provides a more straightforward approach when the region is described in terms of x.

Shell Method Overview

The shell method involves integrating cylindrical shells formed by revolving vertical strips of the region about the y-axis. The volume V is given by:

$$V = 2\pi \int_{a}^{b} x \cdot \text{(height of the shell)} \, dx$$

where the height of each shell is the value of the function y(x) for the current x.

Applying to Our Function

- Shell radius: x
- Shell height: y = 2sin(x) + 1

The volume becomes:

$$ V = 2\pi \int_{a}^{b} x \left(2\sin(x) + 1 \right) dx $$

Step-by-Step Solution

1. Define the bounds

Assuming the interval [0, π], which captures one period of sine:

$$ a = 0, \quad b = \pi $$

2. Set up the integral

The volume is:

$$ V = 2 \pi \int_{0}^{\pi} x (2 \sin x + 1) dx $$

3. Expand the integrand

$$ V = 2 \pi \int_{0}^{\pi} (2x \sin x + x) dx $$

which separates into two integrals:

$$ V = 2 \pi \left[2 \int_{0}^{\pi} x \sin x \, dx + \int_{0}^{\pi} x \, dx \right] $$

Evaluating the Integrals

Integral 1: $\int x \sin x \, dx$

This is a standard integral solved via integration by parts.

Let:

- $ u = x \Rightarrow du = dx $
- $ dv = \sin x \, dx \Rightarrow v = -\cos x $

Applying integration by parts:

$$
\int x \sin x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C
$$

Evaluate from 0 to π:

\[

$$
\left[-x \cos x + \sin x \right]_0^{\pi}
$$

At $x = \pi$:

$$
- \pi \cos \pi + \sin \pi = -\pi (-1) + 0 = \pi
$$

At $x = 0$:

$$
- 0 \cdot \cos 0 + \sin 0 = 0 + 0 = 0
$$

Thus:

$$
\int_{0}^{\pi} x \sin x \, dx = \pi - 0 = \pi
$$

Integral 2: $\int_{0}^{\pi} x \, dx$

Straightforward:

$$
\left[\frac{x^2}{2} \right]_0^{\pi} = \frac{\pi^2}{2}
$$

Final Calculation

Putting it all together:

$$
V = 2 \pi \left[2 \times \pi + \frac{\pi^2}{2} \right] = 2 \pi \left(2 \pi + \frac{\pi^2}{2} \right)
$$

Simplify inside the brackets:

$$
2 \pi + \frac{\pi^2}{2} = \frac{4 \pi}{2} + \frac{\pi^2}{2} = \frac{4 \pi + \pi^2}{2}
$$

Multiply by 2π:

$$
V = 2 \pi \times \frac{4 \pi + \pi^2}{2} = \pi (4 \pi + \pi^2) = 4 \pi^2 + \pi^3
$$

Final Answer:

$$

\boxed{
\text{The volume of the solid obtained by revolving the region between }
y=2\sin x + 1 \text{ and the x-axis from } 0 \text{ to } \pi \text{ about the y-axis is } V = 4 \pi^2 + \pi^3
}
\]

Broader Implications and Further Explorations

Extending the Problem

While this analysis considered the interval $[0, \pi]$, similar techniques can be extended to other intervals, especially those where the function remains positive, to analyze the impact of oscillations in $y = 2\sin(x) + 1$ on the volume.

Variable Axis of Rotation

Exploring the same region about different axes—such as the x-axis or lines parallel to the axes—would necessitate alternative methods, like the washer method or cylindrical shells from a different perspective.

Numerical Methods and Approximate Solutions

In cases where the integral becomes complex or involves functions without elementary antiderivatives, numerical integration techniques like Simpson's rule or Monte Carlo methods are invaluable.

Conclusion

This detailed investigation underscores the importance of carefully interpreting the problem statement, selecting appropriate calculus techniques, and executing step-by-step integrations to determine the volume of a solid of revolution. The interplay between trigonometric functions and geometric rotation exemplifies the depth and versatility of integral calculus, providing tools to solve real-world problems involving complex shapes and surfaces.

Understanding these concepts not only reinforces foundational mathematical principles but also equips practitioners with the skills to analyze and design systems where rotational volumes are critical, from engineering components to scientific modeling.

Note: If the original problem's bounds differ from our assumptions, the methodology outlined here can be adapted accordingly, highlighting the flexibility and robustness of calculus-based volume calculations.

Find The Volume When The Region Between 2sin 1 And The X

[Axis For 6 5 6 Is Revolved About The Y Axis]

Find The Volume When The Region Between 2sin 1 And The X Axis For 6 5 6 Is Revolved About The Y Axis

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