Find Values Of X And Y Such Thatfx(x, Y) = 0 And Fy(x, Y) =0 Simultaneously.f(x, Y) = 7x3 6xy + Y3smaller

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Introduction to the Problem

Understanding the behavior of multivariable functions often involves analyzing their critical points, where the partial derivatives with respect to each variable vanish simultaneously. In this context, we are given a specific function:

$$ f(x, y) = 7x^3 - 6xy + y^3 $$

and asked to find the values of x and y such that both partial derivatives, f_x and f_y, are zero at the same point. This involves solving the system:

$$
f_x(x, y) = 0 \quad \text{and} \quad f_y(x, y) = 0
$$

which reveals the critical points of the function, essential for understanding its maxima, minima, and saddle points.

Calculating the Partial Derivatives

Before solving the system, it is necessary to compute the partial derivatives f_x and f_y.

Partial Derivative with respect to x: f_x

Given:

$$ f(x, y) = 7x^3 - 6xy + y^3 $$

Differentiating with respect to x:

$$
f_x = \frac{\partial}{\partial x} (7x^3 - 6xy + y^3) = 21x^2 - 6y + 0 = 21x^2 - 6y
$$

Partial Derivative with respect to y: f_y

Differentiating with respect to y:

$$
f_y = \frac{\partial}{\partial y} (7x^3 - 6xy + y^3) = 0 - 6x + 3y^2 = -6x + 3y^2
$$

Setting the Partial Derivatives to Zero

The critical points are found by solving the system:

$$
\begin{cases}
f_x(x, y) = 21x^2 - 6y = 0 \\
f_y(x, y) = -6x + 3y^2 = 0
\end{cases}
$$

This simplifies to:

$$
\begin{cases}
21x^2 = 6y \quad \Rightarrow \quad y = \frac{21x^2}{6} = \frac{7x^2}{2} \\
-6x + 3y^2 = 0 \quad \Rightarrow \quad 3y^2 = 6x \quad \Rightarrow \quad y^2 = 2x
\end{cases}
$$

Solving the System of Equations

To find the critical points, substitute $y = \frac{7x^2}{2}$ into $y^2 = 2x$.

Substitution and Simplification

Calculate y^2:

$$
\left(\frac{7x^2}{2}\right)^2 = \frac{49x^4}{4}
$$

Set equal to $2x$:

$$
\frac{49x^4}{4} = 2x
$$

Multiply both sides by 4 to clear denominator:

$$
49x^4 = 8x
$$

Rearranged as:

$$
49x^4 - 8x = 0
$$

Factor out x:

$$
x(49x^3 - 8) = 0
$$

This yields two cases:

1. $x = 0$
2. $49x^3 - 8 = 0$

Case 1: $x = 0$

Substitute into $y = \frac{7x^2}{2}$:

$$
y = \frac{7 \times 0^2}{2} = 0
$$

Critical point: $(0, 0)$

Case 2: $49x^3 = 8$

Solve for x:

$$
x^3 = \frac{8}{49}
$$
$$
x = \sqrt[3]{\frac{8}{49}} = \frac{\sqrt[3]{8}}{\sqrt[3]{49}} = \frac{2}{\sqrt[3]{49}}
$$

Now, find y:

$$
y = \frac{7x^2}{2}
$$

Calculate x^2:

$$
x^2 = \left(\frac{2}{\sqrt[3]{49}}\right)^2 = \frac{4}{\left(\sqrt[3]{49}\right)^2}
$$

Express $\sqrt[3]{49}$ as $49^{1/3}$:

$$
x^2 = \frac{4}{(49^{1/3})^2} = 4 \times 49^{-2/3}
$$

Thus,

$$
y = \frac{7}{2} \times 4 \times 49^{-2/3} = 14 \times 49^{-2/3}
$$

Critical point:

$$
x = \frac{2}{49^{1/3}}, \quad y = 14 \times 49^{-2/3}
$$

Alternatively, expressing in radical form:

$$
x = 2 \times 49^{-1/3}
$$
$$
y = 14 \times 49^{-2/3}
$$

which simplifies the notation.

Summary of Critical Points

The solutions where both partial derivatives vanish are:

1. $\boxed{(0, 0)}$
2. $\boxed{\left(\frac{2}{49^{1/3}}, \quad 14 \times 49^{-2/3}\right)}$

These points are potential locations of maxima, minima, or saddle points. To classify these critical points, further analysis such as the second derivative test or evaluating the Hessian matrix is necessary.

Second Derivative Test and Classification of Critical

Points

To classify the nature of these critical points, compute the second derivatives and analyze the Hessian determinant.

Second Partial Derivatives:

$$
f_{xx} = \frac{\partial^2 f}{\partial x^2} = 42x
$$

$$
f_{yy} = \frac{\partial^2 f}{\partial y^2} = 6y
$$

$$
f_{xy} = f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = -6
$$

Hessian Determinant D:

$$
D = f_{xx} \times f_{yy} - (f_{xy})^2
$$

Calculate D at each critical point.

At $(0, 0)$:

$$
f_{xx} = 42 \times 0 = 0
$$

$$
f_{yy} = 6 \times 0 = 0
$$

$$
f_{xy} = -6
$$

$$
D = 0 \times 0 - (-6)^2 = -36 < 0
$$

Since $D<0$, the point $(0,0)$ is a saddle point.

At $\left(\frac{2}{49^{1/3}}, 14 \times 49^{-2/3}\right)$:

Calculate f_{xx}:

$$
f_{xx} = 42x = 42 \times \frac{2}{49^{1/3}} = \frac{84}{49^{1/3}}
$$

Calculate f_{yy}:

$$
f_{yy} = 6y = 6 \times 14 \times 49^{-2/3} = 84 \times 49^{-2/3}
$$

Recall that $49^{1/3} = (7^2)^{1/3} = 7^{2/3}$, so:

$$
f_{xx} = \frac{84}{7^{2/3}}
$$

Similarly, $49^{-2/3} = (7^2)^{-2/3} = 7^{-4/3}$, so:

$$
f_{yy} = 84 \times 7^{-4/3}
$$

Compute the Hessian determinant:

$$
D = \left(\frac{84}{7^{2/3}}\right) \times (84 \times 7^{-4/3}) - (-6)^2
$$

Simplify:

$$
D = 84 \times 84 \times 7^{-(2/3 + 4/3)} - 36 = 7056 \times 7^{-6/3} - 36 = 7056 \times 7
$$

Frequently Asked Questions

How do you find the values of x and y such that both partial derivatives of $f(x, y) = 7x^3 - 6xy + y^3$ are zero simultaneously?

To find the values, first compute the partial derivatives fx and fy, set each equal to zero, and solve the resulting system of equations. Specifically, $fx = 21x^2 - 6y = 0$ and $fy = -6x + 3y^2 = 0$. Solve these equations simultaneously to find the critical points.

What are the steps to solve for x and y when $fx(x, y) = 0$ and $fy(x, y) = 0$ for the function $f(x, y) = 7x^3 - 6xy + y^3$?

First, set $fx = 21x^2 - 6y = 0$ and $fy = -6x + 3y^2 = 0$. From $fx = 0$, express y in terms of x: $y = (21/6) x^2 = 3.5 x^2$. Substitute into $fy = 0$: $-6x + 3(3.5 x^2)^2 = 0$, then solve for x, and subsequently find y.

Are there multiple solutions for x and y satisfying fx = 0 and fy = 0 for the given function?

Yes, solving the system may yield multiple solutions, including points where both derivatives vanish. For instance, solutions can include x = 0 with y = 0, and other potential points derived from solving the equations, such as y proportional to x^2.

What is the significance of finding points where both fx and fy equal zero in the context of the function f(x, y)?

Points where both fx and fy are zero are critical points, which can be local maxima, minima, or saddle points. Identifying these points helps analyze the behavior of the function and its extrema.

How does substitution help in solving for x and y when setting fx = 0 and fy = 0 in this problem?

Substitution allows you to express one variable in terms of the other using one equation, then substitute into the second equation to reduce the system to a single-variable equation. This simplifies solving for the critical points.

Additional Resources

Find Values of X and Y Such That f_x(x, Y) = 0 and f_y(x, Y) = 0 Simultaneously

Understanding the critical points of a multivariable function is a foundational aspect of calculus, particularly in the study of optimization, saddle points, and the behavior of functions in multiple dimensions. When analyzing a function such as \(f(x, y) = 7x^3 - 6xy + y^3\), mathematicians often seek to find simultaneous solutions to the conditions \(f_x(x, y) = 0\) and \(f_y(x, y) = 0\). These points, where both partial derivatives vanish, are critical points that can represent local maxima, minima, or saddle points.

This article provides a comprehensive, detailed exploration of the process involved in finding such points for the specific function \(f(x, y) = 7x^3 - 6xy + y^3\). We will delve into the calculus techniques necessary to compute partial derivatives, analyze the resulting system of equations, and interpret the solutions within the broader context of multivariable calculus.

Understanding the Function and Its Significance

The Function \(f(x, y) = 7x^3 - 6xy + y^3\)

The function provided is a polynomial in two variables, combining cubic and mixed terms. Its form suggests that it may exhibit complex behavior, including multiple critical points, depending on the values of x and y.

Breaking down the function:

- The term $7x^3$ dominates in the x-direction for large magnitudes of x, influencing the function's growth or decay.
- The mixed term $-6xy$ couples the x and y variables, potentially creating saddle points or influencing the shape of the function.
- The y^3 term impacts the function's behavior in the y-direction, especially for large y.

Understanding the interplay of these terms is essential for analyzing the critical points, which can inform us about the function's maxima, minima, or saddle points.

Calculating Partial Derivatives: The First Step

Partial Derivative with Respect to x (f_x)

To find critical points, the first step is to compute the partial derivative of f with respect to x:

$$
f_x(x, y) = \frac{\partial}{\partial x} (7x^3 - 6xy + y^3)
$$

Applying standard differentiation rules:

- For $7x^3$, derivative is $21x^2$.
- For $-6xy$, treat y as a constant: derivative is $-6y$.
- For y^3, treat as constant: derivative is 0.

Thus, the partial derivative with respect to x is:

$$
f_x(x, y) = 21x^2 - 6y
$$

Partial Derivative with Respect to y (f_y)

Similarly, the partial derivative with respect to y is:

$$

f_y(x, y) = \frac{\partial}{\partial y} (7x^3 - 6xy + y^3)
\]

Applying differentiation:

- \(7x^3\) is constant with respect to \(y\): derivative is 0.
- \(-6xy\): treat \(x\) as constant: derivative is \(-6x\).
- \(y^3\): derivative is \(3y^2\).

Therefore:

\[
f_y(x, y) = -6x + 3y^2
\]

Setting the Partial Derivatives to Zero

The critical points are found by solving the system:

\[
\begin{cases}
f_x(x, y) = 0 \\
f_y(x, y) = 0
\end{cases}
\]

which translates to the equations:

\[
\begin{cases}
21x^2 - 6y = 0 \\
-6x + 3y^2 = 0
\end{cases}
\]

This system requires solving for \(x\) and \(y\) simultaneously.

Solving the System of Equations

Equation 1: \(21x^2 - 6y = 0\)

Rearranged:

\[
6y = 21x^2
\]
\[
y = \frac{21x^2}{6} = \frac{7x^2}{2}
\]

Expressed as:

\[
\boxed{y = \frac{7}{2}x^2}
\]

Equation 2: \(-6x + 3y^2 = 0\)

Rearranged:

\[
3y^2 = 6x
\]
\[
y^2 = 2x
\]

Expressed as:

\[
\boxed{y^2 = 2x}
\]

Substituting to Find \(x\) and \(y\)

Using the expression for \(y\) from the first equation in the second:

\[
y = \frac{7}{2}x^2
\]

Substitute into \(y^2 = 2x\):

$$
\left(\frac{7}{2}x^2\right)^2 = 2x
$$

Calculating the left side:

$$
\left(\frac{7}{2}\right)^2 x^4 = 2x
$$

$$
\frac{49}{4} x^4 = 2x
$$

Assuming $x \neq 0$, divide both sides by x:

$$
\frac{49}{4} x^3 = 2
$$

Solve for x^3:

$$
x^3 = \frac{2 \times 4}{49} = \frac{8}{49}
$$

Thus, the solutions for x:

$$
x = \sqrt[3]{\frac{8}{49}}
$$

Note that cube roots can be real and complex, but in the context of real critical points, we consider the real cube root:

$$
x = \left(\frac{8}{49}\right)^{1/3}
$$

Calculate x:

$$
x \approx \left(\frac{8}{49}\right)^{1/3}
$$

Since $8 = 2^3$, and $49 = 7^2$:

$$
x \approx \frac{2}{7^{2/3}}
$$

Next, find y:

\[
y = \frac{7}{2} x^2
\]

Calculate \(x^2\):

\[
x^2 = \left(\left(\frac{8}{49}\right)^{1/3} \right)^2 = \left(\frac{8}{49}\right)^{2/3}
\]

Expressed as:

\[
x^2 = \frac{8^{2/3}}{49^{2/3}} = \frac{(2^3)^{2/3}}{(7^2)^{2/3}} = \frac{2^{2}}{7^{4/3}}
= \frac{4}{7^{4/3}}
\]

Now, substitute into \(y\):

\[
y = \frac{7}{2} \times \frac{4}{7^{4/3}} = \frac{7 \times 4}{2 \times 7^{4/3}} = \frac{28}{2
\times 7^{4/3}} = \frac{14}{7^{4/3}}
\]

Expressing \(7^{4/3}\):

\[
7^{4/3} = 7^{1 + 1/3} = 7 \times 7^{1/3}
\]

So,

\[
y = \frac{14}{7 \times 7^{1/3}} = \frac{14}{7 \times 7^{1/3}} = \frac{2}{7^{1/3}}
\]

Therefore, the critical point is approximately:

\[
x \approx \frac{2}{7^{2/3}}, \quad y \approx \frac{2}{7^{1/3}}
\]

Special Case: When \(x=0\)

Recall the earlier step involved dividing by \(x\). We should verify the case where \(x=0\):

- From the first derivative \(f_x=0\):

\[
21x^2 - 6y=0
\]
\[
0 - 6y=0 \Rightarrow y=0
\]

- From the second derivative \(f_y=0\):

\[
-6x + 3 y^2=0
\]
\[
0 + 0=0
\]

Thus, the point \((0, 0)\) satisfies both equations.

Conclusion: The point \(\boxed{(0

[Find Values Of X And Y Such Thatfx X Y 0 And Fy X Y 0 Simultaneously F X Y 7x3 6xy Y3smaller]

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