Give The Appropriate Form Of The Partial Fraction Decomposition For The Following Function 4x/(x-7)^2(x^2

Give The Appropriate Form Of The Partial Fraction Decomposition For The Following Function 4x/(x-7)^2(x^2 is a common problem encountered in calculus, particularly when dealing with integration of rational functions. Understanding how to decompose a complex rational expression into partial fractions is essential for simplifying the process of integration, especially when the denominator contains repeated linear factors and irreducible quadratic factors. In this article, we will explore the step-by-step method to find the appropriate partial fraction decomposition for the given function, along with key concepts, formulas, and examples to enhance your understanding.

Understanding Partial Fraction Decomposition

Partial fraction decomposition is a technique used to express a rational function as a sum of simpler fractions. This method is especially useful for integrating rational functions, as it allows us to break down complicated expressions into manageable parts.

Purpose of Partial Fraction Decomposition

- Simplify complex rational functions.
- Facilitate easier integration.
- Break down functions into elementary fractions that are easier to handle.

Types of Factors in Denominator

When performing partial fraction decomposition, the nature of the factors in the denominator determines the form of the decomposition:
- Linear factors (e.g., (x - a))
- Repeated linear factors (e.g., (x - a)^n)
- Irreducible quadratic factors (e.g., x^2 + bx + c, where the quadratic cannot be factored further over the reals)
- Repeated quadratic factors

In our case, the denominator contains a repeated linear factor, (x - 7)^2, and an irreducible quadratic, x^2.

Step-by-Step Approach to Decompose 4x / [(x - 7)^2 x^2]

1. Factor the denominator completely

The denominator is already factored:
- $(x - 7)^2$ (a repeated linear factor)
- x^2 (an irreducible quadratic)

2. Write the general form of the partial fractions

Based on the types of factors, the partial fractions are structured as follows:

- For the repeated linear factor $(x - 7)^2$:
- $\frac{A}{x - 7}$
- $\frac{B}{(x - 7)^2}$

- For the quadratic factor x^2 (which is irreducible over the reals):
- $\frac{Cx + D}{x^2}$

Therefore, the general form of the partial fraction decomposition is:

$$
\frac{4x}{(x - 7)^2 x^2} = \frac{A}{x - 7} + \frac{B}{(x - 7)^2} + \frac{Cx + D}{x^2}
$$

3. Combine the right-hand side over a common denominator

To solve for the constants A, B, C, and D, multiply both sides by the common denominator, $(x - 7)^2 x^2$:

$$
4x = A \cdot (x - 7) \cdot x^2 + B \cdot x^2 + (Cx + D) \cdot (x - 7)^2
$$

Expanding each term:

- $A \cdot (x - 7) \cdot x^2 = A \cdot (x \cdot x^2 - 7 x^2) = A(x^3 - 7 x^2)$
- $B \cdot x^2$ remains as is.
- $(Cx + D) \cdot (x - 7)^2$

Note that $(x - 7)^2 = x^2 - 14x + 49$, so:

$$
(Cx + D)(x^2 - 14x + 49) = Cx(x^2 - 14x + 49) + D(x^2 - 14x + 49)
$$

Expanding:

$$
C x^3 - 14 C x^2 + 49 C x + D x^2 - 14 D x + 49 D
$$

Now, rewrite the entire right-hand side:

$$
A x^3 - 7 A x^2 + B x^2 + C x^3 - 14 C x^2 + 49 C x + D x^2 - 14 D x + 49 D
$$

Group like terms:

- x^3: $A x^3 + C x^3 = (A + C) x^3$
- x^2: $-7 A x^2 + B x^2 - 14 C x^2 + D x^2 = (-7A + B - 14 C + D) x^2$
- x: $49 C x - 14 D x = (49 C - 14 D) x$
- Constant term: $49 D$

The left side is simply $4x$. Equate coefficients of corresponding powers of x:

$$
4x = (A + C) x^3 + (-7A + B - 14 C + D) x^2 + (49 C - 14 D) x + 49 D
$$

Since the left side has only an x term, the coefficients of x^3, x^2, and the constant term on the right must be zero:

$$
\begin{cases}
A + C = 0 \quad & \text{(coefficient of } x^3) \\
-7A + B - 14 C + D = 0 \quad & \text{(coefficient of } x^2) \\
49 C - 14 D = 4 \quad & \text{(coefficient of } x) \\
49 D = 0 \quad & \text{(constant term)}
\end{cases}
$$

4. Solve for the constants

From the last equation:

$$
49 D = 0 \Rightarrow D = 0
$$

Using $D = 0$, the third equation becomes:

$$
49 C - 14 \times 0 = 4 \Rightarrow 49 C = 4 \Rightarrow C = \frac{4}{49}
$$

From the first equation:

$$
A + C = 0 \Rightarrow A = - C = - \frac{4}{49}
$$

Now, substitute \(D = 0\) and \(C = \frac{4}{49}\) into the second equation:

\[
-7A + B - 14 C + D = 0
\]
\[
-7 \left(- \frac{4}{49}\right) + B - 14 \times \frac{4}{49} + 0 = 0
\]

Simplify:

\[
\frac{28}{49} + B - \frac{56}{49} = 0
\]
\[
\frac{28}{49} - \frac{56}{49} + B = 0
\]
\[
- \frac{28}{49} + B = 0
\]
\[
B = \frac{28}{49} = \frac{4}{7}
\]

Summary of constants:

\[
A = - \frac{4}{49}, \quad B = \frac{4}{7}, \quad C = \frac{4}{49}, \quad D = 0
\]

Final Partial Fraction Decomposition

Plugging the constants back into the general form:

\[
\frac{4x}{(x - 7)^2 x^2} = \frac{A}{x - 7} + \frac{B}{(x - 7)^2} + \frac{Cx + D}{x^2}
\]

which becomes:

\[
\boxed{
\frac{4x}{(x - 7)^2 x^2} = -\frac{4}{49} \cdot \frac{1}{x - 7} + \frac{4}{7} \cdot \frac{1}{(x - 7)^2} + \frac{\frac{4}{49} x}{x^2}
}
\]

or equivalently,

\[

\frac{4x}{(x - 7)^2 x^2} = -\frac{4}{49} \cdot \frac{1}{x - 7} + \frac{4}{7} \cdot \frac{1}{(x - 7)^2} + \frac{4}{49} \cdot \frac{x}{x^2}
\]

which simplifies further to:

\[
\frac{4x}{(x - 7)^2 x^2} = -\frac{4}{49} \cdot \frac{1}{x - 7} + \frac{4}{7} \cdot \frac{1}{(x - 7)^2} + \frac{4}{49} \cdot \frac{1}{x}
\]

Conclusion and Applications

Frequently Asked Questions

What is the appropriate form of partial fraction decomposition for 4x/[(x-7)^2 x^2]?

It is A/(x-7) + B/(x-7)^2 + C/x + D/x^2, where A, B, C, and D are constants to be determined.

Why do we include terms like A/(x-7) and B/(x-7)^2 in the partial fractions?

Because the denominator contains a repeated quadratic factor (x-7)^2, we include terms for each power of the factor to account for all parts of the decomposition.

How do you set up the partial fraction decomposition for the given function?

Express 4x/[(x-7)^2 x^2] as a sum of A/(x-7) + B/(x-7)^2 + C/x + D/x^2, then multiply both sides by the common denominator to solve for A, B, C, and D.

What is the first step to find the constants in the partial fraction decomposition?

Multiply both sides of the equation by the common denominator (x-7)^2 x^2 to clear fractions, resulting in a polynomial equation in terms of x.

How can I determine the values of A, B, C, and D after setting up the equations?

Use methods like equating coefficients of powers of x or plugging in specific values of x that simplify the equation to solve for each constant.

What is the significance of choosing specific x-values when solving for constants?

Selecting values that cancel out some terms (like $x=7$ or $x=0$) simplifies the equations, making it easier to find individual constants directly.

Are there any common mistakes to avoid when performing partial fraction decomposition?

Yes, common mistakes include missing terms for repeated factors, miscalculating constants, or forgetting to multiply through by the common denominator when clearing fractions.

Can partial fraction decomposition be used for all rational functions?

Partial fraction decomposition is generally applicable to proper rational functions with denominators factored into linear or quadratic factors; improper functions require polynomial division first.

What is the importance of partial fraction decomposition in calculus?

It simplifies integration of rational functions by breaking them into simpler fractions, making it easier to evaluate integrals involving complex rational expressions.

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Introduction

Partial fraction decomposition is a fundamental technique in algebra and calculus, enabling the integration and simplification of rational functions. When faced with complex rational functions—especially those involving repeated linear factors and irreducible quadratic factors—the process becomes more nuanced, requiring a careful approach to determine the appropriate decomposition form.

This article provides an in-depth investigation into identifying the correct partial fraction form for the function:

$$
\frac{4x}{(x-7)^2 \, x^2}
$$

We will explore the principles underlying partial fraction decomposition, analyze the structure of the given function, and systematically derive the appropriate partial fraction form. This comprehensive review aims to serve as an authoritative guide for students, educators, and mathematicians involved in algebraic manipulation and calculus applications.

Understanding the Structure of the Rational Function

Before commencing the partial fraction decomposition, a clear understanding of the structure and factors of the rational function is essential.

Denominator Factorization

The denominator is:

$$
(x - 7)^2 \, x^2
$$

This indicates:

- A repeated linear factor $(x - 7)^2$
- A repeated linear factor x^2

The factors are:

- $(x - 7)$ with multiplicity 2
- x with multiplicity 2

Thus, the denominator is composed of linear factors with multiplicities:

Factor	Multiplicity
$x - 7$	2
x	2

Principles of Partial Fraction Decomposition

To decompose the rational function:

$$
\frac{4x}{(x - 7)^2 \, x^2}
$$

the general form of the partial fractions depends on the types and multiplicities of the factors in the denominator.

Rules for Decomposition

1. Linear factors with multiplicity n:

For each linear factor $(ax + b)^n$, include terms:

$$
\frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \dots + \frac{A_n}{(ax + b)^n}
$$

2. Irreducible quadratic factors with multiplicity n:

For each quadratic factor $ax^2 + bx + c$, include terms:

$$
\frac{B_1 x + C_1}{ax^2 + bx + c} + \frac{B_2 x + C_2}{(ax^2 + bx + c)^2} + \dots + \frac{B_n x + C_n}{(ax^2 + bx + c)^n}
$$

3. Repeated factors:

Each multiplicity contributes a chain of terms with increasing powers in the denominator.

Applying Principles to the Given Function

Given the denominator:

$$
(x - 7)^2 \, x^2
$$

which consists entirely of linear factors with multiplicities 2 for both x and $x - 7$, the partial fraction decomposition should include:

- Terms for $(x - 7)$ and $(x - 7)^2$
- Terms for x and x^2

Thus, the general form is:

$$
\frac{4x}{(x - 7)^2 \, x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 7} + \frac{D}{(x - 7)^2}
$$

where A, B, C, D are constants to be determined.

Constructing the Partial Fraction Decomposition

Step 1: Write the assumed form

$$
\frac{4x}{(x - 7)^2 \, x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 7} + \frac{D}{(x - 7)^2}
$$

Step 2: Combine the right-hand side over the common denominator

Multiplying through both sides by the common denominator $(x - 7)^2 x^2$:

$$
4x = A \cdot (x - 7)^2 + B \cdot (x - 7)^2 \cdot x + C \cdot x^2 + D \cdot x^2 \cdot (x - 7)
$$

This expansion provides an algebraic equation to solve for the unknowns A, B, C, D.

Step 3: Expand each term

- $A (x - 7)^2 = A (x^2 - 14x + 49)$
- $B (x - 7)^2 x = B x (x^2 - 14x + 49) = B (x^3 - 14x^2 + 49x)$
- $C x^2$ remains as is
- $D x^2 (x - 7) = D (x^3 - 7x^2)$

Putting it all together:

$$
4x = A (x^2 - 14x + 49) + B (x^3 - 14x^2 + 49x) + C x^2 + D (x^3 - 7x^2)
$$

Equating Coefficients

Arrange the right side in powers of x:

$$
4x = (B x^3 + D x^3) + (-14A x^2 -14 B x^2 - 7 D x^2 + A x^2 + C x^2) + (49 B x + 49 A x)
$$

More systematically:

$$
4x = (B + D) x^3 + (-14A -14 B + C -7 D) x^2 + (49 B + 49 A) x
$$

Compare coefficients for each power of x:

- x^3: $B + D = 0$
- x^2: $-14A -14 B + C -7 D = 0$
- x^1: $49 B + 49 A = 4$
- Constant term: No constant term on the left, so the sum of constants is zero.

Now, solve for the unknowns:

1. From $B + D = 0 \Rightarrow D = -B$

2. From $49 B + 49 A = 4 \Rightarrow B + A = \frac{4}{49}$

3. From $-14A -14 B + C -7 D = 0$:

Substitute $D = -B$:

$$
-14A -14 B + C -7 (-B) = 0 \Rightarrow -14A -14 B + C + 7 B = 0 \Rightarrow -14A -7 B + C = 0
$$

Rearranged:

$$
C = 14A + 7 B
$$

4. Recall $B + A = \frac{4}{49}$, so:

$$
A = \frac{4}{49} - B
$$

5. Substitute A into C:

$$
C = 14 \left(\frac{4}{49} - B \right) + 7 B = \frac{14 \times 4}{49} - 14 B + 7 B = \frac{56}{49} - 7 B
$$

Simplify:

$$
C = \frac{8}{7} - 7 B
$$

6. Now, the only remaining variable is B, which is free unless further constraints are applied. Since the original function is known, evaluating at specific x values or solving for A, B, C, D directly from the original can be used for precise values.

Final Partial Fraction Form

Based on the analysis, the appropriate partial fraction decomposition for the function:

$$
\frac{4x}{(x - 7)^2 \, x^2}
$$

is:

$$
\boxed{
\frac{4x}{(x - 7)^2 \, x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 7} + \frac{D}{(x - 7)^2}
}
$$

where the constants A, B, C, D satisfy the relations:

- $D = -B$
- $A = \frac{4}{49} - B$
- $C = \frac{8}{7} - 7 B$

and can be explicitly determined by substituting specific x values or solving the system of equations.

Significance of Choosing the Correct Form

Selecting the proper partial fraction form is crucial, especially when integrating rational functions, as improper decomposition can lead to

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