Given The First 5 Terms Below, Write Down The General Term Of The Following Sequences:4/5, - 8/11, 16/17,

Given The First 5 Terms Below, Write Down The General Term Of The Following Sequences: 4/5, -8/11, 16/17, the task involves analyzing the sequence to identify a pattern and then deriving a formula that can generate any term in the sequence. Sequences are fundamental in mathematics and appear in various fields, from algebra to calculus, and understanding how to find a general term helps in predicting future terms and understanding the sequence's behavior. In this article, we will explore the sequence step-by-step, examine possible patterns, and derive a general term formula that accurately describes the sequence.

Understanding the Given Sequence

Listing the First Terms

Let's first write down the sequence as given:
- Term 1: 4/5
- Term 2: -8/11
- Term 3: 16/17

Notice that the sequence consists of fractions with numerator and denominator, and the signs alternate. Our goal is to find a formula for the nth term, denoted as a_n, that can generate these terms and subsequent terms.

Observing the Pattern in the Terms

To identify the pattern, analyze the numerators and denominators separately:

Term n	Value	Numerator	Denominator	Sign
1	4/5	4	5	+
2	-8/11	-8	11	-
3	16/17	16	17	+

Numerator Pattern:
- 4, -8, 16

Denominator Pattern:
- 5, 11, 17

Notice the numerators:
- 4, -8, 16
- The absolute values: 4, 8, 16
- The pattern in absolute value seems to be doubling: 4, 8, 16

The signs alternate, starting with positive, then negative, then positive again. The numerators' absolute values are powers of 2 multiplied by 2:
- 4 = 2^2
- 8 = 2^3
- 16 = 2^4

But the signs alternate, with the first term positive, second negative, third positive, and so on.

Denominator Pattern:
- 5, 11, 17
- The pattern in denominators appears to be increasing by 6:
- 5 to 11: +6
- 11 to 17: +6

Thus, the denominators follow the sequence:
- 5, 11, 17, 23, ...

which can be expressed as an arithmetic sequence with first term 5 and common difference 6.

Deriving the General Term

Numerator Pattern

Based on the observations:

- The numerator's absolute value doubles each time: 4, 8, 16, ...
- These are powers of 2 starting from 2^2, then 2^3, then 2^4.

In general, the absolute value of the numerator at term n can be expressed as:

$$
|\text{numerator}| = 2^{n+1}
$$

because:
- For $n=1$: $2^{1+1} = 2^2 = 4$
- For $n=2$: $2^{2+1} = 2^3 = 8$
- For $n=3$: $2^{3+1} = 2^4 = 16$

The sign alternates starting with positive for $n=1$. Therefore, the sign pattern can be modeled using $(-1)^{n+1}$, which is positive when n is odd, negative when n is even.

Thus, the numerator N_n is:

$$
N_n = (-1)^{n+1} \times 2^{n+1}
$$

Explanation:

- When n is odd, $(-1)^{n+1} = +1$, numerator positive.
- When n is even, $(-1)^{n+1} = -1$, numerator negative.

Denominator Pattern

The denominators follow the sequence:

$$
d_n = 5 + 6(n - 1)
$$

Which simplifies to:

$$
d_n = 6n - 1
$$

Check:
- For $n=1$: $6(1) - 1 = 5$
- For $n=2$: $12 - 1 = 11$
- For $n=3$: $18 - 1 = 17$

This matches the observed pattern perfectly.

Final General Term Formula

Combining the numerator and denominator expressions, the nth term a_n can be written as:

$$
a_n = \frac{N_n}{d_n} = \frac{(-1)^{n+1} \times 2^{n+1}}{6n - 1}
$$

This formula generates the sequence's terms accurately and can be used to find any term in the sequence.

Verification and Examples

Verify with initial terms:

- For $n=1$:

$$
a_1 = \frac{(-1)^{2} \times 2^{2}}{6(1) - 1} = \frac{1 \times 4}{5} = \frac{4}{5}
$$

Matches the first term.

- For $n=2$:

$$
a_2 = \frac{(-1)^{3} \times 2^{3}}{6(2) - 1} = \frac{-1 \times 8}{12 - 1} = \frac{-8}{11}
$$

Matches the second term.

- For $n=3$:

$$
a_3 = \frac{(-1)^{4} \times 2^{4}}{6(3) - 1} = \frac{1 \times 16}{18 - 1} = \frac{16}{17}
$$

Matches the third term.

Additional Applications and Insights

Predicting Future Terms

Using the formula, you can compute the 4th, 5th, or any subsequent terms:

- For $n=4$:

$$
a_4 = \frac{(-1)^{5} \times 2^{5}}{6(4) - 1} = \frac{-1 \times 32}{24 - 1} = \frac{-32}{23}
$$

- For $n=5$:

$$
a_5 = \frac{(-1)^{6} \times 2^{6}}{6(5) - 1} = \frac{1 \times 64}{30 - 1} = \frac{64}{29}
$$

These calculations confirm the pattern's consistency and the formula's accuracy.

Understanding the Sequence Behavior

- The numerator grows exponentially with 2^{n+1}, causing the absolute value of the terms to increase rapidly.
- The denominator increases linearly, so the overall magnitude of the terms is dominated by the numerator.
- The sign alternates, making the sequence oscillate between positive and negative values.

Conclusion

The sequence given by the first three terms demonstrates a clear pattern that can be expressed succinctly through a general term formula. By analyzing the numerators and denominators separately, recognizing the exponential and linear growth patterns, and accounting for the alternating signs, we derived the formula:

$$
\boxed{
a_n = \frac{(-1)^{n+1} \times 2^{n+1}}{6n - 1}
}
$$

This formula not only generates the initial sequence terms but also enables the prediction of subsequent terms. Understanding such sequences deepens our grasp of mathematical patterns and provides essential tools for tackling complex problems in algebra, calculus, and beyond. Whether for academic purposes, research, or practical applications, mastering the derivation of general terms is a valuable skill that enhances problem-solving capabilities and mathematical insight.

Frequently Asked Questions

What is the pattern in the given sequence 4/5, -8/11, 16/17?

The sequence alternates signs and the numerator doubles each time, while the denominator increases by 6 each step: numerator 4, -8, 16; denominator 5, 11, 17.

How do you determine the general term of the sequence 4/5, -8/11, 16/17?

Identify the pattern in numerator and denominator separately, then express the sequence term as a function of n, including the sign alternation.

What is the formula for the numerator in the sequence?

The numerator doubles each time and alternates sign: numerator = 4 (-2)^{n-1}.

How is the denominator of the sequence terms expressed in the general term?

The denominator increases by 6 each step starting from 5, so denominator = 5 + 6(n-1).

What is the complete general term formula for the sequence?

The nth term is given by: a_n = ((-1)^{n+1}) 4 2^{n-1} / (5 + 6(n-1)).

Can you verify the general term formula by computing the 4th term?

Yes. For n=4, a_4 = (-1)^{5} 4 2^{3} / (5 + 63) = -1 4 8 / (5 + 18) = -32/23, which continues the pattern.

How does understanding the pattern help in predicting subsequent terms?

By identifying the formulas for numerator and denominator and the sign pattern, you can compute any term directly without listing all previous terms.

Additional Resources

Given The First 5 Terms Below, Write Down The General Term Of The Following Sequences: 4/5, -8/11, 16/17,

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Introduction

Sequences form the backbone of mathematical analysis, providing foundational insights into patterns, limits, and functions. Among these, rational sequences—those whose terms are ratios of integers—are particularly intriguing due to their straightforward structure yet often complex behavior. The ability to discern a general term, or explicit formula, for a sequence from its initial terms is a crucial skill in mathematical problem-solving and theoretical exploration.

This article delves into a specific sequence, beginning with the terms:

- 4/5
- -8/11
- 16/17

and aims to derive the general term that governs the sequence's behavior. The pursuit of such a formula involves recognizing patterns, understanding underlying relationships, and applying mathematical tools such as recursive relations, ratio analysis, and pattern recognition.

Significance of Identifying a General Term

Understanding the general term of a sequence allows mathematicians and students to:

- Predict subsequent terms with certainty.
- Analyze the sequence's behavior as $n \to \infty$.
- Explore properties such as convergence, divergence, and oscillation.
- Connect the sequence to broader concepts like functions, series, and limits.

In educational contexts, deriving the general term enhances problem-solving skills, fosters pattern recognition, and deepens understanding of algebraic and analytical techniques.

Initial Observations and Data

Let's examine the initial terms:

Term Number n	Term a_n
1	4/5
2	-8/11
3	16/17

At a glance, these terms suggest some form of pattern, possibly involving alternating signs, powers of 2, and denominators evolving in a particular manner.

Deep Dive: Pattern Recognition in the Sequence

Sign Pattern

- $a_1 = \frac{4}{5}$ (positive)
- $a_2 = -\frac{8}{11}$ (negative)
- $a_3 = \frac{16}{17}$ (positive)

The signs alternate: positive, negative, positive. This hints at a factor of $(-1)^{n+1}$, where the sign depends on the term number.

Numerator Pattern

- Numerator of a_1: 4
- Numerator of a_2: -8
- Numerator of a_3: 16

Ignoring signs, the sequence of numerators is: 4, 8, 16. These are powers of 2:

- 4 = 2^2
- 8 = 2^3
- 16 = 2^4

Notice the pattern:

$$
\text{Numerator } = 2^{n+1}
$$

for $n=1, 2, 3$:

- $n=1 \Rightarrow 2^{2} = 4$

- $n=2 \Rightarrow 2^{3} = 8$
- $n=3 \Rightarrow 2^{4} = 16$

Thus, numerator can be expressed as:

$$
\text{Numerator} = 2^{n+1}
$$

with the sign incorporated through $(-1)^{n+1}$.

Denominator Pattern

- Denominator of a_1: 5
- Denominator of a_2: 11
- Denominator of a_3: 17

The denominators are 5, 11, 17. These are all prime candidates for an arithmetic pattern.

Calculate the differences:

- $11 - 5 = 6$
- $17 - 11 = 6$

The denominators increase by 6 each time, suggesting an arithmetic sequence:

$$
d_n = 5 + (n - 1) \times 6
$$

which simplifies to:

$$
d_n = 6n - 1
$$

Verify for $n=1, 2, 3$:

- $n=1 \Rightarrow 6(1) - 1 = 5$
- $n=2 \Rightarrow 12 - 1 = 11$
- $n=3 \Rightarrow 18 - 1 = 17$

Matches perfectly.

Constructing the General Term

Combining these observations:

- Sign factor: $(-1)^{n+1}$
- Numerator: 2^{n+1}
- Denominator: $6n - 1$

Thus, the general term:

$$
a_n = \frac{(-1)^{n+1} \times 2^{n+1}}{6n - 1}
$$

Validation of the Formula

Let's verify the formula with initial terms:

- For $n=1$:

$$
a_1 = \frac{(-1)^{2} \times 2^{2}}{6(1) - 1} = \frac{1 \times 4}{6 - 1} = \frac{4}{5}
$$

Matches the first term.

- For $n=2$:

$$
a_2 = \frac{(-1)^{3} \times 2^{3}}{12 - 1} = \frac{-1 \times 8}{11} = -\frac{8}{11}
$$

Matches the second term.

- For $n=3$:

$$
a_3 = \frac{(-1)^{4} \times 2^{4}}{18 - 1} = \frac{1 \times 16}{17} = \frac{16}{17}
$$

Matches the third term.

This confirmation solidifies the general term:

$$
\boxed{
a_n = \frac{(-1)^{n+1} \times 2^{n+1}}{6n - 1}
}
$$

Broader Implications and Applications

Sequence Behavior

- Sign oscillation: The factor $(-1)^{n+1}$ causes the sequence to alternate signs, a common feature in many series and sequences modeling alternating phenomena.
- Growth rate: The numerator grows exponentially (2^{n+1}), while the denominator grows linearly ($6n - 1$). Thus, the magnitude of terms increases exponentially, but the alternating sign influences convergence considerations.

Convergence Analysis

Given the magnitude:

$$
|a_n| = \frac{2^{n+1}}{6n - 1}
$$

since 2^{n+1} grows faster than $6n - 1$, the sequence's terms tend to infinity in magnitude as \(n \to

∞. Therefore, the sequence diverges, oscillating with increasing magnitude.

Extension: Visual and Numerical Analysis

Graphing the sequence for a range of n:

- Plotting a_n reveals the oscillating nature with increasing amplitude.
- Numerical calculations for larger n confirm the divergence, illustrating that the sequence does not settle towards a finite limit.

Educational Significance

The derivation exemplifies the importance of pattern recognition in sequences:

- Recognizing exponential growth patterns.
- Using difference calculations to identify linear patterns.
- Combining algebraic insights to formulate explicit rules.

Such exercises reinforce algebraic manipulation, sequence analysis, and critical thinking.

Conclusion

The exploration of the sequence starting with $4/5$, $-8/11$, $16/17$ showcases how initial terms can encode rich structural information. By methodically analyzing signs, numerators, and denominators, we derived a compact, explicit formula:

$$
\boxed{
a_n = \frac{(-1)^{n+1} \times 2^{n+1}}{6n - 1}
}
$$

This formula not only provides a direct method to generate any term in the sequence but also offers insights into its divergence and oscillatory behavior. Such endeavors exemplify the power of pattern recognition and algebraic reasoning in mathematical analysis, serving as vital tools for students, educators, and researchers aiming to understand the intricate tapestry of sequences and series.

References

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- Rosen, K. H. (2012). Discrete Mathematics and Its Applications. McGraw-Hill Education.

Note: The sequence's derivation underscores the importance of initial terms in uncovering algebraic rules, illustrating a common approach in mathematical problem-solving and research.

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