Given Triangle ABC With Vertices A(3, 0), B(0, 6), And C(4, 6).cFind The Equations Of The Three Altitudes

Given Triangle ABC With Vertices A(3, 0), B(0, 6), And C(4, 6).cFind The Equations Of The Three Altitudes

Understanding the properties and equations of altitudes in a triangle is fundamental in analytical geometry. An altitude of a triangle is a perpendicular segment from a vertex to the line containing the opposite side. In this article, we will find the equations of the three altitudes of triangle ABC with vertices at A(3, 0), B(0, 6), and C(4, 6). We will proceed step-by-step, calculating slopes, equations of sides, and then the altitudes themselves.

Step 1: Identifying the Vertices and Sides of Triangle ABC

Vertices of Triangle ABC

The vertices are given as:
- A(3, 0)
- B(0, 6)
- C(4, 6)

Sides of Triangle ABC

The sides are the segments:
- AB: between points A(3, 0) and B(0, 6)
- BC: between points B(0, 6) and C(4, 6)
- AC: between points A(3, 0) and C(4, 6)

Step 2: Calculating the Equations of the Sides

To find the altitudes, we need the equations of the sides, because the altitudes are perpendicular to these sides.

Equation of Side AB

- Points: A(3, 0), B(0, 6)

Calculate the slope (m_AB):
\[

m_{AB} = \frac{6 - 0}{0 - 3} = \frac{6}{-3} = -2
\]

Using point-slope form with point A(3, 0):
\[
y - 0 = -2(x - 3)
\]
\[
y = -2x + 6
\]

Equation of AB:
\[
\boxed{y = -2x + 6}
\]

Equation of Side BC

- Points: B(0, 6), C(4, 6)

Calculate the slope (m_BC):
\[
m_{BC} = \frac{6 - 6}{4 - 0} = \frac{0}{4} = 0
\]

This is a horizontal line:
\[
y = 6
\]

Equation of BC:
\[
\boxed{y = 6}
\]

Equation of Side AC

- Points: A(3, 0), C(4, 6)

Calculate the slope (m_AC):
\[
m_{AC} = \frac{6 - 0}{4 - 3} = \frac{6}{1} = 6
\]

Using point-slope form with A(3, 0):
\[
y - 0 = 6(x - 3)
\]
\[
y = 6x - 18
\]

Equation of AC:
\[
\boxed{y = 6x - 18}
\]

Step 3: Finding the Equations of the Altitudes

An altitude from a vertex is perpendicular to the opposite side. To find the equations of altitudes:

- Determine the slope of the side opposite the vertex.
- Find the negative reciprocal of that slope (since perpendicular lines have slopes that are negative reciprocals).
- Use the vertex point to write the equation of the altitude.

Altitude from Vertex A to Side BC

- Opposite side of A: BC (which is $y = 6$)
- Slope of BC: 0 (horizontal line)

- The altitude from A must be perpendicular to BC:
- Since BC is horizontal (slope 0), the altitude from A is vertical.

- Equation of the altitude from A:
- Vertical line passing through A(3, 0):

$$
\boxed{x = 3}
$$

Altitude from Vertex B to Side AC

- Opposite side of B: AC (which is $y = 6x - 18$)
- Slope of AC: 6

- Slope of the altitude from B:
- Negative reciprocal of 6 is $-\frac{1}{6}$.

- Using point B(0, 6) and slope $-\frac{1}{6}$:

$$
y - 6 = -\frac{1}{6}(x - 0)
$$
$$
y - 6 = -\frac{1}{6}x
$$
$$
\boxed{y = -\frac{1}{6}x + 6}
$$

Altitude from Vertex C to Side AB

- Opposite side of C: AB (which is $y = -2x + 6$)
- Slope of AB: -2

- Slope of the altitude from C:
- Negative reciprocal of -2 is $\frac{1}{2}$.

- Using point C(4, 6) and slope $\frac{1}{2}$:

$$
y - 6 = \frac{1}{2}(x - 4)
$$
$$
y - 6 = \frac{1}{2}x - 2
$$
$$
y = \frac{1}{2}x + 4
$$

Step 4: Final Equations of the Three Altitudes

Summarizing the results:

- Altitude from A(3, 0): **$x = 3$**
- Altitude from B(0, 6): **$y = -\frac{1}{6}x + 6$**
- Altitude from C(4, 6): **$y = \frac{1}{2}x + 4$**

Conclusion

In this analysis, we have systematically derived the equations of the three altitudes of triangle ABC with vertices A(3, 0), B(0, 6), and C(4, 6). Understanding how to compute these lines is essential in geometric constructions and proofs. The process involves calculating side equations, slopes, and then applying perpendicularity conditions to find the altitude equations. The altitudes intersect at a common point called the orthocenter, which can be further investigated by solving the equations of any two altitudes simultaneously to find this intersection point.

Additional Notes

- The orthocenter of the triangle can be found by solving the intersection of any two altitudes.
- If needed, one can verify the concurrency of the altitudes by substituting the intersection point into the third altitude's equation.
- These concepts are fundamental in coordinate geometry and help in understanding triangle centers and properties.

This comprehensive approach provides a clear methodology for finding the equations of altitudes in any triangle given its vertices, reinforcing fundamental principles of analytical geometry.

Frequently Asked Questions

How do you find the equation of the altitude from vertex A in triangle ABC with vertices A(3,0), B(0,6), and C(4,6)?

First, find the slope of side BC, then determine the negative reciprocal to get the slope of the altitude from A. Next, use point A and this slope to write the equation of the altitude.

What is the slope of side BC in triangle ABC with vertices B(0,6) and C(4,6)?

Since B and C have the same y-coordinate, the slope of BC is 0, indicating that BC is a horizontal line.

How do you find the equation of the altitude from vertex B in triangle ABC?

Calculate the slope of side AC, then find its negative reciprocal for the altitude from B. Use point B and this slope to write the altitude's equation.

What is the slope of side AC in triangle ABC with vertices A(3,0) and C(4,6)?

The slope of AC is $(6 - 0)/(4 - 3) = 6/1 = 6$.

How do you determine the equation of the altitude from vertex C in triangle ABC?

Find the slope of side AB, then take its negative reciprocal for the altitude from C. Use point C and this slope to write the equation.

What is the slope of side AB in triangle ABC with vertices A(3,0) and B(0,6)?

The slope of AB is (6 - 0)/(0 - 3) = 6/(-3) = -2.

How do you find the equations of all three altitudes in triangle ABC?

Calculate the slope of each side, determine the negative reciprocal for each altitude, then use the corresponding vertex to write each altitude's equation.

What is the process to verify that the three altitudes in triangle ABC are concurrent?

After finding the equations of the three altitudes, solve any two to find their point of intersection. Check if the third altitude passes through this point to confirm concurrency.

Additional Resources

Given Triangle ABC With Vertices A(3, 0), B(0, 6), and C(4, 6): Find The Equations Of The Three Altitudes

Understanding the properties of a triangle, especially its altitudes, is fundamental in coordinate geometry. The problem at hand involves a triangle with specific vertices and requires deriving the equations of its three altitudes. This task not only reinforces concepts such as slopes, perpendicular lines, and systems of equations but also demonstrates the application of algebraic methods to geometric figures. Accurate calculation and interpretation of altitudes are essential skills in geometry, and this comprehensive analysis aims to elucidate each step involved in finding these key lines.

Understanding the Triangle and Its Vertices

The given vertices of triangle ABC are:

- A(3, 0)
- B(0, 6)
- C(4, 6)

Plotting these points helps visualize the triangle’s shape. Point A is at (3, 0), B at (0, 6), and C at (4, 6). Notably, points B and C share the same y-coordinate, which indicates that segment BC is horizontal. This insight simplifies some calculations, particularly when finding altitudes related to side BC.

Key features of the triangle:

- Side AB connects points A(3,0) and B(0,6).

- Side AC connects points A(3,0) and C(4,6).
- Side BC connects points B(0,6) and C(4,6).

Understanding the orientation and relative positions of these vertices sets the stage for calculating the equations of the altitudes.

Defining Altitudes in a Triangle

An altitude of a triangle is a perpendicular segment from a vertex to the line containing the opposite side. For triangle ABC:

- Altitude from A is perpendicular to BC.
- Altitude from B is perpendicular to AC.
- Altitude from C is perpendicular to AB.

These altitudes intersect at a common point called the orthocenter. Our goal is to find the equations of these three lines.

Step-by-Step Approach to Find the Altitudes

The general method involves:

1. Finding the equation of the side to which the altitude is perpendicular.
2. Determining the slope of that side.
3. Calculating the slope of the altitude (which is the negative reciprocal of the side's slope).
4. Using the point-slope form with the relevant vertex to find the altitude's equation.

This systematic approach ensures clarity and accuracy.

Finding the Equation of Side BC

Since side BC is horizontal, this makes the process straightforward.

- Points: B(0, 6) and C(4, 6)
- Slope of BC: $m_{BC} = \frac{6 - 6}{4 - 0} = 0$

The equation of BC (a horizontal line) is:

$$ y = 6 $$

This line will be used to find the altitude from A, which is perpendicular to BC.

Equation of Altitude from A (Perpendicular to BC)

- Since BC is horizontal (slope 0), the altitude from A must be vertical, with an undefined slope.
- The point A is (3, 0).

The vertical line passing through A:

$$ x = 3 $$

Equation of altitude from A:

$$ x = 3 $$

This is a straightforward result because the perpendicular to a horizontal line is vertical.

Finding the Equation of Side AC

Points: A(3, 0) and C(4, 6).

- Slope of AC:

$$ m_{AC} = \frac{6 - 0}{4 - 3} = \frac{6}{1} = 6 $$

- Equation of AC using point A:

$$ y - 0 = 6(x - 3) $$

$$ y = 6x - 18 $$

Equation of Altitude from B (Perpendicular to AC)

- Slope of AC is 6.
- The altitude from B must be perpendicular to AC, so its slope:

$$ m_{perp} = -\frac{1}{6} $$

- Point B is (0, 6).

Using point-slope form:

$$ y - 6 = -\frac{1}{6}(x - 0) $$

$$ y - 6 = -\frac{1}{6}x $$

$$ y = -\frac{1}{6}x + 6 $$

Equation of altitude from B:

$$ y = -\frac{1}{6}x + 6 $$

Finding the Equation of Side AB

Points: A(3, 0) and B(0, 6).

- Slope of AB:

$$ m_{AB} = \frac{6 - 0}{0 - 3} = \frac{6}{-3} = -2 $$

- Equation of AB using point A:

$$ y - 0 = -2(x - 3) $$

$$ y = -2x + 6 $$

Equation of Altitude from C (Perpendicular to AB)

- Slope of AB is -2.
- The perpendicular slope:

$$ m_{perp} = \frac{1}{2} $$

- Point C: (4, 6).

Using point-slope form:

$$ y - 6 = \frac{1}{2}(x - 4) $$

$$ y - 6 = \frac{1}{2}x - 2 $$

$$ y = \frac{1}{2}x + 4 $$

Equation of altitude from C:

$$ y = \frac{1}{2}x + 4 $$

Summary of the Equations of the Three Altitudes

- Altitude from A (perpendicular to BC):

$$ x = 3 $$

- Altitude from B (perpendicular to AC):

$$ y = -\frac{1}{6}x + 6 $$

- Altitude from C (perpendicular to AB):

$$ y = \frac{1}{2}x + 4 $$

These three lines intersect at a common point, the orthocenter of the triangle, which could be further verified by solving the equations pairwise.

Verification and Significance

To verify the correctness, one can:

- Find the intersection points of the altitudes pairwise.
- Confirm that all three intersect at a single point, validating the intersection as the orthocenter.

This process involves solving systems of two equations at a time:

Example: Intersection of altitude from A and from B

- From A: $x = 3$
- From B: $y = -\frac{1}{6}x + 6$

Substitute $x = 3$:

$$ y = -\frac{1}{6} \times 3 + 6 = -\frac{1}{2} + 6 = 5.5 $$

Point of intersection:

$$ (3, 5.5) $$

Similarly, check with the altitude from C:

$$ y = \frac{1}{2}x + 4 $$

At $x=3$:

$$ y = \frac{1}{2} \times 3 + 4 = 1.5 + 4= 5.5 $$

Matches the previous intersection point, confirming that all three altitudes meet at (3, 5.5), the orthocenter.

Conclusion and Final Remarks

The process of calculating the equations of the altitudes of triangle ABC with vertices A(3, 0), B(0, 6), and C(4, 6) showcases the interplay between algebra and geometry. By carefully analyzing each side, determining slopes, and applying the point-slope form, we derived the equations of all three altitudes. The key steps involved recognizing the special case of side BC being horizontal, which simplified the altitude from A to a vertical line. The other altitudes followed from perpendicular slopes and point-slope equations. The confirmation of the orthocenter at (3, 5.5) underscores the consistency of the calculations.

Features and Pros:

- Clear step-by-step methodology makes the process accessible.
- Use of coordinate geometry simplifies the derivation of line equations.
- Verification via intersection points enhances confidence in results.
- Highlights the importance of understanding slopes and perpendicularity.

Limitations:

- Assumes familiarity with algebraic manipulation and coordinate geometry.
- For more complex triangles, calculations may become more involved.
- The approach is primarily algebraic; geometric intuition can complement but not replace the calculations.

In summary, this exploration illustrates the power of coordinate geometry in solving classical geometric problems. Finding altitudes through systematic algebraic methods provides precise results and deepens understanding of triangle properties. Whether for academic purposes or practical applications, mastering these techniques is invaluable for anyone studying geometry or related fields.

Given Triangle Abc With Vertices A 3 0 B 0 6 And C 4 6 Cfind The Equations Of The Three Altitudes

Given Triangle Abc With Vertices A 3 0 B 0 6 And C 4 6 Cfind The Equations Of The Three Altitudes

Related Articles

- Sensory Memory Lasts A Fraction Of A Second Before The Information Is Lost. Please Select The Best Answer
- Question 8 A Force F Produces An Acceleration A On An Object Of Mass M. A Force 3F Is Exerted On A Second
- Given The Following Explicit Rule An=3(4)n-1. Find The 9th Term In The Geometric Sequence A786432 B196608

Back to Home