If 0 Is An Angle In Standard Position And Its Terminal Side Passes Through The Point(9,8), Find The Exact

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Understanding angles in standard position is fundamental in trigonometry, especially when working with points on the coordinate plane. When an angle's vertex is at the origin and its initial side aligns with the positive x-axis, its terminal side can pass through various points, giving us the opportunity to determine the angle's measure and related trigonometric functions. In this comprehensive guide, we will explore how to find the exact measure of the angle in standard position whose terminal side passes through the point (9,8). We will break down the problem into manageable steps, involving calculating the radius, the angle itself, and the sine, cosine, and tangent ratios, along with their exact values.

Understanding the Problem and Key Concepts

Before diving into calculations, it's essential to understand the critical concepts involved:

Angles in Standard Position

- An angle in standard position has its vertex at the origin (0,0).
- Its initial side lies along the positive x-axis.
- The terminal side is the ray starting from the origin and passing through a specific point.

Point (9,8)

- The point lies in the first quadrant since both x and y are positive.
- The coordinates will help us determine the angle's measure and the trigonometric ratios.

What Is Being Asked?

- To find the exact value of the angle in standard position, usually expressed in radians or degrees.
- To find the exact values of the sine, cosine, and tangent of that angle.

Step 1: Visualizing the Problem and Coordinates

Visualizing the point (9,8) on the coordinate plane helps clarify the problem:
- The point is located 9 units along the x-axis and 8 units along the y-axis from the origin.
- The terminal side of the angle passes through this point.

This visualization indicates that the angle is formed between the positive x-axis and the line connecting the origin to (9,8).

Step 2: Calculating the Radius (r)

The radius, or the distance from the origin to the point (9,8), is essential in calculating trigonometric ratios.

Distance Formula

The distance from the origin to the point is given by the Pythagorean theorem:

$r = \sqrt{x^2 + y^2}$

Applying the coordinates:
- $x = 9$
- $y = 8$

Calculating:
$r = \sqrt{9^2 + 8^2} = \sqrt{81 + 64} = \sqrt{145}$

Exact Value of r

Thus, the radius is:
- $r = \sqrt{145}$

This is an irrational number, and it will be used to find the exact trigonometric ratios.

Step 3: Finding the Exact Measure of the Angle

The angle θ in standard position can be associated with the point (9,8). To find θ, we use the tangent function:

$\tan(\theta) = y / x$

Calculating tan(θ)

$\tan(\theta) = 8 / 9$

Since both numerator and denominator are integers, the tangent ratio is a rational number.

Expressing θ in Exact Form

To find the exact value of θ, we need to take the inverse tangent (arctangent) of the ratio:

$\theta = \text{arctangent}(8/9)$

While this value can be approximated numerically, for an exact expression, we note:

- $\theta = \text{arctangent}(8/9)$

This is the exact measure of the angle in radians in terms of the inverse tangent function.

Step 4: Expressing the Angle in Degrees and Radians

Depending on the requirement, you may want to express θ in degrees or radians.

Approximate in Degrees

Using a calculator:
$\theta \approx \text{arctangent}(8/9) \approx 0.73$ radians

Converting to degrees:
$\theta \approx 0.73 \times (180/\pi) \approx 41.81°$

Note: This is an approximate value. The exact value remains as $\theta = \text{arctangent}(8/9)$.

Expressing in Exact Radians

The exact measure is simply $\theta = \text{arctangent}(8/9)$.

Step 5: Calculating the Trigonometric Ratios

Using the point (9,8) and the radius $r = \sqrt{145}$, we can now find the sine, cosine, and tangent ratios exactly.

Sine of θ

$\sin(\theta) = y / r = 8 / \sqrt{145}$

Cosine of θ

$\cos(\theta) = x / r = 9 / \sqrt{145}$

Tangent of θ

$\tan(\theta) = y / x = 8 / 9$

Step 6: Rationalizing the Denominator for Exact Values

To present the ratios in a more simplified exact form, we rationalize the denominators where necessary.

Sine

$\sin(\theta) = 8 / \sqrt{145}$
- Rationalize:
$\sin(\theta) = (8 / \sqrt{145}) \times (\sqrt{145} / \sqrt{145}) = (8\sqrt{145}) / 145$

Cosine

$\cos(\theta) = 9 / \sqrt{145}$
- Rationalize:
$\cos(\theta) = (9 / \sqrt{145}) \times (\sqrt{145} / \sqrt{145}) = (9\sqrt{145}) / 145$

Summary of Trigonometric Ratios

- $\sin(\theta) = (8\sqrt{145}) / 145$
- $\cos(\theta) = (9\sqrt{145}) / 145$
- $\tan(\theta) = 8 / 9$

All these ratios are expressed exactly, with the radicals rationalized in the denominators.

Step 7: Interpreting the Results

The calculated ratios and angle measure provide comprehensive information:

- The exact angle in radians: $\theta = \text{arctangent}(8/9)$
- Approximate in degrees: $\approx 41.81°$
- Exact sine: $(8\sqrt{145}) / 145$
- Exact cosine: $(9\sqrt{145}) / 145$
- Exact tangent: $8 / 9$

These values can be used in further trigonometric calculations involving the point (9,8), such as finding the sine, cosine, or tangent of related angles, or solving triangle problems.

Additional Tips for Trigonometry Problems Involving Coordinates

- Always find the radius (r) first when working with points on the coordinate plane.
- Rationalize denominators to express ratios in exact form.
- Use inverse tangent to find angles from ratios, remembering the domain and quadrant considerations.
- Recognize special right triangles and known ratios for quick calculations, though in this case, the ratios are not part of a common special triangle.

Conclusion

When given a point like (9,8) through which the terminal side of an angle in standard position passes, the process to find the exact measure of the angle and its trigonometric ratios involves calculating the radius (distance from the origin to the point), determining the tangent ratio for the angle, and then rationalizing the ratios for exact expressions. The key takeaways include:

- The radius is $r = \sqrt{145}$.
- The exact angle in radians is $\theta = \text{arctangent}(8/9)$.
- The exact trigonometric ratios are $\sin(\theta) = (8\sqrt{145})/145$, $\cos(\theta) = (9\sqrt{145})/145$, and $\tan(\theta) = 8/9$.

This approach ensures precise and exact solutions, which are critical in advanced mathematics and applications involving angles and coordinate geometry.

Keywords: standard position, angle, coordinate point, (9,8), exact value, trigonometry, sine, cosine, tangent, arctangent, rationalize denominator, radians, degrees, coordinate geometry.

Frequently Asked Questions

What is the measure of the angle in radians if its terminal side passes through the point (9,8) and its initial side is on the positive x-axis?

The exact measure of the angle in radians is θ = arctangent(8/9), which is approximately 0.73 radians.

How do you find the exact value of the angle in degrees for a point (9,8) in standard position?

Calculate θ = arctangent(8/9), then convert to degrees: $\theta \approx$ arctangent(8/9) $\times$ (180/π) $\approx$ 39.81°.

What is the significance of the point (9,8) in determining the angle in standard position?

The point (9,8) lies on the terminal side of the angle, allowing us to find the angle's measure using its coordinates and the tangent function.

How do you calculate the exact value of the angle if the terminal side passes through (9,8)?

Use the tangent ratio: $\tan \theta = y/x = 8/9$, so θ = arctangent(8/9).

Does the point (9,8) indicate that the angle is acute or obtuse?

Since both x and y are positive, the angle is in the first quadrant and therefore acute, with a measure less than 90°.

How can I verify the calculation of the angle using the

point (9,8)?

Calculate θ = arctangent(8/9), then verify by checking that cosine and sine values correspond to the point: cos θ = 9/√145, sin θ = 8/√145.

What is the exact value of the cosine and sine of the angle passing through (9,8)?

cos θ = 9/√145 and sin θ = 8/√145, where √145 is the distance from the origin to the point.

How do I find the exact value of the angle in radians from the point (9,8)?

Calculate θ = arctangent(8/9). Since arctangent yields an exact value in terms of inverse tangent, the exact angle in radians is θ = arctangent(8/9).

Additional Resources

If 0 Is An Angle In Standard Position And Its Terminal Side Passes Through The Point (9,8), Find The Exact Value of the Angle is a compelling problem that combines fundamental concepts of coordinate geometry and trigonometry. This type of question often appears in advanced math courses, standardized tests, and practical applications involving navigation, physics, and engineering. Understanding how to analyze angles in standard position, especially when given a specific point through which the terminal side passes, is essential in developing a comprehensive grasp of trigonometric functions and their geometric interpretations.

In this guide, we will explore the process step by step, breaking down the problem into manageable parts. You will learn how to determine the exact value of the angle in standard position based on the given point, using concepts such as the distance formula, trigonometric ratios, and inverse functions. By the end of this article, you will have a clear methodology for solving similar problems efficiently and accurately.

Understanding the Problem

Before diving into calculations, it’s crucial to interpret what the problem is asking and identify the key components.

- Given: An initial angle of 0 radians (or degrees) in standard position, with its terminal side passing through the point (9,8).
- Goal: Find the exact value of the angle in standard position that corresponds to this terminal side passing through that point.

Key Concepts:

- Standard Position: An angle in standard position has its vertex at the origin (0,0) and its

initial side along the positive x-axis.
- Terminal Side: The ray that results from rotating the initial side by the given angle.
- Point (9,8): Lies somewhere in the coordinate plane, and the terminal side of the angle passes through this point, indicating the angle's measure.

Step 1: Visualizing the Scenario

Imagine the coordinate plane with the origin at the center. The initial side of the angle is along the positive x-axis, pointing to the right. The terminal side, after rotation, passes through the point (9,8). Visualizing this helps understand what the angle looks like:

- The terminal side starts at the origin.
- It extends outward, passing through the point (9,8).
- The angle between the initial side and the terminal side is what we need to find.

Step 2: Determine the Radius (Distance from the Origin to the Point)

The first step in analyzing this point relative to the angle is to find the distance from the origin to the point (9,8). This distance, often called the radius (or r), is crucial because it relates to the hypotenuse of a right triangle formed by the x and y coordinates.

Calculating the Radius:

Use the distance formula:

$$
r = \sqrt{(x)^2 + (y)^2}
$$

Plugging in the given coordinates:

$$
r = \sqrt{(9)^2 + (8)^2} = \sqrt{81 + 64} = \sqrt{145}
$$

Thus, the radius is:

$$
r = \sqrt{145}
$$

This distance represents the hypotenuse of a right triangle with legs of lengths 9 and 8.

Step 3: Find the Trigonometric Ratios

Since the terminal side passes through (9,8), the point's coordinates give us the x and y components relative to the origin. These components can be used to find the angle θ in standard position.

Basic Definitions:

- $\cos \theta$ = adjacent / hypotenuse = x / r
- $\sin \theta$ = opposite / hypotenuse = y / r
- $\tan \theta$ = opposite / adjacent = y / x

Using the values:

$$
\cos \theta = \frac{9}{\sqrt{145}}, \quad \sin \theta = \frac{8}{\sqrt{145}}
$$

$$
\tan \theta = \frac{8}{9}
$$

These ratios are crucial in determining the exact value of the angle.

Step 4: Determine the Quadrant

The signs of the x and y coordinates tell us the quadrant where the terminal side lies:

- $x = 9$ (positive)
- $y = 8$ (positive)

Since both are positive, the point is in Quadrant I. This simplifies the process because:

- The angle θ is between 0° and 90° (or 0 and $\pi/2$ radians).
- The inverse tangent function will yield the principal value directly, and no adjustment for quadrant is necessary.

Step 5: Calculate the Exact Angle

Now, we proceed to find the exact value of the angle θ.

Using the inverse tangent:

$$
\theta = \arctan \left(\frac{y}{x} \right) = \arctan \left(\frac{8}{9} \right)
$$

- The ratio 8/9 is a known value, but it does not correspond to a standard angle with a simple radical form.

- Therefore, the exact value of θ remains in terms of the inverse tangent function.

Step 6: Expressing the Exact Value

Given that the problem asks for the exact value of the angle, and the ratio 8/9 is not one of the special angles (like 30°, 45°, 60°, etc.), the most precise way is to leave the answer in terms of inverse tangent:

$$
\boxed{
\theta = \arctan \left(\frac{8}{9} \right)
}
$$

This is the exact value in radians (or degrees, if you convert), representing the measure of the angle in standard position.

Step 7: Optional: Approximate Numerical Value

If a decimal approximation is needed:

$$
\theta \approx \arctan \left(\frac{8}{9} \right) \approx \arctan(0.8889)
$$

Using a calculator:

$$
\theta \approx 41.81^\circ \quad \text{or} \quad 0.73 \text{ radians}
$$

But remember, the exact answer remains:

$$
\boxed{
\theta = \arctan \left(\frac{8}{9} \right)
}
$$

Additional Considerations and Clarifications

What if the Point Was in a Different Quadrant?

- If the point had negative x or y, the quadrant would change, and you'd need to adjust the inverse tangent accordingly, possibly adding π (or 180°).

How about angles greater than 90°?

- If the terminal side passes through points in Quadrants II, III, or IV, you’d need to consider the signs of x and y to determine the correct angle measure.

Role of the Sign of the Angle

- Since the point (9,8) is in Quadrant I, the angle θ is directly given by $\arctan(y/x)$.
- For other quadrants, you might need to add π or 2π to get the correct angle in standard position.

Summary and Key Takeaways

- To find the exact angle in standard position passing through a point (x, y):

1. Calculate the distance from the origin to the point: $r = \sqrt{x^2 + y^2}$.
2. Find the trigonometric ratios: $\sin θ = y/r$, $\cos θ = x/r$, $\tan θ = y/x$.
3. Determine the quadrant based on the signs of (x, y).
4. Use the inverse tangent function ($\arctan$) for the ratio y/x to find θ.
5. Adjust the angle based on the quadrant if necessary.

- The exact value of the angle is:

$$
\boxed{
\theta = \arctan \left(\frac{8}{9} \right)
}
$$

- To convert to degrees or radians, use a calculator or conversion formulas.

Final Remarks

This problem exemplifies the harmony between coordinate geometry and trigonometry, illustrating how geometric insights lead to precise analytical solutions. Mastering these steps not only helps solve similar problems but also deepens understanding of the fundamental relationships in the unit circle and the coordinate plane. Remember, always consider the quadrant and the signs of x and y when determining the exact angle, and keep in mind that some ratios may require leaving your answer in inverse tangent form for true mathematical precision.

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