If The Point (6,10) Is On The Graph Of Y = F(x), Find A Point On The Graph Of A) Y = F(x+2) B) Y= 1/2f(x)

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Understanding how transformations affect the graph of a function is fundamental in algebra and calculus. When given a specific point on the original graph of a function $y = F(x)$, such as (6, 10), it becomes possible to determine corresponding points on transformed graphs. These transformations include horizontal shifts, vertical stretches or compressions, and other modifications that change the appearance and position of the graph. In this article, we will explore methods to find points on the graphs of transformed functions based on the known point, focusing on the specific cases of $y = F(x+2)$ and $y = \frac{1}{2}f(x)$.

Understanding the Original Point and Its Significance

Before delving into transformations, it’s essential to interpret what the point (6, 10) signifies in relation to the original function $y = F(x)$.

The Point (6, 10) on $y = F(x)$

This notation indicates that when $x = 6$, the function $F(x)$ takes the value 10:

$$
F(6) = 10
$$

This is a key piece of information, as it anchors our understanding of the function's behavior at that specific point.

Why is This Important for Transformation?

Transformations modify the graph by shifting, stretching, or compressing it horizontally or vertically. Knowing a specific point on the original graph allows us to find its corresponding point after transformation by applying the rules of each transformation directly to the point’s coordinates.

Transformation A: $y = F(x + 2)$ – Horizontal Shift

The transformation $y = F(x + 2)$ involves a horizontal shift of the original graph. To understand how it affects points, let's analyze the effect on the coordinates.

Effect of $y = F(x + 2)$ on the Graph

This transformation shifts the graph of $F(x)$ to the left by 2 units. The reasoning is as follows:

- For a given x, the value of y is determined by $F(x + 2)$.
- To find the y-value at a particular x, you evaluate F at $x + 2$.

This means that the point on the original graph at $x = a$ corresponds to the point on the transformed graph at $x = a - 2$. Conversely, the point at $x = a - 2$ on the transformed graph corresponds to the original point at $x = a$.

Finding the Corresponding Point

Given $F(6) = 10$, to find the corresponding point on $y = F(x + 2)$:

- We set:

$$
x + 2 = 6
$$

- Solve for x:

$$
x = 6 - 2 = 4
$$

- The y-value remains the same:

$$
y = F(6) = 10
$$

Therefore, the point $(4, 10)$ lies on the graph of $y = F(x + 2)$.

Summary of Transformation A

Original point (x, y)	Transformed point (x', y')	Explanation
$(6, 10)$	$(4, 10)$	Shift left by 2 units

Transformation B: $y = \frac{1}{2}f(x)$ – Vertical Compression

The second transformation involves multiplying the original function by $\frac{1}{2}$, which affects the graph vertically.

Understanding Vertical Compression

- Multiplying $F(x)$ by $\frac{1}{2}$ compresses the graph toward the x-axis.
- For any x, the y-value on the transformed graph becomes half the original y-value.

Mathematically, if (x, y) is on $y = F(x)$, then:

$$
\text{On } y = \frac{1}{2}F(x), \quad (x, y') \quad \text{where} \quad y' = \frac{1}{2} y
$$

Applying the Transformation to Find the Point

Given $F(6) = 10$, the corresponding point on the graph of $y = \frac{1}{2}F(x)$ will be:

- Same x-coordinate: $x = 6$

- y-coordinate scaled by $\frac{1}{2}$:

$$
y' = \frac{1}{2} \times 10 = 5
$$

Thus, the point $(6, 5)$ lies on the graph of $y = \frac{1}{2}f(x)$.

Summary of Transformation B

Original point (x, y)	Transformed point (x, y')	Explanation
$(6, 10)$	$(6, 5)$	Vertically compressed by factor of 1/2

Key Takeaways and Summary

Understanding how to manipulate points based on transformations is essential in graph analysis and function study. Here are the critical points from our exploration:

- Horizontal shifts involve changing the input x value before evaluation. For $y = F(x + c)$, the graph shifts left by c units; for $y = F(x - c)$, it shifts right.
- Vertical stretches or compressions involve multiplying the entire function by a constant. If $y = aF(x)$:
- $a > 1$: vertical stretch (amplification)
- $0 < a < 1$: vertical compression
- Applying transformations to known points involves solving for the new x-coordinate or adjusting the y-value accordingly.

Additional Practice and Applications

To reinforce these concepts, consider the following exercises:

1. Given $F(4) = 7$, find the corresponding points on:
- $y = F(x - 3)$
- $y = 3F(x)$

2. If $F(2) = 5$, determine the point on the graph of $y = F(x + 5)$,

and $y = -2F(x)$.

3. Graph transformations in real-world contexts:
- How do these transformations model real-world phenomena like resizing images, shifting data points, or adjusting signals?

Conclusion

Knowing how to analyze and determine corresponding points after transformations is invaluable for understanding complex functions and their graphs. Starting from a known point, such as (6, 10) on $y = F(x)$, you can effectively find related points on transformed functions through simple algebraic adjustments. Whether shifting graphs horizontally or vertically, stretching or compressing them, these techniques form the foundation of graph analysis in algebra and calculus. Mastery of these concepts empowers students and professionals to interpret and manipulate functions with confidence, fostering deeper comprehension of mathematical behavior and its applications.

Frequently Asked Questions

If the point (6,10) is on the graph of y = f(x), what is the corresponding point on the graph of y = f(x + 2)?

The point on $y = f(x + 2)$ is (4, 10). This is because replacing x with (x + 2) shifts the graph 2 units to the left; thus, when $x = 4$, $f(4 + 2) = f(6) = 10$.

Given that (6,10) is on y = f(x), what point lies on y = (1/2)f(x)?

The point is (6, 5). Multiplying the output by 1/2 scales the graph vertically by a factor of 1/2, so the y-coordinate becomes 10 (1/2) = 5 at x = 6.

How does shifting the function to y = f(x + 2) affect the graph if (6,10) is on y = f(x)?

Shifting to $y = f(x + 2)$ moves the graph 2 units to the left. Therefore, the point that was at $x = 6$ on $y = f(x)$ moves to $x = 4$ on $y = f(x + 2)$.

If (6,10) is on y = f(x), what is the point on the graph of y = (1/2)f(x)?

The point is (6, 5), because scaling the output by 1/2 halves the y-coordinate from 10 to 5.

What is the effect on the graph when transforming y = f(x) to y = f(x + 2)?

The graph shifts 2 units to the left because adding 2 inside the function argument moves the graph horizontally in that direction.

How do you find the new point on y = (1/2)f(x) if the original point is (6, 10) on y = f(x)?

Multiply the original y-coordinate by 1/2: 10 (1/2) = 5. So, the new point is (6, 5).

Why does the point (6,10) on y = f(x) correspond to (4,10) on y = f(x + 2)?

Because replacing x with (x + 2) shifts the graph 2 units to the left, so to find the point on y = f(x + 2), subtract 2 from the original x-coordinate, resulting in (4, 10).

Additional Resources

Understanding Transformations of Functions: A Deep Dive into Shifting and Scaling with Respect to the Point (6,10)

When working with functions and their graphs, one of the most fundamental skills is understanding how alterations to the function—such as shifting or scaling—affect the graph's position and shape. This exploration becomes particularly insightful when we are given specific points on the original graph and asked to find corresponding points on transformed graphs.

In this detailed analysis, we'll examine the problem: If the point (6,10) is on the graph of $y = f(x)$, find a point on the graph of

a) $y = f(x + 2)$
b) $y = \frac{1}{2} f(x)$

We'll unpack each transformation's effect on the graph and demonstrate step-by-step how to determine the corresponding points, emphasizing the core concepts of horizontal shifts and vertical scalings.

Foundational Concepts in Function Transformations

Before tackling the specific problem, it's essential to understand the fundamental types of transformations applied to functions:

- Horizontal Shifts: Moving the graph left or right without changing its shape.

- Vertical Shifts: Moving the graph up or down without changing its shape.

- Vertical Scalings (Stretching or Compression): Changing the height of the graph proportionally.

- Horizontal Scalings: Altering the width of the graph proportionally.

In the given problem, the focus is on horizontal shifts and vertical scalings, which are among the most common transformations in function analysis.

Analyzing the Original Point (6,10) on $y = f(x)$

Given that (6,10) lies on the graph of $y = f(x)$, this immediately tells us:

$$
f(6) = 10
$$

This piece of information acts as a reference point that helps us determine the corresponding points on the transformed graphs.

Transformation A: $y = f(x + 2)$ – Horizontal Shift

Understanding the Transformation

The transformation $y = f(x + 2)$ involves replacing x with $x + 2$. This is a horizontal shift of the original graph.

- Effect on the graph: The graph of $y = f(x + 2)$ is shifted 2 units to the left relative to the graph of $y = f(x)$.

- Why left? Because for a given x, $f(x + 2)$ evaluates the original function at $x + 2$. To find the point on the new graph corresponding to a particular x, you look at the value of the original function at $x + 2$. Conversely, to find the x-coordinate corresponding to a particular point, you need to consider how the input shifts.

Determining the Corresponding Point

Given the original point:

$$
f(6) = 10
$$

we want to find a point (x, y) on the graph of $y = f(x + 2)$ such that:

$$
y = f(x + 2)
$$

Since $f(6) = 10$, to relate this to the transformed function, note that:

$$
f(x + 2) = 10 \quad \Rightarrow \quad x + 2 = 6 \quad \Rightarrow \quad x = 4
$$

Therefore, when $x = 4$, the value of y on the transformed graph is:

$$
y = f(4 + 2) = f(6) = 10
$$

Result: The point on the graph of $y = f(x + 2)$ is (4, 10).

Summary of Transformation A

- The point \((6, 10) \) on \(y = f(x) \) corresponds to the point (4, 10) on \(y = f(x + 2) \).

- Interpretation: The graph shifts 2 units to the left; thus, the point moves from \(x = 6 \) to \(x = 4 \), maintaining the same \(y \)-value.

Transformation B: \(y = \frac{1}{2} f(x) \) — Vertical Scaling (Compression)

Understanding the Transformation

The function \(y = \frac{1}{2} f(x) \) involves multiplying the original function's output by \(\frac{1}{2} \). This results in a vertical compression of the graph by a factor of \(\frac{1}{2} \).

- Effect on the graph: All \(y \)-values are scaled down by half, making the graph "flatter" vertically.

- Intuition: For the same \(x \)-value, the output is halved, decreasing the height of the graph at every point.

Determining the Corresponding Point

Given \(f(6) = 10 \), for the transformed function:

\[
y = \frac{1}{2} f(x)
\]

we evaluate at \(x = 6 \):

\[
y = \frac{1}{2} \times f(6) = \frac{1}{2} \times 10 = 5
\]

Thus, the point \((6, 10) \) on the original graph corresponds to (6, 5) on the scaled graph.

Result: The point on the graph of \(y = \frac{1}{2} f(x) \) is (6, 5).

Summary of Transformation B

- The original point $(6, 10)$ becomes (6, 5) after the vertical compression.

- Interpretation: The entire graph is squeezed vertically by a factor of $\frac{1}{2}$, reducing the y-coordinate from 10 to 5 at the same x-coordinate.

Broader Implications and Additional Insights

Understanding these transformations at a deeper level leads to several important insights about function behavior and graph manipulation:

1. Horizontal Shifts and Their Effect on Input-Output Relationships

- When the function is modified as $y = f(x + k)$, the graph shifts k units to the left if $k > 0$, and $|k|$ units to the right if $k < 0$.

- The key to understanding this is the inner transformation: $x \rightarrow x + k$. To find the corresponding point, set $x + k = x_{original}$ from the original graph.

2. Vertical Scalings and Their Effect on Function Values

- When the function is scaled as $y = a \times f(x)$, the entire graph is scaled vertically by a factor of a.

- For $a > 1$, the graph stretches vertically, increasing the height proportionally.

- For $0 < a < 1$, the graph compresses vertically, reducing the height.

- When a is negative, it reflects the graph across the x-axis in addition to scaling.

3. Combining Transformations

- More complex transformations involve combining shifts and scalings, such as:

$$
y = a \times f(x + k) + c
$$

- Each transformation affects the graph in a predictable way:

- $x + k$: horizontal shift

- a: vertical scaling

- c: vertical shift

- To find a corresponding point, apply each transformation step-by-step, starting from the original point.

4. Application in Graph Sketching and Analysis

- These principles are crucial in graphing functions quickly and accurately.

- Recognizing how a point on the original graph maps to the transformed graph simplifies the process of sketching and analyzing complex functions.

5. Real-World Relevance

- These transformations model real-world phenomena:

- Horizontal shifts can represent delays or advances in processes.

- Vertical scalings can indicate amplification or attenuation of signals.

- Combining transformations helps in modeling complex systems with multiple factors.

Conclusion: The Power of Transformation in Function Analysis

This in-depth examination of the problem involving the point $(6, 10)$ on

\(y = f(x) \) and its transformations underscores the importance of understanding how shifts and scalings affect a graph's position and shape.

Key takeaways include:

- Horizontal shifts are handled by adjusting the input variable and understanding the inverse relationship between input and shifts.

- Vertical scalings directly affect the output values, scaling the entire graph proportionally.

- The original point provides a reference to determine corresponding points after transformation, highlighting the importance of function evaluation at specific points.

- Combining these transformations enables the analysis of more complex functions and their graphs, essential for advanced mathematics, engineering, physics, and data modeling.

Practical Application Summary:

| Transformation

If The Point 6 10 Is On The Graph Of Y F X Find A Point On The Graph Of A Y F X 2 B Y 1 2f X

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