If Using The Method Of Completing The Square To Solve The Quadratic Equationx - 13x +31 = 0, Which Number

If Using The Method Of Completing The Square To Solve The Quadratic Equation x - 13x + 31 = 0, Which Number is a question that often arises when students and math enthusiasts attempt to understand the process of solving quadratic equations by completing the square. This method is a fundamental algebraic technique that transforms a quadratic equation into a perfect square trinomial, making it easier to solve for the variable. In this article, we will explore the concept thoroughly, explaining each step with clarity, and answer the key question: which number should be added to complete the square in the given quadratic equation?

Understanding the Quadratic Equation

What Is a Quadratic Equation?

A quadratic equation is any second-degree polynomial equation of the form:

```
ax^2 + bx + c = 0
```

where:
- a ≠ 0
- b and c are coefficients

In our case, the original equation appears to be:

```
x - 13x + 31 = 0
```

However, this expression simplifies to:

```
-12x + 31 = 0
```

which is linear, not quadratic.

Important Clarification:
To apply the method of completing the square effectively, the equation must be quadratic, i.e., it should have an \(x^2 \) term. If the original problem is \(x^2 - 13x + 31 = 0 \), then it is a quadratic equation suitable for completing the square.

Assumption:
Given the context, it's likely that the intended equation is:

```
x^2 - 13x + 31 = 0
```

which is quadratic and suitable for the method.

Rewriting the Equation for Completing the Square

Step 1: Ensure the Equation Is in Standard Form

The standard form for completing the square is:

$$ax^2 + bx + c = 0$$

For our case:

$$x^2 - 13x + 31 = 0$$

Here:
- $a = 1$
- $b = -13$
- $c = 31$

Step 2: Isolate the Quadratic and Linear Terms

To complete the square, move the constant term to the other side:

$$x^2 - 13x = -31$$

Completing the Square: The Core Concept

What Does Completing the Square Mean?

Completing the square involves rewriting a quadratic expression in the form:

$$(x + d)^2$$

which expands to:

$$x^2 + 2dx + d^2$$

This form makes it straightforward to solve for x by taking square roots.

Step 3: Find the Number to Complete the Square

To transform $x^2 - 13x$ into a perfect square trinomial, identify the coefficient of x, which is -13.

The process involves:
- Taking half of the coefficient of x: $\frac{-13}{2}$
- Squaring this value: $\left(\frac{-13}{2}\right)^2$

Calculations:

1. Half of -13: $\frac{-13}{2} = -6.5$

2. Square of -6.5: $(-6.5)^2 = 42.25$

Therefore, the number to add to the left side to complete the square is 42.25.

Transforming the Equation

Step 4: Add the Number to Both Sides

To maintain equality, add 42.25 to both sides:

$$x^2 - 13x + 42.25 = -31 + 42.25$$

Simplify the right side:

$$-31 + 42.25 = 11.25$$

Step 5: Express as a Perfect Square

Now, the left side can be written as:

$$(x - 6.5)^2$$

since:

$$(x - 6.5)^2 = x^2 - 13x + 42.25$$

Resulting Equation:

$$(x - 6.5)^2 = 11.25$$

Solving the Equation

Step 6: Take the Square Root of Both Sides

Applying square root:

$$x - 6.5 = \pm \sqrt{11.25}$$

Calculate $\sqrt{11.25}$:

$$\sqrt{11.25} \approx 3.3541$$

Step 7: Final Solutions for x

Adding 6.5 to both sides:

$$x = 6.5 \pm 3.3541$$

which yields two solutions:

1. $x \approx 6.5 + 3.3541 = 9.8541$

2. $x \approx 6.5 - 3.3541 = 3.1459$

Summary:
The number you add to complete the square is 42.25.

Key Takeaways and Tips

Understanding the Process

- The critical step in completing the square is halving the coefficient of x and squaring it.
- This number ensures the quadratic can be expressed as a perfect square trinomial.
- Always perform the same operation on both sides to maintain equality.

Common Mistakes to Avoid

- Confusing the original quadratic form; ensure the equation is quadratic (x^2 term present).
- Forgetting to add the number to both sides.
- Incorrect calculation of the square of half the coefficient.

Additional Examples of Completing the Square

- For quadratic: $x^2 + 4x + 1 = 0$
- Half of 4 is 2, square is 4.
- Add 4 to both sides.
- Rewrite as $(x + 2)^2 = 4 - 1 = 3$.
- Solve for x: $x + 2 = \pm\sqrt{3}$.
- Final solutions: $x = -2 \pm \sqrt{3}$.

- For quadratic: $2x^2 - 8x + 6 = 0$
- Divide entire equation by 2: $x^2 - 4x + 3 = 0$.
- Half of -4 is -2, square is 4.
- Add 4 to both sides: $x^2 - 4x + 4 = -3 + 4 = 1$.
- Rewrite as $(x - 2)^2 = 1$.
- Solve: $x - 2 = \pm 1$, so solutions are $x = 3$ and $x = 1$.

Conclusion

Using the method of completing the square to solve quadratic equations is an elegant and systematic approach. For the specific equation $x^2 - 13x + 31 = 0$, the number that must be added to complete the square is 42.25. This process not only simplifies solving quadratic equations but also deepens understanding of algebraic structures. Remember, the key is always to take half of the linear coefficient, square it, and add that to both sides of the equation. This transforms the quadratic into a perfect square trinomial, making the solution straightforward. With practice, completing the square becomes a powerful tool in your algebraic toolkit.

Frequently Asked Questions

What is the first step when using completing the square to solve the quadratic equation $x^2 - 13x + 31 = 0$?

The first step is to move the constant term to the other side, so the equation becomes $x^2 - 13x = -31$.

Which number should be added to both sides to complete the square for $x^2 - 13x = -31$?

You add $(13/2)^2 = (13/2)(13/2) = 169/4$ to both sides to complete the square.

What is the value of the number used to complete the square in the equation $x^2 - 13x + 31 = 0$?

The number is 169/4.

After completing the square, what is the next step to find the solutions of the quadratic?

Rewrite the left side as a perfect square $(x - 13/2)^2$ and then solve for x by taking the square root of both sides.

How do you determine the solutions of the quadratic equation after completing the square?

You take the square root of both sides, remember to consider both positive and negative roots, then solve for x to find the solutions.

Additional Resources

If Using The Method Of Completing The Square To Solve The Quadratic Equation $x^2 - 13x + 31 = 0$, Which Number?

Quadratic equations are fundamental components of algebra, serving as essential tools across mathematics, science, engineering, and various applied fields. Among the many methods to solve quadratic equations, the technique of completing the square holds a distinguished place due to its conceptual clarity and historical significance. This article explores the process of solving the quadratic equation $x^2 - 13x + 31 = 0$ using completing the square, with a keen focus on identifying the critical "number" involved—namely, the constant term added and subtracted during the process. We will analyze the method step-by-step, examine the underlying mathematical principles, and discuss the significance of the key number that emerges in this context.

Understanding the Quadratic Equation and Its Components

Before delving into the method of completing the square, it is essential to parse the structure of the quadratic equation in question:

$x^2 - 13x + 31 = 0$

This standard form involves three components:

- The quadratic term: x^2
- The linear term: $-13x$
- The constant term: 31

The goal is to manipulate this equation into a form that makes it straightforward to solve for x, typically by isolating the quadratic expression and then taking square roots. The method of completing the square transforms the original quadratic into a perfect square trinomial, which can be expressed as a binomial squared.

The Method of Completing the Square: An Overview

Completing the square is a technique that rewrites a quadratic expression in the form:

$$ax^2 + bx + c = 0$$

as

$$a(x + d)^2 + e = 0$$

where d and e are constants determined through algebraic manipulation. The core idea involves adding and subtracting a specific number to the original quadratic to form a perfect square trinomial.

Historical Context:
This method dates back to ancient mathematicians, including the Babylonians and Indian scholars, and was later formalized in the work of European mathematicians such as Al-Khwarizmi and Descartes. It is not only a solution technique but also provides insight into the geometric interpretation of quadratics.

The Critical "Number" in Completing the Square

The key "number" in completing the square pertains to the value added and subtracted to the quadratic expression to form a perfect square. For a quadratic in the form:

$$x^2 + bx + c$$

the "completing the square" process involves adding and subtracting $(b/2)^2$. This number, $(b/2)^2$, is central because it ensures the expression becomes a perfect square trinomial.

In our specific quadratic:

$$x^2 - 13x + 31$$

the coefficient b is -13, so the critical number is:

$$(-13/2)^2 = (-6.5)^2 = 42.25$$

This value, 42.25, is the "number" that is added and subtracted within the process to facilitate the perfect square formation.

Step-by-Step Solution Using Completing the Square

Applying the method to the given quadratic involves systematic steps:

Step 1: Isolate the quadratic and linear terms

The quadratic is already in standard form:

$x^2 - 13x + 31 = 0$

Step 2: Move the constant to the other side

$x^2 - 13x = -31$

Step 3: Determine the number to complete the square

Calculate:

$(-13/2)^2 = 42.25$

Step 4: Add and subtract this number on the left side

$x^2 - 13x + 42.25 = -31 + 42.25$

This maintains equality because adding and subtracting the same number keeps the expression balanced.

Step 5: Write the left side as a perfect square trinomial

$(x - 6.5)^2 = 11.25$

Note:
- The perfect square trinomial is $(x + d)^2$, where d is half the coefficient of x with the sign adjusted appropriately.
- Since $b = -13$, $d = -13/2 = -6.5$.

Step 6: Solve for x by taking square roots

$x - 6.5 = \pm\sqrt{11.25}$

Calculate $\sqrt{11.25}$:

$\sqrt{11.25} \approx 3.3541$

Thus:

$x = 6.5 \pm 3.3541$

Step 7: Write the solutions explicitly

$x \approx 6.5 + 3.3541 \approx 9.8541$

$x \approx 6.5 - 3.3541 \approx 3.1459$

The Significance of the "Number" 42.25

The number 42.25 emerges as the crux of the completing the square process for the quadratic $x^2 - 13x + 31 = 0$. It is the "key number" that bridges the original quadratic to its perfect square form. This number is computed from half the linear coefficient squared—specifically, $(b/2)^2$.

Why is this number important?

- It transforms the quadratic into a perfect square, enabling straightforward extraction of roots.
- It embodies the geometric interpretation of the quadratic as a parabola's vertex form.
- It reflects the "distance" from the original quadratic to a perfect square, encapsulating the essence of the method.

Mathematical Implications of the Key Number

The number 42.25 is not arbitrary; it is deeply tied to the structure of the quadratic. Its derivation and role exemplify the algebraic harmony underpinning quadratic equations. The process reveals that:

- The coefficient b directly influences the "completing the square" number.
- The constant term c, in conjunction with this number, determines the solutions' nature.
- The method emphasizes the symmetry and quadratic nature of the problem.

Alternative Perspectives and Broader Context

While the completing the square method is often introduced as a solution technique, its conceptual depth extends further:

- Graphical interpretation: The number 42.25 corresponds to the horizontal shift of the parabola's vertex from the origin.
- Connection to quadratic formula: The discriminant $(b^2 - 4ac)$ is related to the square of this number, illustrating how completing the square and quadratic formula are intertwined.
- Historical significance: Understanding the critical number deepens appreciation for algebra's development and its geometric roots.

Conclusion: Which Number? The Answer Lies in the Completing the Square

In solving the quadratic equation $x^2 - 13x + 31 = 0$ using completing the square, the pivotal number is 42.25. This number arises from $(b/2)^2$, where $b = -13$, and is the key to transforming the quadratic into a perfect square trinomial. Its role is fundamental: it unlocks the solution by enabling the extraction of square roots and revealing the roots of the quadratic.

The process underscores that the "number" in question is not merely a numerical artifact but a reflection of the algebraic structure and geometric interpretation of quadratic equations. Recognizing and understanding this number enhances our conceptual grasp of algebra and illustrates the elegance and power of completing the square as a solution method.

In broader mathematical discourse, this number exemplifies how a simple algebraic manipulation can unveil profound insights into the nature of quadratic functions and their solutions. Whether for educational purposes, theoretical exploration, or practical problem-solving, the "number" that completes the square remains a cornerstone of quadratic analysis.

References

- Stewart, J. (2012). Precalculus: Mathematics for Calculus. Brooks Cole.
- Larson, R., & Edwards, B. H. (2013). Precalculus. Cengage Learning.
- Stillwell, J. (2010). Mathematics and Its History. Springer.
- Halmos, P. R. (1974). Naive Set Theory. Springer.

Note: The above exploration emphasizes a detailed understanding of the key number involved in completing the square for the quadratic equation $x^2 - 13x + 31 = 0$, offering insights into both the algebraic process and its broader mathematical significance.

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