# In A Certain Shipment Of 12 Computers, 6 Are Defective. 5 Of The Computers Are Selected At Random Without

## Understanding the Scenario: Computers in a Shipment

**In A Certain Shipment Of 12 Computers, 6 Are Defective. 5 Of The Computers Are Selected At Random Without** replacement, a common problem in statistics and probability theory, particularly in quality control and sampling methods. This scenario provides an excellent example to explore concepts such as probability, combinations, and the analysis of defective items within a batch. In this article, we will delve into the details of this problem, explain how to approach it using probability formulas, and highlight important concepts relevant to similar real-world situations.

## Breaking Down the Problem

Before diving into calculations, it's essential to understand the components of the problem:

- Total computers in the shipment: 12
- Number of defective computers: 6
- Number of non-defective (good) computers: 6
- Number of computers selected: 5
- Selection method: Without replacement (once a computer is selected, it is not put back into the pool)

This setting involves probability calculations related to selecting defective and non-defective computers, which can be approached using combinatorics and hypergeometric distribution.

## Key Concepts in Probability and Combinatorics

### 1. Combinations

Combinations are used when selecting items where the order does not matter. The number of ways to choose 'k' items from 'n' items is given by:

$$
C(n, k) = \frac{n!}{k!(n - k)!}
$$

where:
- $n!$ denotes factorial of n

- $C(n, k)$ is called "n choose k"

## 2. Hypergeometric Distribution

This distribution models the probability of drawing a specific number of defective items in a sample, without replacement. Its probability mass function (PMF) is:

$$
P(X = k) = \frac{C(D, k) \times C(N - D, n - k)}{C(N, n)}
$$

where:
- $N$ = total population size
- $D$ = total number of defective items
- $n$ = number of items drawn
- $k$ = number of defective items in the sample

This formula calculates the probability that exactly $k$ defective items are in the sample.

# Applying the Concepts to the Computer Shipment

Given the initial problem:

- Total computers, $N = 12$
- Defective computers, $D = 6$
- Non-defective computers, $N - D = 6$
- Sample size, $n = 5$

Let's explore different scenarios, such as calculating the probability that exactly $k$ defective computers are in the sample.

# Calculating Probabilities for Different Numbers of Defective Computers

## 1. Probability of Selecting Exactly 0 Defective Computers

This scenario involves choosing all 5 computers from the non-defective group:

$$
P(X = 0) = \frac{C(6, 0) \times C(6, 5)}{C(12, 5)}
$$

Calculations:

- $C(6, 0) = 1$
- $C(6, 5) = 6$
- $C(12, 5) = \frac{12!}{5! \times 7!} = 792$

Thus,

$$
P(X=0) = \frac{1 \times 6}{792} = \frac{6}{792} = \frac{1}{132} \approx 0.00758
$$

Interpretation: There's approximately a 0.76% chance that none of the selected computers are defective.

## 2. Probability of Selecting Exactly 1 Defective Computer

This involves selecting 1 defective and 4 non-defective:

$$
P(X = 1) = \frac{C(6, 1) \times C(6, 4)}{C(12, 5)}
$$

Calculations:

- $C(6, 1) = 6$
- $C(6, 4) = 15$
- $C(12, 5) = 792$

So,

$$
P(X=1) = \frac{6 \times 15}{792} = \frac{90}{792} = \frac{15}{132} \approx 0.1136
$$

Interpretation: There's about an 11.36% chance exactly one defective computer is selected.

## 3. Probability of Selecting Exactly 2 Defective Computers

This involves choosing 2 defective and 3 non-defective:

$$
P(X=2) = \frac{C(6, 2) \times C(6, 3)}{C(12, 5)}
$$

Calculations:

- $C(6, 2) = 15$
- $C(6, 3) = 20$

Thus,

$$
P(X=2) = \frac{15 \times 20}{792} = \frac{300}{792} = \frac{25}{66} \approx 0.3788
$$

Interpretation: About 37.88% chance of selecting exactly 2 defective computers.

## 4. Probability of Selecting 3 or More Defective Computers

We can sum the probabilities for 3, 4, and 5 defective computers, but given the small total, it's often easier to compute each:

- For 3 defective:

$$
P(X=3) = \frac{C(6, 3) \times C(6, 2)}{C(12, 5)} = \frac{20 \times 15}{792} = \frac{300}{792} \approx 0.3788
$$

- For 4 defective:

$$
P(X=4) = \frac{C(6, 4) \times C(6, 1)}{C(12, 5)} = \frac{15 \times 6}{792} = \frac{90}{792} = \frac{15}{132} \approx 0.1136
$$

- For 5 defective (all selected are defective):

$$
P(X=5) = \frac{C(6, 5) \times C(6, 0)}{C(12, 5)} = \frac{6 \times 1}{792} = \frac{6}{792} = \frac{1}{132} \approx 0.00758
$$

Adding these:

$$
P(\text{3 or more defective}) \approx 0.3788 + 0.1136 + 0.00758 \approx 0.49998
$$

or roughly 50%.

# Summary of Probabilities

| Number of Defective in Sample | Approximate Probability |
|---|---|
| 0 | 0.76% |

| | |
|---|---|
| 1 | 11.36% |
| 2 | 37.88% |
| 3 | 37.88% |
| 4 | 11.36% |
| 5 | 0.76% |

This distribution shows that selecting exactly 2 or 3 defective computers is the most likely scenario, each with nearly 38% chance.

# Applications of the Problem in Quality Control

Understanding how to calculate such probabilities is essential in quality control processes. For instance:

- Assessing the likelihood of defective items in a sample helps in decision-making about shipment quality.
- Determining acceptance criteria: If a certain maximum number of defective units is acceptable, probability calculations inform whether shipments meet quality standards.
- Sampling plans: Knowing the probability distributions guides the design of sampling strategies that balance cost and accuracy.

# Practical Considerations and Extensions

## 1. Larger Shipments and Different Sample Sizes

The same principles apply when the total shipment size or sample size changes. Adjust the values in the hypergeometric formula accordingly.

## 2. Multiple Defect Types

If there are different types of defects, probabilities can be extended to handle multiple categories using multinomial distributions.

## 3. Software Tools for Calculations

Calculators and statistical software such as R, Python (with scipy.stats), or specialized quality control tools can efficiently compute these probabilities, especially for larger numbers or more complex scenarios.

# Conclusion

The problem of selecting a subset of computers from a shipment containing defective units illustrates fundamental concepts in probability and combinatorics. By understanding how to apply the hypergeometric distribution and combinations, one can accurately assess the likelihood of various outcomes in quality control and sampling procedures. Mastery of these techniques enhances decision-making in manufacturing, logistics, and quality assurance, ensuring products meet standards while optimizing resource use.

Whether you're managing inventory, conducting audits, or designing sampling plans, these probability principles are invaluable tools in ensuring product quality and reliability.

# Frequently Asked Questions

## What is the probability that all 5 selected computers are non-defective?

The probability is calculated by choosing all 5 from the 6 non-defective computers out of the total ways to choose any 5 from all 12 computers: (6C5) / (12C5) = 6 / 792 = 1 / 132 ≈ 0.00758.

## What is the probability that exactly 3 of the selected computers are defective?

Number of ways to select 3 defective (from 6) and 2 non-defective (from 6): (6C3) (6C2) = 20 15 = 300. Total ways to select 5 from 12: (12C5) = 792. Thus, probability = 300 / 792 ≈ 0.3788.

## What is the probability that none of the selected computers are defective?

Since there are 6 defective and 6 non-defective, selecting none defective means choosing all 5 from the 6 non-defective: (6C5) / (12C5) = 6 / 792 = 1 / 132 ≈ 0.00758.

## If 2 defective computers are selected, how many different combinations are possible?

Number of combinations selecting 2 defective from 6 and 3 non-defective from 6: (6C2) (6C3) = 15 20 = 300 combinations.

## What is the probability that at least 1 defective computer is selected?

Probability that none are defective is 1/132. Therefore, probability that at least 1 is defective = 1 - (1/132) ≈ 131/132 ≈ 0.9924.

## Is it more likely to select all non-defective computers or at

## least one defective?

It is more likely to select at least one defective computer (probability ≈ 0.9924) than selecting none (probability ≈ 0.00758).

# Additional Resources

In a certain shipment of 12 computers, 6 are defective—this scenario presents a classic problem in probability theory and statistics, often used to illustrate concepts related to hypergeometric distributions, sampling without replacement, and risk assessment. Such problems are not only theoretically interesting but also practically significant for quality control, inventory management, and decision-making processes in manufacturing and supply chain operations. This article explores the various facets of the problem, including the underlying probability concepts, methods of calculation, real-world applications, and implications for quality assurance.

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# Understanding the Problem Context

The given problem involves a shipment of 12 computers, with exactly 6 defective units and 6 non-defective units. When selecting 5 computers at random without replacement, questions naturally arise: What is the probability that a certain number of defective computers are among the selected units? How does the composition of the shipment influence these probabilities? These questions are central to quality control practices, where the aim is often to determine the likelihood of selecting defective items in a sample, thereby informing decisions such as acceptance or rejection of entire shipments.

## Core Elements of the Scenario

- Total units in the shipment: 12
- Number of defective units: 6
- Number of non-defective units: 6
- Sample size: 5
- Sampling method: Without replacement

This setup emphasizes the importance of the hypergeometric distribution, which models the probability of drawing a specific number of defective items in a sample when selecting without replacement.

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# Hypergeometric Distribution: The Mathematical

# Backbone

The hypergeometric distribution is pivotal in calculating probabilities in sampling without replacement scenarios involving two categories (defective and non-defective). Its probability mass function (PMF) is expressed as:

$$
P(X = k) = \frac{\binom{D}{k} \binom{N - D}{n - k}}{\binom{N}{n}}
$$

where:
- $N$ = total population size (12 in this case)
- $D$ = total number of defective items (6)
- $n$ = size of the sample drawn (5)
- $k$ = number of defective items in the sample

This formula calculates the probability that exactly $k$ defective computers are in the selected sample.

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# Calculating Probabilities for Different Outcomes

One of the key questions in such a scenario is: What is the probability that exactly $k$ defective computers are selected? For $k$ ranging from 0 up to 5 (since the sample size is 5), the probabilities can be computed explicitly.

## Example Calculations

- Probability of selecting zero defective units ($k=0$):

$$
P(X=0) = \frac{\binom{6}{0} \binom{6}{5}}{\binom{12}{5}} = \frac{1 \times 6}{792} \approx 0.0076
$$

- Probability of selecting exactly one defective unit ($k=1$):

$$
P(X=1) = \frac{\binom{6}{1} \binom{6}{4}}{\binom{12}{5}} = \frac{6 \times 15}{792} \approx 0.1136
$$

- Probability of selecting exactly two defective units ($k=2$):

\[

$$
P(X=2) = \frac{\binom{6}{2} \binom{6}{3}}{\binom{12}{5}} = \frac{15 \times 20}{792} \approx 0.3788
$$

- Probability of selecting exactly three defective units ($k=3$):

$$
P(X=3) = \frac{\binom{6}{3} \binom{6}{2}}{\binom{12}{5}} = \frac{20 \times 15}{792} \approx 0.3788
$$

- Probability of selecting exactly four defective units ($k=4$):

$$
P(X=4) = \frac{\binom{6}{4} \binom{6}{1}}{\binom{12}{5}} = \frac{15 \times 6}{792} \approx 0.1136
$$

- Probability of selecting all five defective units ($k=5$):

$$
P(X=5) = \frac{\binom{6}{5} \binom{6}{0}}{\binom{12}{5}} = \frac{6 \times 1}{792} \approx 0.0076
$$

These calculations reveal that the most probable outcomes are selecting 2 or 3 defective units in the sample, each with nearly 38% probability.

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# Applications and Implications in Quality Control

Understanding these probabilities has practical significance in quality control processes within manufacturing, logistics, and supply chain management. The insights derived from hypergeometric calculations guide decisions such as acceptance sampling, inspection strategies, and risk assessments.

## Acceptance Sampling

Acceptance sampling involves inspecting a subset of units from a shipment and deciding whether to accept or reject the entire batch based on the number of defective items found. Knowing the probabilities of different defect counts in samples helps determine appropriate sampling sizes and acceptance criteria to balance risks of passing defective shipments or rejecting good ones.

Pros:
- Efficiently reduces inspection costs.
- Provides statistical assurance about shipment quality.

- Can be tailored based on risk tolerance levels.

Cons:
- May miss certain defect patterns if sampling is too small.
- Does not guarantee complete defect detection.
- Depends heavily on the accuracy of initial defect estimates.

## Risk Management

Quantitative probability analysis enables manufacturers to quantify risks associated with accepting shipments with a certain defect rate. For instance, if the acceptable defect rate is set at a threshold, the probability calculations inform the likelihood of accepting a shipment that exceeds this threshold.

Features:
- Facilitates informed decision-making.
- Allows for setting statistically justified quality thresholds.
- Supports compliance with industry standards.

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# Limitations and Assumptions

While the hypergeometric model offers precise calculations under ideal conditions, real-world scenarios often involve additional complexities.

## Assumptions:
- Random sampling without bias.
- Known and fixed defect rate.
- No correlation between defective units.

## Limitations:
- Assumes perfect randomness, which may not be achievable.
- Does not account for heterogeneity in defect distribution.
- Real shipments may have clusters of defects, deviating from model assumptions.

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# Extensions and Related Concepts

The problem can be extended or related to other statistical models and scenarios:

- Sampling with Replacement: Transitioning to binomial distribution models when items are sampled with replacement.
- Multiple Shipments: Comparing probabilities across batches with varying defect rates.
- Bayesian Updating: Incorporating prior knowledge about defect rates and updating beliefs based on sample results.
- Quality Improvement Strategies: Using data-driven insights to reduce defect rates over time.

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## Conclusion and Practical Takeaways

The problem of selecting 5 computers from a shipment of 12, where 6 are defective, exemplifies fundamental probability principles with significant practical implications. The hypergeometric distribution provides a rigorous framework for understanding the likelihood of various defect counts in samples, thereby enabling more effective quality control and risk management strategies.

Key points to remember:
- The most probable outcomes are selecting 2 or 3 defective units.
- The probabilities are symmetric around the midpoint, reflecting the balanced composition of the shipment.
- Proper sampling plans can significantly reduce the risk of accepting defective shipments or rejecting good ones.

In real-world applications, combining such probabilistic analysis with practical inspection policies and quality management systems enhances the ability to maintain high standards, reduce costs, and ensure customer satisfaction. As manufacturing processes evolve and data collection becomes more sophisticated, integrating statistical models like the hypergeometric distribution will remain central to effective quality assurance.

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Final thoughts:
Understanding the probabilistic nature of defective item selection in small batches is vital. It allows managers and quality engineers to design better sampling plans, interpret inspection results accurately, and make data-driven decisions that uphold product quality and operational efficiency.

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