Let C Be The Curve Of Intersections Of The Cylinder Y2+z2=9 And The Plane X=4. (a) Find A Parametrization

Let C Be The Curve Of Intersections Of The Cylinder Y2+z2=9 And The Plane X=4. (a) Find A Parametrization

---

Understanding the Intersection of a Cylinder and a Plane in 3D Geometry

In the realm of three-dimensional calculus and analytical geometry, understanding the intersection of different surfaces offers deep insights into spatial relationships. One common problem involves finding the intersection curve of a cylinder and a plane, which not only enhances our comprehension of geometric forms but also has practical applications in engineering, computer graphics, and physics.

This article will explore the process of parametrizing the curve of intersection between the cylinder defined by $y^2 + z^2 = 9$ and the plane $x = 4$. We will systematically analyze the problem, derive the parametric equations, and discuss the significance of this intersection curve.

Defining the Surfaces: Cylinder and Plane

Before diving into the parametrization process, it is crucial to understand the surfaces involved:

The Cylinder $y^2 + z^2 = 9$

- This is a right circular cylinder aligned along the x-axis.
- The equation indicates that for any point on this surface, the y and z coordinates satisfy the circle of radius 3 in the yz-plane.
- The cylinder extends infinitely along the x-axis, but in this problem, the intersection occurs at a specific plane.

The Plane $x = 4$

- This is a vertical plane parallel to the yz-plane.
- It slices through the cylinder at the position where the x-coordinate is fixed at 4.

Visualizing the Intersection Curve

Visualizing the intersection helps in understanding the shape and nature of the curve:

- The cylinder extends infinitely along the x-axis, with a circular cross-section in the yz-plane.
- The plane $x = 4$ cuts through the cylinder perpendicular to the x-axis, resulting in a circular intersection.
- The intersection curve is, therefore, a circle lying in the plane $x=4$, with its shape determined by the cylinder's cross-sectional circle.

Step-by-Step Process to Find the Parametrization of the Curve

The key goal is to find a set of parametric equations that describe the curve C, which is the intersection of the two surfaces.

Step 1: Recognize the Fixed Coordinate

- Since the plane $x=4$ is fixed, the x-coordinate remains constant along the intersection curve.
- Therefore, any parametric form of the intersection curve will include $x=4$.

Step 2: Parametrize the Cylinder $y^2 + z^2 = 9$

- The cylinder's cross-section is a circle of radius 3 in the yz-plane.
- Standard parametrization of a circle of radius r in the yz-plane is:

$$
y = r \cos t \quad \text{and} \quad z = r \sin t
$$

- For our cylinder, $r=3$, so:

$$
y = 3 \cos t, \quad z = 3 \sin t
$$

- The parameter t varies from 0 to 2π to trace the entire circle.

Step 3: Incorporate the Fixed x-coordinate

- Since $x=4$ for all points on the intersection curve, the parametrization of the whole curve becomes:

$$
\boxed{
\begin{aligned}
x &= 4 \\
y &= 3 \cos t \\
z &= 3 \sin t
\end{aligned}
}
$$

Step 4: Write the Complete Parametric Equation

- The parametric equations describing the intersection curve C are:

$$
\boxed{
\mathbf{r}(t) = (4, 3 \cos t, 3 \sin t), \quad t \in [0, 2\pi]
}
$$

This parametrization fully describes the intersection curve as a circle of radius 3 in the plane $x=4$.

Significance of the Parametrization in Calculus and Geometry

Understanding the parametrization of intersection curves is fundamental in various mathematical and engineering contexts:

- Calculus Applications:
- Computing line integrals along the curve.
- Determining the length of the curve.
- Evaluating surface integrals where the curve acts as a boundary.

- Geometry and Visualization:
- Visualizing the intersection in 3D modeling software.
- Analyzing the spatial relationship between different surfaces.

- Physics and Engineering:
- Calculating flux or circulation along the curve.
- Designing mechanical parts where such curves define edges or boundaries.

Extending the Concept: Other Intersection Curves and Their Parametrizations

The approach used for the cylinder $y^2 + z^2 = 9$ and the plane $x=4$ can be generalized to other surface intersections:

1. Cylinder and Arbitrary Plane:
- When intersecting a cylinder with a plane not parallel to the axes, the intersection might be an ellipse, parabola, or hyperbola, depending on the orientation.

2. Cylinder with Other Surfaces:
- Intersection with spheres, cones, or more complex surfaces can lead to various conic sections.

3. Parametric Equations for Different Geometries:
- Use appropriate parametrizations for each surface, such as spherical coordinates for spheres or hyperbolic functions for hyperbolas.

Practical Applications of Cylinder-Plane Intersection Parametrizations

Understanding and computing the intersection of cylinders and planes are vital in many real-world applications:

- CAD and 3D Modeling:
- Designing complex mechanical parts involving intersecting cylindrical and planar surfaces.

- Robotics and Kinematics:
- Planning movement paths along intersection curves.

- Structural Engineering:
- Analyzing stress points where different structural components intersect.

- Optics and Light Engineering:
- Designing reflective surfaces or lenses with intersecting geometries.

Summary of Key Points

- The intersection of the cylinder $y^2 + z^2 = 9$ with the plane $x=4$ results in a circle of radius 3.
- The parametric equations for this intersection curve are:

$$
\boxed{
\mathbf{r}(t) = (4, 3 \cos t, 3 \sin t), \quad t \in [0, 2\pi]
}
$$

- This parametrization simplifies the analysis and visualization of the intersection curve.
- The process demonstrates how understanding simple geometric parametrizations is essential for tackling more complex 3D problems.

Conclusion

The process of finding the parametrization of the intersection curve between a cylinder and a plane is a foundational skill in multivariable calculus and 3D geometry. By recognizing the geometric shapes involved and applying standard parametrizations, we can accurately describe complex curves in three-dimensional space. This understanding not only enriches mathematical knowledge but also enables practical applications across various engineering, scientific, and design fields. Whether visualizing the intersection for a CAD model or calculating physical properties along the curve, the ability to parametrize such curves is a powerful tool in the mathematician's toolkit.

Frequently Asked Questions

How do you parametrize the intersection of the cylinder $y^2 + z^2 = 9$ and the plane $x = 4$?

Since the plane $x = 4$ is a constant, and the cylinder $y^2 + z^2 = 9$ has a standard parametrization $y = 3\cos t$, $z = 3\sin t$, the curve can be parametrized as $r(t) = (4, 3\cos t, 3\sin t)$, where $t \in [0, 2\pi]$.

What is the geometric interpretation of the intersection of the cylinder $y^2 + z^2 = 9$ and the plane $x = 4$?

The intersection is a circle of radius 3 located at $x = 4$, lying on the plane parallel to the yz-plane, formed where the cylinder cuts through the plane $x = 4$.

Can the parametrization of the curve be generalized for different cylinders and planes?

Yes, if the cylinder is given by $y^2 + z^2 = r^2$ and the plane is $x = c$, the parametrization is $r(t) = (c, r \cos t, r \sin t)$, where $t \in [0, 2\pi]$, assuming the cylinder is aligned with the x-axis.

What is the significance of choosing the parameters $y = 3\cos t$ and $z = 3\sin t$ in the parametrization?

These parametric equations represent a circle of radius 3 in the yz-plane, which is the cross-section of the cylinder $y^2 + z^2 = 9$, facilitating straightforward parametrization of the intersection curve.

How would the parametrization change if the plane was given by $x = a$ (for some constant a) and the cylinder $y^2 + z^2 = r^2$?

The parametrization would be $r(t) = (a, r \cos t, r \sin t)$, with $t \in [0, 2\pi]$, representing the circle at $x = a$ where the cylinder intersects the plane.

Additional Resources

Let C Be The Curve Of Intersections Of The Cylinder $Y^2 + z^2 = 9$ And The Plane X = 4. (a) Find A Parametrization

Understanding the intersection of surfaces in three-dimensional space is a fundamental skill in multivariable calculus, with applications ranging from engineering to computer graphics. In this guide, we will focus on Let C Be The Curve Of Intersections Of The Cylinder $Y^2 + z^2 = 9$ And The Plane X = 4, and specifically, how to find a parametrization of this curve. This problem elegantly combines the geometry of cylinders and planes, providing insight into how to approach intersection problems systematically.

Introduction to the Problem

The problem involves two surfaces:

1. Cylinder: $y^2 + z^2 = 9$
This describes a circular cylinder of radius 3 centered along the x-axis. Its cross-sections perpendicular to the x-axis are circles of radius 3.

2. Plane: $x = 4$
This is a vertical plane parallel to the yz-plane, located at x = 4.

The intersection of these two surfaces results in a space curve, which is essentially where the cylinder and the plane meet. Our goal is to find a parametrization of this curve, which means expressing the coordinates (x, y, z) as functions of one or more parameters, typically a variable like t.

Understanding the Geometry

Before diving into the parametrization, it's important to visualize the scenario:

- The cylinder $y^2 + z^2 = 9$ extends infinitely along the x-axis, forming a "tunnel" of radius 3.
- The plane $x = 4$ slices through this cylinder at a specific x-coordinate.
- The intersection curve will be a circle lying on the plane $x = 4$, which is the intersection of the cylinder with this plane.

Since the cylinder is symmetric around the x-axis, and the plane is fixed at $x = 4$, the intersection should be a circle in the yz-plane at $x=4$.

Step-by-Step Guide to Find the Parametrization

Step 1: Recognize the Geometry of the Intersection

The intersection of a cylinder $y^2 + z^2 = 9$ with a plane $x = 4$ results in a circle of radius 3 lying in the plane $x=4$.

- This circle is centered at the point $(4, 0, 0)$ in 3D space.
- The circle lies in the plane $x=4$.

Step 2: Express the Cylinder's Cross-Section

The cylinder's equation:

$$
y^2 + z^2 = 9
$$

is the equation of a circle in the yz-plane with radius 3, centered at the origin $(0, 0)$.

- For any fixed x, the cross-section is this circle in yz-plane.
- Since we're fixing $x = 4$, the intersection curve will have:

$$
x = 4
$$

- The corresponding circle in yz-plane at $x=4$ has the same radius and center, just shifted to $x=4$.

Step 3: Parametrize the Circle

To parametrize the circle $y^2 + z^2 = 9$, use the standard parametrization:

$$
\begin{cases}
y = 3 \cos t \\
z = 3 \sin t
\end{cases}
$$

where $t \in [0, 2\pi)$.

This parametrization traces out the circle as t varies from 0 to 2π.

Step 4: Incorporate the Fixed x-coordinate

Since the intersection occurs at $x=4$, and the circle lies in the plane $x=4$, the parametrized points become:

$$
\boxed{
\begin{cases}
x(t) = 4 \\
y(t) = 3 \cos t \\
z(t) = 3 \sin t
\end{cases}
}
$$

for $t \in [0, 2\pi)$.

Final Parametrization of the Curve C

Putting it all together, the parametrization of the intersection curve C is:

$$
\boxed{
\mathbf{r}(t) = (4,\, 3 \cos t,\, 3 \sin t), \quad t \in [0, 2\pi)
}
$$

This simple, elegant parametrization captures the entire intersection curve between the cylinder and the plane.

Visualizing the Curve

- The curve is a circle of radius 3, lying in the plane $x=4$.
- The circle is centered at $(4, 0, 0)$ in 3D space.
- As t varies from 0 to 2π, the point moves around the circle, providing a complete description of the intersection.

Additional Insights and Applications

Why Is This Parametrization Useful?

- Calculus Applications: You can now compute the length of the curve, evaluate line integrals along C, or analyze properties like curvature.
- Visualization: The parametrization makes it easier to plot the intersection in software like MATLAB, GeoGebra, or Desmos.
- Further Analysis: Extending this approach helps in more complex surface intersections, such as those involving ellipsoids, paraboloids, or other cylinders.

Extending the Approach

If the problem involved a different plane, for example $x = a$, or a different cylinder, such as $y^2 + z^2 = r^2$, the approach remains similar:

- Find where the surface intersects the plane.
- Recognize the geometric shape of the intersection (circle, ellipse, etc.).
- Use standard parametrizations of that shape.
- Incorporate the fixed coordinate from the plane.

Summary

- The intersection of the cylinder $y^2 + z^2 = 9$ with the plane $x=4$ yields a circle in 3D space.
- The circle's parametrization is straightforward using trigonometric functions:

$$
\mathbf{r}(t) = (4,\, 3 \cos t,\, 3 \sin t), \quad t \in [0, 2\pi)
$$

- This parametrization succinctly describes the entire intersection curve, making it a powerful tool for further calculations or visualizations.

Final Remarks

Mastering the parametrization of intersection curves is an essential skill in multivariable calculus, offering both theoretical insights and practical tools for analysis. By understanding the geometry involved, recognizing standard shapes like circles and ellipses, and applying trigonometric parametrizations, you can efficiently tackle a wide class of intersection problems involving surfaces in three-dimensional space.

Let C Be The Curve Of Intersections Of The Cylinder Y2 Z2 9 And The Plane X 4 A Find A Parametrization

Let C Be The Curve Of Intersections Of The Cylinder Y2 Z2 9 And The Plane X 4 A Find A Parametrization

Related Articles

- In A System Of 100-percent-reserve Banking, Question 8 Options: Deposits Are Banks' Only Assets. Currency
- If The Point (6,10) Is On The Graph Of Y = F(x), Find A Point On The Graph Of A) Y = F(x+2) B) Y= 1/2f(x)
- Select The Correct Answer From The Drop-down Menu.Omar Is Writing A Draft Of A Research Paper About UFOs.