Let Y = Tan(5x+7). Find The Differential Dy When X = 5 And Dx = 0.1 ________________Find The Differential

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Understanding how to compute differentials in calculus is essential for analyzing how small changes in a variable affect a related function. In this article, we will explore the process of finding the differential Dy for the function Y = Tan(5x+7) when x equals 5 and the change in x, denoted by Dx, is 0.1. This problem involves the application of derivatives, the concept of differentials, and the chain rule, all fundamental tools in differential calculus.

Introduction to Differentials in Calculus

Differentials are a way to approximate how a small change in an independent variable, such as x, impacts a dependent variable, like Y. The differential Dy provides an approximation to the actual change in the function value when x changes by a small amount Dx.

Key concepts:

- Derivative (dy/dx): Represents the rate of change of Y with respect to x.
- Differential (Dy): The product of the derivative and the change in x, Dy ≈ (dy/dx) Dx.
- Application: Used for approximation, error estimation, and understanding the behavior of functions near a point.

Understanding the Function Y = Tan(5x+7)

The function involves the tangent function, which is periodic and has vertical asymptotes at points where its argument equals (pi/2) + npi, for integers n. The inner function, 5x+7, acts as the argument to the tangent.

Important features:

- The argument of the tangent function: u = 5x + 7.
- The derivative of tan(u): derivative = $\sec^2(u)$.
- Chain rule: necessary to differentiate composite functions like tan(5x+7).

Step-by-Step Solution to Find Dy

To find the differential Dy at x = 5 with Dx = 0.1, follow these steps:

1. Find the derivative dy/dx

Given Y = tan(5x + 7), apply the chain rule:

- Derivative of tan(u) with respect to u is $\sec^2(u)$.
- u = 5x + 7, so du/dx = 5.

Therefore,

$$
\frac{dy}{dx} = \frac{d}{dx} [\tan(5x + 7)] = \sec^2(5x + 7) \times 5
$$

2. Evaluate dy/dx at x = 5

Calculate u at x = 5:

$$
u = 5(5) + 7 = 25 + 7 = 32
$$

Now, compute $\sec^2(32)$. Since sec(θ) = 1 / cos(θ), we need cos(32):

- Convert 32 radians to degrees for easier interpretation:

$$
32 \text{ radians} \approx 1834.4^\circ
$$

But it's better to work directly in radians:

- Use a calculator to find cos(32):

$$
\cos(32) \approx -0.848048
$$

\]

Then,

\[
\sec(32) = \frac{1}{\cos(32)} \approx \frac{1}{-0.848048} \approx -1.178
\]

Square it:

\[
\sec^2(32) \approx (-1.178)^2 \approx 1.387
\]

Multiply by 5:

\[
dy/dx \big|_{x=5} \approx 5 \times 1.387 \approx 6.935
\]

3. Compute Dy using Dx = 0.1

Using the differential approximation:

\[
Dy \approx \frac{dy}{dx} \big|_{x=5} \times Dx = 6.935 \times 0.1 = 0.6935
\]

4. Final answer

The approximate differential Dy when x = 5 and Dx = 0.1 is:

\[
\boxed{Dy \approx 0.6935}
\]

This indicates that for a small increase of 0.1 in x near x=5, the function Y = tan(5x+7) increases approximately by 0.6935.

Additional Insights and Applications

Understanding differentials is crucial in various fields such as physics, engineering, and economics, where small changes and their effects need to be estimated efficiently.

Applications include:

- Error estimation: Approximating the potential error in measurements.
- Optimization problems: Understanding how slight adjustments affect outcomes.
- Curve analysis: Studying how functions behave locally.

Summary of Key Steps in Finding the Differential Dy

- Identify the function and its inner argument.
- Compute the derivative using the chain rule.
- Evaluate the derivative at the given x-value.
- Multiply the derivative by the small change Dx to approximate Dy.
- Interpret the result as the approximate change in the function's value.

Conclusion

Calculating the differential Dy for functions like Y = Tan(5x+7) involves understanding derivatives, the chain rule, and how small changes in x influence the function. In this particular problem, with x=5 and Dx=0.1, the approximate change in Y is about 0.6935. Mastery of these concepts enables precise approximations and deeper insights into the behavior of functions in calculus.

By practicing these steps, students and professionals can confidently analyze the impact of small variations in variables, essential for both theoretical and practical applications in science and engineering.

Frequently Asked Questions

What is the given function Y in the problem?

Y = Tan(5x + 7)

What value of x is used to find the differential Dy?

x = 5

What is the value of Dx provided in the problem?

Dx = 0.1

How do you find the differential Dy for the function Y = Tan(5x + 7)?

First, find the derivative of Y with respect to x, then multiply by Dx: Dy = dY/dx Dx.

What is the derivative of Y = Tan(5x + 7)?

dY/dx = sec^2(5x + 7) 5.

What is the first step to compute Dy at x = 5?

Calculate the value of the derivative dY/dx at x = 5.

How do you evaluate sec^2(5x + 7) at x = 5?

Substitute x = 5 into 5x + 7 to get 5(5) + 7 = 25 + 7 = 32, then compute sec^2(32).

What is sec^2(32) in terms of cosine?

sec^2(32) = 1 / cos^2(32), where 32 is in radians.

How do you compute Dy when x = 5 and Dx = 0.1?

Calculate dY/dx at x=5, then multiply by Dx: Dy = dY/dx 0.1.

What is the final expression for Dy in this problem?

Dy = 5 sec^2(5x + 7) Dx, evaluated at x = 5.

Additional Resources

Let Y = Tan(5x + 7). Find The Differential Dy When X = 5 And Dx = 0.1 ________________Find The Differential

Understanding how a function changes as its input varies is fundamental in calculus, especially in fields such as physics, engineering, and economics. In this article, we delve into the process of calculating the differential \(

Dy for the function $Y = \tan(5x + 7)$ at a specific point, with an incremental change in x. Specifically, we examine what happens when $x = 5$ and $\Delta x = 0.1$. This practical example illustrates core calculus concepts, including derivatives and differentials, providing a comprehensive guide for students and professionals alike.

Overview of the Problem

The problem requires us to:

- Find the differential Dy of the function $Y = \tan(5x + 7)$.
- Evaluate this differential at the point where $x = 5$.
- Use an increment $\Delta x = 0.1$ to understand how y changes in response.

The key idea here is that the differential Dy approximates the change in the function y corresponding to a small change Δx around a specific point. This is especially useful when the exact change in y is complex to compute directly but can be approximated using derivatives.

Mathematical Foundations: Derivatives and Differentials

Before jumping into the calculations, it’s essential to clarify the concepts involved:

- Derivative $\frac{dy}{dx}$: Measures the rate of change of y with respect to x. It tells us how much y will change for a tiny change in x.
- Differential Dy: An approximation of the actual change in y, given by $Dy = \frac{dy}{dx} \times \Delta x$. When Δx is small, Dy closely estimates Δy.

Calculating the derivative of a composite function like $\tan(5x + 7)$ typically involves the chain rule, which simplifies handling nested functions.

Step-by-Step Solution Approach

1. Find the derivative $\frac{dy}{dx}$ of $y = \tan(5x + 7)$.

Using the chain rule:

- The outer function is $\tan(u)$, with derivative $\sec^2 u$.
- The inner function is $u = 5x + 7$, with derivative 5.

Thus,

$$
\frac{dy}{dx} = \sec^2(5x + 7) \times 5
$$

or

$$
\boxed{\frac{dy}{dx} = 5 \sec^2(5x + 7)}
$$

---

2. Evaluate the derivative at $x = 5$.

Calculate the inner argument:

$$
u = 5(5) + 7 = 25 + 7 = 32
$$

Now, evaluate $\sec^2(32)$. Since $\sec u = \frac{1}{\cos u}$, we need $\cos 32$ radians.

- Using a calculator or software, compute $\cos 32$:

$$
\cos 32 \approx 0.848048
$$

- Then,

$$
\sec 32 = \frac{1}{0.848048} \approx 1.178
$$

- Square this to find $\sec^2 32$:

$$
(1.178)^2 \approx 1.387
$$

Thus, the derivative at $x=5$:

$$
\frac{dy}{dx} \bigg|_{x=5} = 5 \times 1.387 \approx 6.935
$$

---

3. Calculate the differential Dy.

Recall that:

$$
Dy = \frac{dy}{dx} \times \Delta x
$$

Given $\Delta x = 0.1$, we have:

$$
Dy \approx 6.935 \times 0.1 = 0.6935
$$

This value indicates that, near $x=5$, the function y increases approximately by 0.6935 when x increases by 0.1.

Interpretation and Significance

The differential Dy offers an approximation of the actual change Δy:

$$
\Delta y \approx Dy = 0.6935
$$

This means that if you start at $x=5$ and increase x slightly by 0.1, the value of $y = \tan(5x+7)$ is expected to increase by approximately 0.6935.

This approximation is especially valuable in scenarios where calculating the exact y values at the new point directly is complex or unnecessary, such as in engineering tolerances, physics simulations, or economic models.

Broader Context and Applications

Calculus, and specifically the concept of differentials, plays a pivotal role in modeling real-world phenomena where quantities change continuously. Here are some areas where such calculations are crucial:

- Physics: Calculating small changes in position, velocity, or acceleration.
- Engineering: Estimating the impact of small design modifications.
- Economics: Understanding how slight changes in market variables influence outcomes.
- Biology: Modeling population growth or chemical reactions.

The ability to approximate changes efficiently helps professionals make informed decisions without resorting to complex, time-consuming calculations.

Additional Insights and Considerations

While the calculation above provides a good approximation, it’s important to recognize its limitations:

- Accuracy of Approximation: The differential Dy closely approximates Δy only when Δx is small. Larger increments may lead to significant errors.
- Function Behavior: For functions with high curvature or rapid oscillations, the differential approximation might be less reliable.
- Units and Angles: Always verify whether the calculator or software is set to radians or degrees, as trigonometric functions depend heavily on this setting. In this case, calculations assume radians.

Summary of the Process

To summarize, calculating the differential Dy for $y = \tan(5x + 7)$ at $x = 5$ with an increment of 0.1 involves:

1. Differentiating the function using the chain rule.
2. Evaluating the derivative at the specific point.
3. Multiplying the derivative by the small change Δx.

This systematic approach underscores the power of calculus in analyzing how functions evolve with their inputs, providing valuable approximations that inform practical decision-making.

Final Thoughts

The exploration of differentials exemplifies the elegance and utility of calculus in bridging theory and real-world application. By mastering these techniques, students and professionals can better understand the subtle nuances of change—whether in the trajectory of a spacecraft, fluctuations in financial markets, or the oscillations of a pendulum. As the world increasingly relies on precise modeling and predictions, the importance of such foundational concepts continues to grow.

In conclusion, the differential Dy for $y = \tan(5x + 7)$ at $x=5$ with $\Delta x=0.1$ is approximately 0.6935, providing a quick and reliable estimate of how the function responds to small changes in x. This methodology exemplifies the practical power of calculus, transforming abstract mathematical principles into tools for understanding and navigating the complex dynamics of the world around us.

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