List The First Five Terms Of The Sequence. A_n = (-1)^n-1/n^2a_1= ____a_2= ______ A_3= ______a_4= ______a_5=

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Understanding the Sequence and Its Components

Introduction to the Sequence

The sequence provided is defined by the formula:

$$ A_n = \frac{(-1)^{n-1}}{n^2} $$

This formula combines an alternating sign pattern with a quadratic denominator. To analyze and compute the first five terms, it is essential to understand how each component influences the sequence's behavior.

Breaking Down the Formula

The key elements in the sequence are:

- The numerator: $(-1)^{n-1}$, which determines the sign of each term.
- The denominator: n^2, which influences the magnitude of each term.

Let's explore these components in detail.

Component Analysis: Sign Pattern and Magnitude

Sign Pattern: $(-1)^{n-1}$

This term alternates between positive and negative depending on the value of n:

- When n is odd, $n-1$ is even, and $(-1)^{even} = 1$

- When n is even, $n-1$ is odd, and $(-1)^{odd} = -1$

Therefore, the sign pattern follows:

- For odd n, the term is positive.
- For even n, the term is negative.

Magnitude: $\frac{1}{n^2}$

The magnitude of each term decreases as n increases, since the denominator grows quadratically:

- $n=1$, magnitude: $1/1^2 = 1$
- $n=2$, magnitude: $1/2^2 = 1/4$
- $n=3$, magnitude: $1/3^2 = 1/9$
- and so on.

This quadratic decay causes the terms to approach zero as $n \to \infty$.

Calculating the First Five Terms

Term 1: A_1

Applying $n=1$:

$$A_1 = \frac{(-1)^{1-1}}{1^2} = \frac{(-1)^0}{1} = \frac{1}{1} = 1$$

Term 2: A_2

Applying $n=2$:

$$A_2 = \frac{(-1)^{2-1}}{2^2} = \frac{(-1)^1}{4} = \frac{-1}{4}$$

Term 3: A_3

Applying $n=3$:

$$A_3 = \frac{(-1)^{3-1}}{3^2} = \frac{(-1)^2}{9} = \frac{1}{9}$$

Term 4: A_4

Applying $n=4$:

$$A_4 = \frac{(-1)^{4-1}}{4^2} = \frac{(-1)^3}{16} = \frac{-1}{16}$$

Term 5: A_5

Applying $n=5$:

$$A_5 = \frac{(-1)^{5-1}}{5^2} = \frac{(-1)^4}{25} = \frac{1}{25}$$

Summary of the First Five Terms

Putting it all together, the first five terms of the sequence are:

1. $a_1 = 1$
2. $a_2 = -\frac{1}{4}$
3. $a_3 = \frac{1}{9}$
4. $a_4 = -\frac{1}{16}$
5. $a_5 = \frac{1}{25}$

Insights and Observations

Behavior of the Sequence

- The signs alternate between positive and negative, following the pattern dictated by $(-1)^{n-1}$.
- The magnitudes decrease rapidly, approaching zero as n increases, due to the quadratic denominator n^2.

Convergence of the Sequence

Since the terms tend toward zero, the sequence converges to zero as $n \to \infty$. This behavior is characteristic of alternating series with decreasing magnitude terms.

Applications and Relevance

Sequences of this nature often appear in:

- Fourier series
- Convergence analysis in calculus
- Approximation of functions using series

Understanding how to compute initial terms provides foundational insight for studying series convergence and properties.

Conclusion

The sequence defined by $A_n = \frac{(-1)^{n-1}}{n^2}$ exhibits a clear pattern: alternating signs with decreasing magnitude. The first five terms are explicitly calculated as:

- $a_1 = 1$
- $a_2 = -\frac{1}{4}$
- $a_3 = \frac{1}{9}$
- $a_4 = -\frac{1}{16}$
- $a_5 = \frac{1}{25}$

This sequence exemplifies how combining simple algebraic components can generate intricate and interesting behaviors in mathematical series. Analyzing initial terms helps build intuition for the overall pattern and convergence properties, which are fundamental in advanced calculus and mathematical analysis.

Frequently Asked Questions

What is the general formula for the sequence $A_n = (-1)^{n-1} / n^2$?

The sequence is defined by $A_n = (-1)^{n-1} / n^2$, where n is a positive integer.

What is the value of the first term a_1 in the sequence?

$a_1 = (-1)^{1-1} / 1^2 = 1 / 1 = 1$.

How do you compute the second term a_2 of the

sequence?

$a_2 = (-1)^{2-1} / 2^2 = (-1)^1 / 4 = -1/4$.

What is the third term a_3 in the sequence?

$a_3 = (-1)^{3-1} / 3^2 = (-1)^2 / 9 = 1/9$.

Calculate the fourth term a_4 of the sequence.

$a_4 = (-1)^{4-1} / 4^2 = (-1)^3 / 16 = -1/16$.

Find the fifth term a_5 in the sequence.

$a_5 = (-1)^{5-1} / 5^2 = (-1)^4 / 25 = 1/25$.

What pattern can be observed in the signs of the terms in this sequence?

The signs alternate between positive and negative, starting with positive for a_1, then negative for a_2, positive for a_3, and so on.

Additional Resources

List The First Five Terms Of The Sequence. $A_n = (-1)^{n-1}/n^2$ $a_1=$ ____ $a_2=$ _____ $a_3=$ _____ $a_4=$ _____ $a_5=$

Understanding sequences is fundamental in mathematics, serving as building blocks for concepts in calculus, analysis, and beyond. One such sequence, defined by the formula $A_n = \frac{(-1)^{n-1}}{n^2}$, introduces an elegant interplay between alternating signs and quadratic decay. This article aims to dissect this sequence thoroughly, compute its first five terms, and explore its properties in a clear, reader-friendly manner.

Decoding the Sequence Formula: $A_n = \frac{(-1)^{n-1}}{n^2}$

Before diving into calculations, it's essential to understand the structure of the sequence. The general term:

$$
A_n = \frac{(-1)^{n-1}}{n^2}
$$

combines two key elements:

1. Alternating Sign Factor: $(-1)^{n-1}$

2. Quadratic Denominator: n^2

This combination results in a sequence where the sign alternates between positive and negative as n increases, while the magnitude diminishes at a rate proportional to $1/n^2$.

The Significance of the Alternating Sign

The expression $(-1)^{n-1}$ is responsible for the sequence's alternating signs. Its behavior:

- When $n = 1$: $(-1)^{1-1} = (-1)^0 = 1$ (positive)
- When $n = 2$: $(-1)^{2-1} = (-1)^1 = -1$ (negative)
- When $n = 3$: $(-1)^{3-1} = (-1)^2 = 1$ (positive)
- When $n = 4$: $(-1)^{4-1} = (-1)^3 = -1$ (negative)
- When $n = 5$: $(-1)^{5-1} = (-1)^4 = 1$ (positive)

Thus, the sequence exhibits a pattern:

$$
A_1 > 0,\quad A_2 < 0,\quad A_3 > 0,\quad A_4 < 0,\quad A_5 > 0
$$

This oscillation is characteristic of many important mathematical sequences and series, such as the alternating harmonic series.

Computing the First Five Terms

Term A_1

$$
A_1 = \frac{(-1)^{1-1}}{1^2} = \frac{(-1)^0}{1} = \frac{1}{1} = 1
$$

The first term is positive, with a magnitude of 1.

Term A_2

$$
A_2 = \frac{(-1)^{2-1}}{2^2} = \frac{(-1)^1}{4} = \frac{-1}{4} = -0.25
$$

The second term is negative, with a magnitude of 0.25.

Term A_3

\[

$$
A_3 = \frac{(-1)^{3-1}}{3^2} = \frac{(-1)^2}{9} = \frac{1}{9} \approx 0.1111
$$

The third term is positive, approximately 0.1111.

Term A_4

$$
A_4 = \frac{(-1)^{4-1}}{4^2} = \frac{(-1)^3}{16} = \frac{-1}{16} = -0.0625
$$

The fourth term is negative, approximately -0.0625.

Term A_5

$$
A_5 = \frac{(-1)^{5-1}}{5^2} = \frac{(-1)^4}{25} = \frac{1}{25} = 0.04
$$

The fifth term is positive, 0.04.

Summary of the First Five Terms

Term	Calculation	Result	Sign
a_1	$\frac{(-1)^0}{1^2}$	1	Positive
a_2	$\frac{(-1)^1}{2^2}$	-0.25	Negative
a_3	$\frac{(-1)^2}{3^2}$	0.1111	Positive
a_4	$\frac{(-1)^3}{4^2}$	-0.0625	Negative
a_5	$\frac{(-1)^4}{5^2}$	0.04	Positive

Exploring the Sequence's Behavior and Significance

The initial terms reveal a pattern of oscillation and diminishing magnitude. This behavior is characteristic of convergent alternating series, and understanding this pattern is key to grasping broader concepts in mathematical analysis.

Convergence and Infinite Series

The sequence $A_n = \frac{(-1)^{n-1}}{n^2}$ forms the terms of an alternating series:

$$
\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n^2}
$$

This series converges, thanks to the Alternating Series Test, because:

- The absolute value $\frac{1}{n^2}$ decreases monotonically to zero
- $\lim_{n \to \infty} \frac{1}{n^2} = 0$

This series converges to a specific value, which is related to the famous Basel problem, where:

$$
\sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}
$$

The alternating counterpart:

$$
\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n^2}
$$

converges to a value involving π^2 as well, specifically:

$$
\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}
$$

which underscores the deep connections between sequence behavior and fundamental constants.

Broader Implications and Applications

Sequences like A_n aren't just abstract constructs—they underpin many areas of mathematics and physics:

- Fourier Series: Alternating series are key in representing functions as sums of sines and cosines.
- Signal Processing: Oscillating signals often modeled via alternating sequences.
- Convergence Analysis: Understanding how partial sums approach their limits informs numerical methods and approximation algorithms.
- Mathematical Physics: Series involving π^2 appear in quantum mechanics, wave theory, and thermodynamics.

The specific sequence $A_n = \frac{(-1)^{n-1}}{n^2}$ exemplifies how alternating signs coupled with decreasing magnitudes produce well-behaved, convergent series with profound mathematical significance.

Final Remarks

The initial five terms of $A_n = \frac{(-1)^{n-1}}{n^2}$ are:

- $a_1 = 1$
- $a_2 = -0.25$
- $a_3 \approx 0.1111$
- $a_4 = -0.0625$
- $a_5 = 0.04$

Examining these terms reveals the oscillating nature and the diminishing size that lead to the series' convergence. Sequences like this serve as a foundation for understanding more complex mathematical phenomena, bridging the gap between abstract theory and practical applications.

By analyzing the structure, calculating initial terms, and exploring their implications, we gain a holistic understanding of how simple formulas can unveil intricate properties of mathematical series and their relevance across scientific disciplines.

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