Maximize The Objective Function P=3x+5y For The Given Constraints. X>=0 Y>=0 X+y<=5 3x+6y<=21

Maximize The Objective Function P=3x+5y For The Given Constraints. X≥0, Y≥0, X + Y ≤ 5, 3x + 6y ≤ 21

Optimizing a mathematical function within certain constraints is a fundamental problem in operations research and linear programming. In this article, we will explore how to maximize the objective function P=3x+5y given the constraints: X ≥ 0, Y ≥ 0, X + Y ≤ 5, and 3x + 6y ≤ 21. By understanding these constraints and applying methods such as graphing and corner point analysis, we can determine the optimal solution that yields the maximum value of P.

Understanding the Objective Function and Constraints

What is the Objective Function?

The objective function P=3x+5y represents a linear expression where the goal is to find values of x and y that maximize P. This kind of problem is typical in linear programming, where we seek the best outcome (maximize or minimize) subject to certain restrictions.

Breaking Down the Constraints

The constraints limit the feasible region where solutions can exist:

- X ≥ 0: The variable x cannot be negative, representing a non-negativity constraint.
- Y ≥ 0: Similarly, y must be non-negative.
- X + Y ≤ 5: The sum of x and y cannot exceed 5, defining a boundary line in the coordinate plane.
- 3x + 6y ≤ 21: This inequality further restricts the feasible region, representing a line with a slope and intercept limiting the variables.

Understanding these constraints allows us to visualize the feasible region where the optimal solution must lie.

Graphical Method for Linear Programming

Plotting the Constraints

The graphical method involves plotting the inequality constraints on a coordinate plane:

- Plot the line $X + Y = 5$. This line passes through points (5, 0) and (0, 5).
- Plot the line $3x + 6y = 21$, which simplifies to $x + 2y = 7$, passing through points (7, 0) and (0, 3.5).

The feasible region is where all the inequalities overlap, including $x \geq 0$ and $y \geq 0$, which restricts the region to the first quadrant.

Identifying the Feasible Region

The feasible region is a polygon bounded by:

- The axes ($x=0$ and $y=0$).
- The lines $X + Y = 5$ and $X + 2Y = 7$.

By shading the area that satisfies all constraints, we identify the region where solutions are valid.

Finding Corner Points of the Feasible Region

Why Corner Points Matter

In linear programming, the maximum or minimum of a linear objective function within a convex feasible region always occurs at one of the polygon's vertices (corner points). Therefore, we evaluate $P=3x+5y$ at each corner point.

Calculating the Intersection Points

The corner points are:

1. Origin: (0, 0)

2. Intersection of X + Y = 5 and axes: (0, 5) and (5, 0)

3. Intersection of X + 2Y = 7 and axes: (0, 3.5) and (7, 0)

4. Intersection of the lines X + Y = 5 and X + 2Y = 7

Let's find the intersection point of the two lines:

Solve the system:
- X + Y = 5 ...(1)
- X + 2Y = 7 ...(2)

Subtract (1) from (2):
- (X + 2Y) - (X + Y) = 7 - 5
- X + 2Y - X - Y = 2
- Y = 2

Substitute Y=2 into (1):
- X + 2 = 5
- X = 3

Therefore, the intersection point is (3, 2).

Check if (3, 2) lies within the feasible region:
- X ≥ 0? Yes.
- Y ≥ 0? Yes.
- X + Y = 3 + 2 = 5 ≤ 5? Yes.
- 3x + 6y = 33 + 62 = 9 + 12 = 21 ≤ 21? Yes.

Thus, (3, 2) is a valid corner point.

Summary of corner points:
- (0, 0)
- (0, 3.5)
- (3, 2)
- (5, 0)

Evaluating the Objective Function at Corner Points

Calculate P=3x+5y at each point:

- (0, 0): P=30 + 50= 0

- (0, 3.5): P=30 + 53.5= 0 + 17.5= 17.5
- (3, 2): P=33 + 52= 9 + 10= 19
- (5, 0): P=35 + 50= 15 + 0= 15

Maximum value of P occurs at (3, 2) with P=19.

Conclusion: Optimal Solution and Interpretation

The optimal solution to maximize the objective function P=3x+5y, given the constraints, is at the point (x=3, y=2), with a maximum value of P=19.

Implications:
- This solution suggests allocating resources or making decisions corresponding to x=3 and y=2 will achieve the highest possible objective value.
- The constraints are satisfied at this point, ensuring feasibility in real-world applications such as production planning, resource allocation, or logistics.

Additional Insights and Applications

Linear Programming in Real-World Scenarios

The principles demonstrated here are applicable in various fields:

- Manufacturing: Maximizing profit based on resource constraints.
- Transportation: Optimal routing and scheduling.
- Finance: Portfolio optimization within risk limits.
- Supply Chain Management: Inventory and production decisions.

Tools for Solving Linear Programming Problems

While graphical methods are effective for problems involving two variables, more complex problems often require:

- Simplex Algorithm

- Linear programming software (e.g., LINDO, MATLAB, Excel Solver)

These tools facilitate solving higher-dimensional problems efficiently.

Summary

To maximize the objective function $P=3x+5y$ under the constraints:
- $X \geq 0$
- $Y \geq 0$
- $X + Y \leq 5$
- $3x + 6y \leq 21$

the best approach is to graph the constraints, identify the feasible region, evaluate the objective function at each corner point, and select the point with the highest value. In this case, the optimal point is (3,2), yielding a maximum P of 19.

This process exemplifies the power of linear programming in decision-making and optimization, providing clear guidance for resource allocation and strategic planning across various industries.

Keywords for SEO:
Linear programming, maximize objective function, constraints, feasible region, corner point method, optimization, resource allocation, mathematical modeling, linear inequalities, graphical solution, linear optimization problems

Frequently Asked Questions

What is the objective function in the given problem?

The objective function is $P = 3x + 5y$.

What are the constraints provided for the optimization problem?

The constraints are: $x \geq 0$, $y \geq 0$, $x + y \leq 5$, and $3x + 6y \leq 21$.

How do you determine the feasible region for this problem?

The feasible region is determined by plotting the inequalities on the xy-plane and identifying the overlapping area that satisfies all constraints simultaneously.

Which points should be checked to find the maximum value of P?

The vertices (corner points) of the feasible region should be checked because the maximum or minimum of a linear objective function occurs at these points.

What are the corner points of the feasible region for this problem?

The corner points are (0,0), (0,5), (3,3), and (5,0).

How do you evaluate the objective function at each corner point?

Substitute each corner point into P = 3x + 5y and calculate the value to determine which gives the maximum P.

What is the maximum value of P, and at which point does it occur?

The maximum value of P is 24, occurring at the point (3,3).

Why is it sufficient to evaluate the objective function at the corner points?

Because linear programming problems have their optimal solutions at the vertices of the feasible region, evaluating at these points ensures the maximum or minimum is found efficiently.

Additional Resources

Maximize The Objective Function P=3x+5y For The Given Constraints is a classic problem in linear programming, often encountered in operations research, economics, and management science. This problem involves finding the optimal values of variables \(x\) and \(y\) that maximize (or minimize) a linear objective function while satisfying a set of linear constraints. In this guide, we will thoroughly analyze how to approach such a problem, interpret the constraints, and systematically find the maximum value of the objective function.

Understanding the Problem

The core task is to maximize the objective function:

$$ P = 3x + 5y $$

subject to the following constraints:

- $x \geq 0$
- $y \geq 0$
- $x + y \leq 5$
- $3x + 6y \leq 21$

This type of problem is a linear programming problem, which can be visualized geometrically or solved algebraically using methods like the Graphical Method, the Corner Point Method, or using Simplex Algorithm techniques.

Step 1: Interpreting the Constraints

Understanding the constraints helps define the feasible region—the set of all points (x, y) that satisfy all inequalities simultaneously.

Non-negativity Constraints

- $x \geq 0$: The feasible region lies on or to the right of the y-axis.
- $y \geq 0$: The feasible region lies on or above the x-axis.

These constraints restrict the solution to the first quadrant.

Other Constraints

1. $x + y \leq 5$

- This inequality represents all points on or below the line $x + y = 5$.
- The boundary line is a straight line that cuts through the axes at $x=5$ and $y=5$.

2. $3x + 6y \leq 21$

- Simplify by dividing through by 3:

$$
x + 2y \leq 7
$$

- The boundary line is $x + 2y = 7$, cutting through the axes at $x=7$ and $y=3.5$.

Step 2: Plotting the Constraints and Feasible Region

Visualizing the Constraints

To visualize the feasible region, plot the boundary lines:

- Line 1: $x + y = 5$

- Intercepts:
- $x=0 \Rightarrow y=5$
- $y=0 \Rightarrow x=5$

- Line 2: $x + 2y = 7$

- Intercepts:
- $x=0 \Rightarrow y=3.5$
- $y=0 \Rightarrow x=7$

Feasible Region

- The region where all inequalities overlap, considering the non-negativity constraints, is bounded by:

- The axes $x=0$ and $y=0$,
- The line $x + y = 5$,
- The line $x + 2y = 7$.

Plotting these lines on a graph (conceptually):

- The line $x + y=5$ goes from (0,5) to (5,0).
- The line $x + 2y=7$ goes from (0,3.5) to (7,0).

The feasible region is the common area satisfying both inequalities:

- Below or on the line $x + y=5$,
- Below or on the line $x + 2y=7$,
- In the first quadrant ($x \geq 0$, $y \geq 0$).

Step 3: Identifying Corner Points

In linear programming, the maximum (or minimum) of the objective function occurs at one or more vertices (corner points) of the feasible region. Therefore, the next step is to find these vertices.

Find the Intersection Points

1. Intersection of the two boundary lines:

Solve the system:

\[

\begin{cases}
x + y = 5 \\
x + 2y = 7
\end{cases}
\]

Subtract the first from the second:

\[
(x + 2y) - (x + y) = 7 - 5 \Rightarrow y = 2
\]

Substitute \(y=2\) into \(x + y=5\):

\[
x + 2=5 \Rightarrow x=3
\]

Vertex A: \((3, 2)\)

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2. Intersection with axes:

- At \(x=0\):

- From \(x + y=5\):

\[
0 + y=5 \Rightarrow y=5
\]

- Check \(x + 2y \leq 7\):

\[
0 + 2(5)=10 \leq 7 \Rightarrow \text{No} \Rightarrow \text{Point (0,5) is outside feasible region}
\]

- Check \(x + 2y=7\) at \(x=0\):

\[
0 + 2y=7 \Rightarrow y=3.5
\]

- Since \(y=3.5\) is less than 5, point (0,3.5) is within the feasible region.

- At \(y=0\):

- From \(x + y=5\):

$$
x=5
$$

- Check $x + 2(0)=x \leq 7$:

$$
x=5 \leq 7 \Rightarrow \text{Feasible}
$$

- Also check $x + 2(0)=5 \leq 7$, yes.

- From $x + 2y=7$:

$$
x=7
$$

- At $y=0$, $x=7$ is feasible, but need to check if it satisfies $x + y \leq 5$:

$$
7+0=7 \nleq 5
$$

So, point (7,0) is outside the feasible region.

Summarizing the Corner Points:

- Point 1: $(0, 0)$ (origin)
- Point 2: $(0, 3.5)$ (intersection of $x=0$ and $x + 2y=7$)
- Point 3: $(3, 2)$ (intersection of the two lines)
- Point 4: $(5, 0)$ (intersection of $x + y=5$ and $y=0$)

Check whether these points satisfy all constraints:

- (0,0): Yes
- (0,3.5):

- $x + y=0+3.5=3.5 \leq 5$? Yes
- $x + 2y=0+7=7 \leq 7$? Yes
- Both inequalities satisfied.

- (3,2):

- $3+2=5 \leq 5$? Yes
- $3+2(2)=3+4=7 \leq 7$? Yes

- (5,0):

- $5+0=5 \leq 5$? Yes
- $5+2(0)=5 \leq 7$? Yes

All points are within the feasible region.

Step 4: Evaluating the Objective Function at Corner Points

To find the maximum value of $P=3x+5y$, compute P at each vertex:

Vertex	Coordinates	$P=3x+5y$
(0, 0)	(0, 0)	$30 + 50=0$
(0, 3.5)	(0, 3.5)	$30 + 53.5=0 + 17.5=17.5$
(3, 2)	(3, 2)	$33 + 52=9 + 10=19$
(5, 0)	(5, 0)	$35 + 50=15 + 0=15$

Maximum value of P is 19 at $(x,y) = (3, 2)$.

Step 5: Conclusion and Interpretation

The optimal solution to maximize the objective function $P=3x+5y$ under the given constraints is:

- Maximum Objective Function Value: 19
- Optimal Values of Variables:

$$
x=3,\quad y=2
$$

This indicates that to achieve the highest value of P, the decision variables should be set

Maximize The Objective Function P 3x 5y For The Given Constraints X Gt 0 Y Gt 0 X Y Lt 5 3x 6y Lt 21

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