On A Coordinate Plane, A Line Goes Through (0, 3) And (3, Negative 1). A Point Is At (negative 3, 2).What

Understanding the Coordinates and Their Significance

On A Coordinate Plane, A Line Goes Through (0, 3) And (3, Negative 1). A Point Is At (−3, 2). What does this information tell us about the relationship between these points and the line? In this article, we will explore the fundamentals of coordinate geometry, analyze the given points, and determine what the question might be asking. By understanding how to interpret the points and the line, students can enhance their skills in graphing, calculating slopes, and finding equations of lines.

Basics of the Coordinate Plane

What Is a Coordinate Plane?

The coordinate plane, also known as the Cartesian plane, is a two-dimensional surface formed by two perpendicular axes: the x-axis (horizontal) and the y-axis (vertical). It allows us to plot points, lines, and shapes using pairs of numbers known as coordinates.

Understanding Coordinates

- Each point on the plane is represented by an ordered pair (x, y).
- The first number, x, indicates the position along the horizontal axis.
- The second number, y, indicates the position along the vertical axis.
- For example, the point (0, 3) is located at 0 on the x-axis and 3 on the y-axis.

Analyzing the Given Points and Line

The Known Points

- Point A: (0, 3)
- Point B: (3, -1)
- Point C: (−3, 2) — the point in question

Plotting the Points

Visualizing these points on the coordinate plane can help us better understand their relationships. Point A at (0, 3) is on the y-axis, three units above the origin. Point B at (3, -1) lies three units to the right and one unit below the x-axis. Point C at (−3, 2) is three units left of the origin and two units above the x-axis.

Calculating the Slope of the Line Through (0, 3) and (3, -1)

What Is Slope?

The slope of a line measures its steepness and is calculated as the change in y divided by the change in x between two points. The slope formula is:

$$\text{slope } (m) = (y_2 - y_1) / (x_2 - x_1)$$

Applying the Formula

Using the points (0, 3) and (3, -1):

1. $y_2 = -1$, $y_1 = 3$
2. $x_2 = 3$, $x_1 = 0$

Calculate the slope:

$$m = (-1 - 3) / (3 - 0) = (-4) / 3 = -4/3$$

Interpretation of the Slope

The slope of -4/3 indicates that for every 3 units the line moves horizontally to the right, it moves 4 units downward vertically.

Finding the Equation of the Line

Using Point-Slope Form

The point-slope form of a line’s equation is:

$$y - y_1 = m(x - x_1)$$

Choosing point (0, 3):

$$y - 3 = -4/3(x - 0)$$

Simplifies to:

$$y = -4/3\ x + 3$$

Equation of the Line

- The line passing through (0, 3) and (3, -1) has the equation:
- **y = -4/3 x + 3**

Determining the Position of the Point (−3, 2)

Is the Point On the Line?

To check whether (−3, 2) lies on the line, substitute x = -3 into the line’s equation:

$$y = -4/3\ (-3) + 3 = (4) + 3 = 7$$

Since the calculated y-value is 7, which does not match the point’s y-coordinate of 2, the point (−3, 2) does not lie on the line.

What Is the Relative Position?

- The point’s y-coordinate (2) is less than 7, the y-value on the line at x = -3.
- This means (−3, 2) is below the line at that x-value.

Interpreting the Question

Possible Questions Based on the Given Data

Given the information, the question might be asking one of the following:

1. What is the equation of the line passing through the points (0, 3) and (3, -1)?
2. Is the point (−3, 2) on, above, or below the line?
3. What is the shortest distance from the point (−3, 2) to the line?
4. Find the foot of the perpendicular from (−3, 2) to the line.

Calculating the Distance from the Point to the Line

Why Find the Distance?

Determining how far the point (−3, 2) is from the line can help in various applications, such as shortest path calculations or geometric proofs.

Distance Formula from a Point to a Line

The distance (d) from a point (x_0, y_0) to a line Ax + By + C = 0 is given by:

$$d = |A x_0 + B y_0 + C| / \sqrt{A^2 + B^2}$$

Expressing the Line in Standard Form

The line's equation:

$$y = -4/3 x + 3$$

Multiply both sides by 3 to clear fractions:

$$3 y = -4 x + 9$$

Rearranged to standard form:

$$4 x + 3 y - 9 = 0$$

Calculating the Distance

Substitute $(x_0, y_0) = (-3, 2)$:

```
d = |4(−3) + 3(2) − 9| / √(4² + 3²)
= |−12 + 6 − 9| / √(16 + 9)
= |−15| / √25
= 15 / 5
= 3
```

Thus, the point (−3, 2) is 3 units away from the line.

Summary and Practical Applications

Key Takeaways

- The line passing through (0, 3) and (3, -1) has the equation $y = -4/3\, x + 3$.
- The point (−3, 2) lies below the line at that x-value, with a perpendicular distance of 3 units.
- Understanding how to find slopes, equations, and distances is crucial in coordinate geometry.

Real-World Uses of Coordinate Geometry

- Designing graphical interfaces and computer graphics
- Navigation and pathfinding
- Engineering and architectural design
- Physics for analyzing trajectories and forces

Conclusion

Analyzing points and lines on a coordinate plane combines algebraic techniques with geometric understanding. By calculating slopes, equations, and distances, students can interpret spatial relationships effectively. Whether the question is about the position of a point relative to a line or finding the shortest distance, mastering these concepts is fundamental in mathematics and its applications.

Frequently Asked Questions

What is the slope of the line passing through points (0, 3) and (3, -1)?

The slope is $(-1 - 3) / (3 - 0) = (-4) / 3 = -4/3$.

What is the equation of the line passing through (0, 3) and (3, -1)?

Using point-slope form with point (0, 3): $y - 3 = (-4/3)(x - 0)$, so $y = (-4/3)x + 3$.

Does the point (-3, 2) lie on the line passing through (0, 3) and (3, -1)?

Substitute $x = -3$ into the line equation $y = (-4/3)x + 3$: $y = (-4/3)(-3) + 3 = 4 + 3 = 7$. Since $y = 2$, the point (-3, 2) does not lie on the line.

What is the distance between the point (-3, 2) and the line passing through (0, 3) and (3, -1)?

Calculate the perpendicular distance from (-3, 2) to the line $y = (-4/3)x + 3$ using the distance formula for point to line.

How do you find the perpendicular distance from a point to a line on a coordinate plane?

Use the formula: $|Ax + By + C| / \sqrt{A^2 + B^2}$, where the line is in $Ax + By + C = 0$ form.

Convert the line y = (-4/3)x + 3 to standard form Ax + By + C = 0.

Multiply both sides by 3 to clear fractions: $3y = -4x + 9$; then, $4x + 3y - 9 = 0$.

What is the slope-intercept form of the line passing through (0, 3) and (3, -1)?

The slope-intercept form is $y = (-4/3)x + 3$.

Is the point (-3, 2) above or below the line y = (-4/3)x + 3?

Substitute $x = -3$: $y = 4 + 3 = 7$. Since $2 < 7$, the point (-3, 2) is below the line.

What is the significance of the points (0, 3) and (3, -1) in determining the line's equation?

They are the given points through which the line passes, allowing us to find the slope and equation of the line.

Additional Resources

On a Coordinate Plane, a Line Goes Through (0, 3) and (3, -1). A Point Is at (-3, 2). What?

Understanding the relationships between points and lines on a coordinate plane is fundamental in geometry and algebra. When analyzing specific configurations—such as a line passing through two known points and a third point located elsewhere—mathematicians and students alike seek to uncover the underlying properties and implications. This article provides an in-depth exploration of such a scenario, examining the equation of the line, the position of the third point relative to the line, and the broader mathematical principles at play. Through detailed explanation and critical analysis, we aim to clarify the question: What is the significance or the resulting property when considering these points and their arrangement?

Understanding the Basic Components

Before delving into the core analysis, it is essential to understand the individual components of the scenario:

- The Points Involved:
- Point A (0, 3): Located at the origin's vertical intersection, 3 units above the x-axis.
- Point B (3, -1): Located 3 units to the right of the origin and 1 unit below the x-axis.
- Point C (-3, 2): Located 3 units to the left of the origin and 2 units above the x-axis.

- Coordinate Plane:
The two-dimensional plane where x and y axes intersect at the origin (0, 0). It provides a visual framework to analyze geometric relationships.

Determining the Equation of the Line Passing Through (0, 3) and (3, -1)

A primary step in analyzing this scenario involves deriving the equation of the line that passes through points A (0, 3) and B (3, -1). This process involves calculating the slope and then the line's equation.

Slope Calculation

The slope (m) of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying this formula:

$$
m = \frac{-1 - 3}{3 - 0} = \frac{-4}{3}
$$

Interpretation:
The line has a slope of $-\frac{4}{3}$, indicating it descends 4 units vertically for every 3 units it moves horizontally to the right.

Line Equation in Slope-Intercept Form

Using the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Choosing point A (0, 3):

$$
y - 3 = -\frac{4}{3}(x - 0)
$$
$$
y - 3 = -\frac{4}{3}x
$$

Adding 3 to both sides:

$$
y = -\frac{4}{3}x + 3
$$

Result:
The equation of the line is:

$$
\boxed{y = -\frac{4}{3}x + 3}
$$

This linear equation succinctly captures the relationship between x and y along the line passing through the given points.

Positioning the Third Point (-3, 2) Relative to the Line

With the line's equation established, the next step is to determine where the third point $(-3, 2)$ lies relative to this line—whether it is above, below, or on the line.

Calculating the Line's y-Value at x = -3

Substitute $x = -3$ into the line's equation:

$$
y = -\frac{4}{3}(-3) + 3
$$
$$
y = 4 + 3 = 7
$$

Comparison with the actual y-coordinate of the point:

- The point C has $y = 2$.
- The line's y-value at $x = -3$ is 7.

Since $2 < 7$, the point (-3, 2) lies below the line.

Implications of the Point's Position

This positional information offers insights into the geometry:

- The point is located below the line $y = -\frac{4}{3}x + 3$ when considering the coordinate plane.
- The vertical difference indicates that the point is not on the line, and knowing its relative position can be useful in various contexts, such as determining whether it is inside or outside a region bounded by the line or in solving inequalities.

Broader Mathematical Significance and Analytical Insights

Beyond the straightforward calculation, this scenario invites several analytical considerations:

1. The Line's Slope and Its Geometric Meaning

The negative slope of $-\frac{4}{3}$ indicates a descending line, slanting downward from left to right. Specifically, for every increase of 3 units along the x-axis, the y-coordinate decreases by 4 units. This slope reflects a particular rate of change, which can relate to real-world phenomena such as rate of decline, depreciation, or decreasing trends in data.

2. The Position of the Third Point and Its Significance

Knowing that $(-3, 2)$ lies below the line allows us to consider:

- Inequalities:
Since the point's y-value (2) is less than the line's y-value at $x = -3$ (which is 7), the point satisfies the inequality:

$$
y < -\frac{4}{3}x + 3
$$

This is useful in problems involving regions, optimization, or boundary conditions.

- Distance from the Line:
The shortest distance from the point to the line can be calculated, which has applications in error measurement or proximity analysis.

3. Exploring Generalizations and Applications

This configuration can serve as a basis for broader mathematical explorations:

- Line-Point Relationships:
Analyzing whether a point lies on, above, or below a line helps in solving inequalities and graphing functions.

- Parallel and Perpendicular Lines:
Comparing slopes can determine if other lines are parallel or perpendicular to the given line, which is critical in geometric constructions.

- Linear Regression and Data Modeling:

Understanding how points relate to lines is foundational in data fitting and predictive modeling.

Additional Analytical Dimensions

To deepen understanding, we can consider more advanced topics connected to this scenario:

1. Equation of a Line Through a Third Point and Known Conditions

Suppose we seek a line passing through $(-3, 2)$ with a particular slope or intersecting the original line at some point. The general approach involves using point-slope form:

$$
y - y_0 = m(x - x_0)
$$

where (x_0, y_0) is the known point, and m is the slope.

2. Distance from a Point to a Line

The shortest distance d from a point (x_0, y_0) to the line $ax + by + c = 0$ is given by:

$$
d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}
$$

Expressing the line in standard form:

$$
y = -\frac{4}{3}x + 3
$$
$$
\Rightarrow \frac{4}{3}x + y - 3 = 0
$$
Multiplying through by 3:

$$
4x + 3y - 9 = 0
$$

Calculating the distance from $(-3, 2)$:

$$
d = \frac{|4(-3) + 3(2) - 9|}{\sqrt{4^2 + 3^2}} = \frac{|-12 + 6 - 9|}{\sqrt{16 + 9}} = \frac{|-15|}{\sqrt{25}} = \frac{15}{5} = 3
$$

Thus, the point is exactly 3 units below the line, emphasizing the significance of the positional relationship.

Real-World Contexts and Applications

While this analysis is theoretical, similar scenarios are prevalent in practical domains:

- Physics and Engineering:
Analyzing slopes and positions helps in understanding trajectories, forces, and linear relationships in systems.

- Economics and Data Science:
Linear models are used to predict trends; understanding the relative positioning of data points informs decision-making.

- Navigation and Geography:
Plotting points and lines on a coordinate plane aids in map reading, route planning, and spatial analysis.

Conclusion: The Significance of the Question "What?"

The question posed—What—invites

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