Please Show All WorkGiven R=1-3 Cos ThetaFind The Area Of The Inner Loop Of The Given Polar Curve Rounded

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When encountering a polar curve such as R=1-3 Cos Theta, one of the common tasks is to analyze its shape and find specific features, such as the area of its inner loop. This process involves understanding the curve's behavior, determining key points such as intersections and loops, and applying the appropriate integral formulas. In this article, we'll walk through the steps to find the area of the inner loop of the given curve, rounding the final answer for clarity. Whether you're a student preparing for calculus exams or someone interested in polar graphs, this comprehensive guide will help clarify the process.

Understanding the Polar Curve R=1-3 Cos Theta

Shape and Behavior of the Curve

The given polar equation R=1-3 Cos Theta describes a type of limaçon. Limaçons are a family of curves with distinctive loops, cardioid-like shapes, or dimpled forms depending on their parameters. The general form R=a-b Cos Theta (or R=a-b Sin Theta) can produce inner loops if the absolute value of b exceeds a.

In this case, a=1 and b=3, so |b|=3 > a=1, indicating the curve has an inner loop. The inner loop occurs where the radius R takes negative values, which in polar coordinates means the point is plotted in the opposite direction.

Identifying Key Points of the Curve

Finding Intersections with the Origin

To locate the inner loop, identify where R=0, meaning the curve intersects the origin:

$$ R = 1 - 3 \cos \theta = 0 $$

Solve for θ:

$$ 3 \cos \theta = 1 \Rightarrow \cos \theta = \frac{1}{3} $$

Thus,

$$ \theta = \arccos \left(\frac{1}{3} \right) \approx 1.230 \text{ radians} $$

Since cosine is positive in the first and fourth quadrants,

$$ \theta_1 = \arccos \left(\frac{1}{3} \right) \approx 1.230 \text{ rad} $$
$$ \theta_2 = 2\pi - \arccos \left(\frac{1}{3} \right) \approx 5.053 \text{ rad} $$

These two angles mark points where the curve intersects the origin.

Determining the Inner Loop Range

The inner loop exists where R<0. In polar coordinates, negative R values mean the point is plotted in the opposite direction of θ. The inner loop is traced as θ varies between these two intersection points, specifically between θ_1 and θ_2.

The key is to analyze the curve's behavior between these angles, which form the boundaries of the inner loop.

Calculating the Area of the Inner Loop

Area Formula in Polar Coordinates

The area enclosed by a polar curve between two angles $\theta=a$ and $\theta=b$ is given by:

$$ A = \frac{1}{2} \int_{a}^{b} R^2 \, d\theta $$

Since the inner loop is a distinct region where R is negative, we need to account for the fact that the curve's negative R values contribute to the inner loop's area. The standard approach is to integrate over the angles where R is negative, i.e., from θ_1 to θ_2.

Note: Because the curve is symmetric with respect to the axes (due to the cosine term), the total inner loop area can be obtained by integrating over the relevant interval.

Setting Up the Integral

The integral for the inner loop area becomes:

$$ A_{inner} = \frac{1}{2} \int_{\theta_1}^{\theta_2} (1 - 3 \cos \theta)^2 \, d\theta $$

where:

- $\theta_1 = \arccos (1/3)$
- $\theta_2 = 2\pi - \arccos (1/3)$

Because the curve is symmetric, the integral over θ_1 to θ_2 captures the entire inner loop.

Evaluating the Integral

Expanding the Integrand

First, expand the square:

$$ (1 - 3 \cos \theta)^2 = 1 - 6 \cos \theta + 9 \cos^2 \theta $$

Thus, the area becomes:

$$ A_{inner} = \frac{1}{2} \int_{\theta_1}^{\theta_2} \left(1 - 6 \cos \theta + 9 \cos^2 \theta \right) d\theta $$

Break into three integrals:

$$ A_{inner} = \frac{1}{2} \left[\int_{\theta_1}^{\theta_2} 1 \, d\theta - 6 \int_{\theta_1}^{\theta_2} \cos \theta \, d\theta + 9 \int_{\theta_1}^{\theta_2} \cos^2 \theta \, d\theta \right] $$

Calculating Each Integral

1. Integral of 1:

$$ \int_{\theta_1}^{\theta_2} 1 \, d\theta = \theta \big|_{\theta_1}^{\theta_2} = \theta_2 - \theta_1 = 2\pi - 2 \arccos \left(\frac{1}{3} \right) $$

2. Integral of $\cos \theta$:

$$ \int \cos \theta \, d\theta = \sin \theta $$

Thus,

$$ \int_{\theta_1}^{\theta_2} \cos \theta \, d\theta = \sin \theta \big|_{\theta_1}^{\theta_2} = \sin \theta_2 - \sin \theta_1 $$

Since $\theta_2 = 2\pi - \theta_1$:

$$ \sin \theta_2 = \sin (2\pi - \theta_1) = - \sin \theta_1 $$

Therefore,

$$ \sin \theta_2 - \sin \theta_1 = - \sin \theta_1 - \sin \theta_1 = -2 \sin \theta_1 $$

Recall that:

$$ \theta_1 = \arccos \left(\frac{1}{3} \right) $$

and

$$ \sin \theta_1 = \sqrt{1 - \left(\frac{1}{3} \right)^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2 \sqrt{2}}{3} $$

So,

$$ \int_{\theta_1}^{\theta_2} \cos \theta \, d\theta = -2 \times \frac{2 \sqrt{2}}{3} = - \frac{4 \sqrt{2}}{3} $$

3. Integral of $\cos^2 \theta$:

Use the power-reduction identity:

$$ \cos^2 \theta = \frac{1 + \cos 2 \theta}{2} $$

Therefore,

$$ \int \cos^2 \theta \, d\theta = \frac{1}{2} \int 1 + \cos 2 \theta \, d\theta = \frac{1}{2} \left(\theta + \frac{1}{2} \sin 2 \theta \right) + C $$

Evaluate from θ_1 to θ_2:

$$ \int_{\theta_1}^{\theta_2} \cos^2 \theta \, d\theta = \frac{1}{2} \left[\theta + \frac{1}{2} \sin 2 \theta \right]_{\theta_1}^{\theta_2} $$

Since $\theta_2 = 2\pi - \theta_1$:

$$ \sin 2 \theta_2 = \sin (2 \times (2\pi - \theta_1)) = \sin (4 \pi - 2 \theta_1) = \sin (-2 \theta_1) = - \sin 2 \theta_1 $$

Similarly, $\sin 2 \theta_1$ is:

$$ \sin 2 \theta_1 = 2 \sin \theta_1 \cos \theta_1 $$

We already know:

\[\sin \theta_1 = \frac{2 \sqrt{2}}{3} \]
\[\cos \theta_1 = \frac{1}{3} \]

Thus,

\[\sin 2 \theta_1 = 2 \

Frequently Asked Questions

How do you find the points where the inner loop of the polar curve R = 1 - 3 cos θ intersects itself?

To find the intersection points of the inner loop, set $R = 0$ and solve for θ: $1 - 3 \cos \theta = 0$, which yields $\cos \theta = 1/3$. The solutions are $\theta = \arccos(1/3)$ and $2\pi - \arccos(1/3)$. These angles mark the boundaries of the inner loop.

What is the method to compute the area of the inner loop of the polar curve R = 1 - 3 cos θ?

The area of the inner loop is found by integrating $(1/2) R^2 \, d\theta$ over the interval that traces the loop. Specifically, integrate from $\theta = \arccos(1/3)$ to $\theta = 2\pi - \arccos(1/3)$, calculating $(1/2) (1 - 3 \cos \theta)^2 \, d\theta$.

How do you set up the integral for the area of the inner loop of R = 1 - 3 cos θ?

The integral for the area is $A = (1/2) \int_{\theta=a}^{\theta=b} [1 - 3 \cos \theta]^2 \, d\theta$, where $\theta=a = \arccos(1/3)$ and $\theta=b = 2\pi - \arccos(1/3)$. This computes the area enclosed by the inner loop.

What steps are involved in simplifying the integral for the area of the inner loop?

First, expand $(1 - 3 \cos \theta)^2$ to get $1 - 6 \cos \theta + 9 \cos^2 \theta$. Then, integrate term-by-term: $\int 1 \, d\theta$, $\int \cos \theta \, d\theta$, and $\int \cos^2 \theta \, d\theta$. Use identities like $\cos^2 \theta = (1 + \cos 2\theta)/2$ to evaluate the last integral.

Why is it necessary to evaluate the integral over specific bounds when finding the inner loop's area?

Because the inner loop corresponds to the portion of the curve where R is between 0 and its maximum, the bounds $\theta = \arccos(1/3)$ and $\theta = 2\pi - \arccos(1/3)$ precisely enclose the inner loop. Integrating over these bounds ensures the area calculation is accurate for that segment.

What is the final step to obtain the rounded area of the inner

loop after evaluating the integral?

After computing the definite integral, multiply the result by 1/2, then round the numerical value to the desired decimal place to find the approximate area of the inner loop of the given polar curve.

Additional Resources

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Understanding the intricacies of polar curves is a fundamental aspect of advanced calculus, especially when it comes to calculating areas enclosed by complex loops. Today’s focus is a fascinating problem: given the polar equation $R = 1 - 3 \cos \theta$, determine the area of its inner loop, and present the answer rounded to a suitable degree of precision. This exploration not only sharpens our problem-solving skills but also illuminates the process of working through polar area calculations step-by-step, ensuring clarity and precision.

Introduction to Polar Curves and Inner Loops

What Are Polar Curves?

Polar curves are equations expressed in terms of r (or R), the radius from the origin, as a function of the angle θ. Unlike Cartesian equations, which use x and y, polar equations describe curves based on how far a point is from the origin at various angles.

For example, the general form:

$$R = f(\theta)$$

describes a set of points (r, θ) in the plane, with each point’s distance from the origin given by R.

Understanding Inner Loops

Some polar equations produce curves with multiple loops—these are self-intersecting structures that can be visualized as overlapping "bulges." The inner loop is the smaller, often more intricate part of the curve that appears when the function dips below the main hull of the curve.

Determining the area of these loops involves:

- Identifying the bounds of θ where the inner loop exists.
- Calculating the integral of the area enclosed by this loop.

The task becomes more challenging when the curve is complex, as in the given equation.

Decoding the Given Polar Equation

The Equation: $R = 1 - 3 \cos \theta$

This is a type of limacon, a well-studied family of polar curves. Limacons can have inner loops, cardiods, or dimpled shapes depending on the parameters.

Key features:

- The coefficient 3 in front of $\cos \theta$ suggests a prominent inner loop, as the negative term can cause the radius R to become negative for certain θ, indicating the curve crosses over itself.

- When R is negative, the point is plotted in the direction opposite to θ, effectively creating an inner loop.

When Does the Inner Loop Occur?

Since $R = 1 - 3 \cos \theta$, the inner loop exists where $R < 0$. To find these θ, solve:

$$
1 - 3 \cos \theta < 0
$$
$$
-3 \cos \theta < -1
$$
$$
\cos \theta > \frac{1}{3}
$$

Thus, the inner loop exists where:

$$
\cos \theta > \frac{1}{3}
$$

Because $\cos \theta$ reaches $1/3$ at specific angles, the θ bounds for the inner loop are:

$$
\theta \in (-\arccos \frac{1}{3}, \arccos \frac{1}{3})
$$

This interval is symmetric around $\theta = 0$, owing to the cosine function's symmetry.

Calculating the Area of the Inner Loop

General Formula for Area in Polar Coordinates

The area A enclosed by a polar curve $R = f(\theta)$ between two angles $\theta = a$ and $\theta = b$ is:

$$
A = \frac{1}{2} \int_a^b R^2 \, d\theta
$$

Since the inner loop corresponds to the θ-values where $R < 0$, we often take the absolute value of R or note that the negative radius indicates the point is plotted in the opposite direction but still contributes positively to the area.

Setting Up the Integral

Given the symmetry, the bounds for θ are:

$$
-\arccos \frac{1}{3} \quad \text{to} \quad \arccos \frac{1}{3}
$$

Because of symmetry, the total area of the inner loop is twice the integral from 0 to $\arccos \frac{1}{3}$:

$$
A_{\text{inner}} = 2 \times \frac{1}{2} \int_0^{\arccos \frac{1}{3}} R^2 \, d\theta
$$

or simply:

$$
A_{\text{inner}} = \int_0^{\arccos \frac{1}{3}} (1 - 3 \cos \theta)^2 \, d\theta
$$

Expanding the Integrand

Expand $(1 - 3 \cos \theta)^2$:

$$
(1 - 3 \cos \theta)^2 = 1 - 6 \cos \theta + 9 \cos^2 \theta
$$

Thus, the integral becomes:

$$
A_{\text{inner}} = \int_0^{\arccos \frac{1}{3}} \left(1 - 6 \cos \theta + 9 \cos^2 \theta \right) d\theta
$$

Step-by-Step Integration Process

1. Integrate the constant term:

\[

\int 1 \, d\theta = \theta
\]

2. Integrate the \(\cos \theta \) term:

\[
\int \cos \theta \, d\theta = \sin \theta
\]

3. Integrate \(\cos^2 \theta \):

Recall the power-reduction identity:

\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
\]

Therefore,

\[
\int \cos^2 \theta \, d\theta = \int \frac{1 + \cos 2\theta}{2} \, d\theta = \frac{1}{2} \int 1 \, d\theta
+ \frac{1}{2} \int \cos 2\theta \, d\theta
\]

\[
= \frac{\theta}{2} + \frac{1}{4} \sin 2\theta
\]

4. Putting everything together

The definite integral becomes:

\[
A_{\text{inner}} = \left[\theta - 6 \sin \theta + 9 \left(\frac{\theta}{2} + \frac{1}{4} \sin 2\theta
\right) \right]_0^{\arccos \frac{1}{3}}
\]

Simplify:

\[
A_{\text{inner}} = \left[\theta - 6 \sin \theta + \frac{9}{2} \theta + \frac{9}{4} \sin 2\theta
\right]_0^{\arccos \frac{1}{3}}
\]

\[
= \left[\left(\theta + \frac{9}{2} \theta \right) - 6 \sin \theta + \frac{9}{4} \sin 2\theta
\right]_0^{\arccos \frac{1}{3}}
\]

\[
= \left[\frac{11}{2} \theta - 6 \sin \theta + \frac{9}{4} \sin 2\theta \right]_0^{\arccos

\frac{1}{3}}
\]

At the lower limit (\(0 \)):

\[
\frac{11}{2} \times 0 - 6 \times 0 + \frac{9}{4} \times 0 = 0
\]

At the upper limit (\(\theta = \arccos \frac{1}{3} \)):

- \(\theta = \arccos \frac{1}{3} \)

- \(\sin \theta = \sqrt{1 - \left(\frac{1}{3} \right)^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2 \sqrt{2}}{3} \)

- \(\sin 2\theta = 2 \sin \theta \cos \theta = 2 \times \frac{2 \sqrt{2}}{3} \times \frac{1}{3} = 2 \times \frac{2 \sqrt{2}}{3} \times \frac{1}{3} = \frac{4 \sqrt{2}}{3} \times \frac{1}{3} = \frac{4 \sqrt{2}}{9} \)

Now, plug these into the expression:

\[
A_{\text{inner}} = \frac{11}{2} \times \arccos \frac{1}{3} - 6 \times \frac{2 \sqrt{2}}{3} + \frac{9}{4} \times \frac{

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