Put The Matrix 1 1 4 5 2 15 5 0132 B = -1 1 2 2 Into Reduced Row Echelon Form. (a) The Homogeneous System

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Understanding how to convert matrices into their Reduced Row Echelon Form (RREF) is fundamental in linear algebra, especially when analyzing systems of linear equations. In this comprehensive guide, we'll explore the process step-by-step, focusing specifically on the matrix given:

$$ \text{Matrix} = \begin{bmatrix} 1 & 1 & 4 & 5 & 2 & 15 & 5 & 0 \\ 1 & 3 & 2 & -1 & 1 & 2 & 2 & 0 \end{bmatrix} $$

and the associated homogeneous system.

Understanding the Homogeneous System and Matrices

What is a Homogeneous System?

A homogeneous system of linear equations is one where all constant terms are zero. It can be written in matrix form as:

$$ A \mathbf{x} = \mathbf{0} $$

where:

- A is the coefficient matrix,
- $\mathbf{x}$ is the vector of variables,
- $\mathbf{0}$ is the zero vector.

Homogeneous systems always have at least one solution, known as the trivial solution (where all variables are zero). The goal often is to determine whether there are infinitely many solutions or a unique solution.

Relevance of Reduced Row Echelon Form (RREF)

Transforming a matrix into RREF allows us to:

- Easily identify pivot positions,
- Determine the rank of the matrix,
- Find solutions to the system,
- Establish whether solutions are unique or infinite.

Step-by-Step Guide to Converting the Matrix into RREF

Initial Matrix Setup

Let's define the matrix \(A\):

\[
A = \begin{bmatrix}
1 & 1 & 4 & 5 & 2 & 15 & 5 & 0 \\
1 & 3 & 2 & -1 & 1 & 2 & 2 & 0
\end{bmatrix}
\]

Since we're examining a homogeneous system, the last column of zeros indicates that the augmented matrix has zero constants.

Step 1: Write the augmented matrix

The augmented matrix is:

\[
\left[\begin{array}{cccccccc}
1 & 1 & 4 & 5 & 2 & 15 & 5 & 0 \\
1 & 3 & 2 & -1 & 1 & 2 & 2 & 0
\end{array}\right]
\]

Step 2: Make the leading coefficient of the first row a 1

The first row already has a leading 1 in position (1,1). No changes are necessary here.

Step 3: Eliminate the first variable from the second row

Subtract the first row from the second row:

$$
R_2 := R_2 - R_1
$$

Calculations:

- $1 - 1 = 0$
- $3 - 1 = 2$
- $2 - 4 = -2$
- $-1 - 5 = -6$
- $1 - 2 = -1$
- $2 - 15 = -13$
- $2 - 5 = -3$
- $0 - 0 = 0$

Updated matrix:

$$
\left[\begin{array}{cccccccc}
1 & 1 & 4 & 5 & 2 & 15 & 5 & 0 \\
0 & 2 & -2 & -6 & -1 & -13 & -3 & 0
\end{array}\right]
$$

Step 4: Make the pivot in the second row a 1

Divide the second row by 2:

$$
R_2 := \frac{1}{2} R_2
$$

Results:

$$
\left[\begin{array}{cccccccc}
1 & 1 & 4 & 5 & 2 & 15 & 5 & 0 \\
0 & 1 & -1 & -3 & -0.5 & -6.5 & -1.5 & 0
\end{array}\right]
$$

Step 5: Eliminate the second variable from the first row

Subtract the second row from the first row multiplied by 1:

$$
R_1 := R_1 - (R_2)
$$

Calculations:

- $1 - 0 = 1$
- $1 - 1 = 0$
- $4 - (-1) = 4 + 1 = 5$
- $5 - (-3) = 5 + 3 = 8$
- $2 - (-0.5) = 2 + 0.5 = 2.5$
- $15 - (-6.5) = 15 + 6.5 = 21.5$
- $5 - (-1.5) = 5 + 1.5 = 6.5$
- $0 - 0 = 0$

Updated matrix:

$$
\left[\begin{array}{cccccccc}
1 & 0 & 5 & 8 & 2.5 & 21.5 & 6.5 & 0 \\
0 & 1 & -1 & -3 & -0.5 & -6.5 & -1.5 & 0
\end{array}\right]
$$

Step 6: Achieve zeros above the leading 1 in the second row

In this case, the first row already has a zero in position (1,2), so no further elimination is needed there.

Final Reduced Row Echelon Form (RREF)

The matrix is now in RREF:

\[
\boxed{
\left[\begin{array}{cccccccc}
1 & 0 & 5 & 8 & 2.5 & 21.5 & 6.5 & 0 \\
0 & 1 & -1 & -3 & -0.5 & -6.5 & -1.5 & 0
\end{array}\right]
}
\]

This form clearly indicates the positions of pivots and free variables, facilitating the solution process.

Analyzing the Homogeneous System in RREF

Key Points from the RREF

- Pivot positions are in columns 1 and 2.
- Remaining columns correspond to free variables.
- The system has infinitely many solutions since there are free variables.

Expressing the Solution

Variables:

- \(x_1\) and \(x_2\) are basic variables.
- \(x_3, x_4, x_5, x_6, x_7\) are free variables.

Expressed in parametric form:

\[
\begin{cases}
x_1 = -5x_3 - 8x_4 - 2.5x_5 - 21.5x_6 - 6.5x_7 \\
x_2 = x_3 + 3x_4 + 0.5x_5 + 6.5x_6 + 1.5x_7 \\

x_3, x_4, x_5, x_6, x_7 \in \mathbb{R}
\end{cases}
\]

This parametric representation shows an infinite set of solutions, characteristic of homogeneous systems with free variables.

Implications and Applications of RREF in Homogeneous Systems

Understanding System Consistency and Solution Sets

- Homogeneous systems always have at least the trivial solution (\(\mathbf{x} = \mathbf{0}\)).
- Infinite solutions occur when the rank of the coefficient matrix is less than the number of variables.
- Unique solutions occur only when all variables are pivot variables (not the case here).

Practical Applications

Transforming matrices into RREF is crucial in many fields, such as:

- Computer graphics (transformations)
- Engineering (system analysis)
- Data science (dimensionality reduction)
- Physics (solving systems of equations)

Summary and Key Takeaways

- The process of converting a matrix to RREF involves elementary row operations: row swapping, scaling, and adding multiples of rows.
- Homogeneous systems always have at least one solution— the trivial solution.
- The RREF form clarifies the structure of solutions, indicating whether solutions are unique or infinite.
- Understanding the positions of pivots and free variables is essential in solving and interpreting linear systems.

Conclusion

Transforming matrices into Reduced Row Echelon Form is a fundamental skill in linear algebra that enables the analysis of systems of equations, especially homogeneous systems. By following systematic row operations, as demonstrated with the matrix example, students and professionals can efficiently

Frequently Asked Questions

What is the first step in converting the matrix [[1, 1, 4, 5], [2, 15, 5, 0], [132, B, -1, 1], [2, 2, 0, 0]] to Reduced Row Echelon Form?

The initial step is to write the augmented matrix and then perform row operations to create a leading 1 in the first row, first column, by dividing the first row by its leading coefficient if necessary.

How do we handle the parameter B when reducing the matrix to RREF?

The parameter B appears in the third row, second column, so during row operations, its value will influence the elimination process. Depending on B's value, certain rows may become dependent or lead to special cases like free variables.

What does the reduced row echelon form tell us about the solutions of the homogeneous system?

The RREF indicates whether the system has a unique solution, infinitely many solutions, or no solution. For a homogeneous system, it always has at least the trivial solution, and the form reveals if there are free variables leading to infinite solutions.

In the context of homogeneous systems, what is the significance of rows of zeros in the RREF?

Rows of zeros imply free variables, which lead to infinitely many solutions. The number of free variables equals the number of columns minus the rank of the matrix.

Can the matrix be reduced to RREF when B equals certain values? If so, which values?

Yes, specific values of B can cause the matrix to be singular or lead to inconsistent systems. For example, if B is chosen such that a row becomes inconsistent (like 0 = non-zero), the system has no solution; otherwise, it can be reduced successfully.

What are the key row operations used in converting the matrix to RREF in this problem?

Key operations include swapping rows, multiplying a row by a scalar to create a leading 1, and adding multiples of one row to another to eliminate entries below and above the leading 1s.

Why is understanding the reduction of this matrix important for solving the homogeneous system?

Reducing the matrix to RREF simplifies the system to identify solutions easily, determine the nature of solutions (trivial or infinite), and analyze the dependence on the parameter B, providing clear insights into the solution structure.

Additional Resources

Matrix Reduction and Homogeneous Systems: An Expert Guide

In the realm of linear algebra, the process of transforming matrices into their simplified forms is fundamental to understanding the structure of systems of equations. Among these forms, the Reduced Row Echelon Form (RREF) stands out as a powerful tool for solving linear systems efficiently. This article delves into a specific example: transforming the matrix

$$
\begin{bmatrix}
1 & 1 & 4 & 5 \\
2 & 15 & 5 & 0 \\
1 & 3 & 2 & -1 \\
1 & 2 & 2 & 1
\end{bmatrix}
$$

and its associated homogeneous system into RREF, with a focus on the homogeneous case. We will explore each step meticulously, providing clarity and insights into the process.

Understanding the Matrix and the Homogeneous System

Matrix Context and Dimensions

The matrix provided appears to be a 4x4 augmented matrix, which typically represents a system of 4 equations with 3 variables (if considering the last column as constants). However, since the question emphasizes the homogeneous system, the last column plays a critical role in defining the structure.

In a homogeneous system, the rightmost column consists entirely of zeros, indicating:

$$
A \mathbf{x} = \mathbf{0}
$$

where A is the coefficient matrix, and $\mathbf{x}$ is the vector of variables.

Given the matrix:

$$
\begin{bmatrix}
1 & 1 & 4 & 5 \\
2 & 15 & 5 & 0 \\
1 & 3 & 2 & -1 \\
1 & 2 & 2 & 1
\end{bmatrix}
$$

the last column contains non-zero entries, which suggests it may be an augmented matrix for a non-homogeneous system. However, since our focus is on the homogeneous system, we will consider the matrix with the last column replaced by zeros:

$$
\begin{bmatrix}
1 & 1 & 4 & 0 \\
2 & 15 & 5 & 0 \\
1 & 3 & 2 & 0 \\
1 & 2 & 2 & 0
\end{bmatrix}
$$

This is the matrix we'll analyze for the homogeneous case.

Step-by-Step Reduction to RREF

Initial Setup and Row Operations

The goal is to convert the matrix into RREF, which satisfies the following conditions:

1. All leading entries (pivots) are 1.
2. Each leading 1 is the only non-zero entry in its column.
3. The leading 1s move to the right as we go down each row.

Let's proceed systematically.

Step 1: Write the matrix

\[
\begin{bmatrix}
1 & 1 & 4 & 0 \\
2 & 15 & 5 & 0 \\
1 & 3 & 2 & 0 \\
1 & 2 & 2 & 0
\end{bmatrix}
\]

Step 2: Use the first row to eliminate below

- The pivot in row 1 is already 1.
- Eliminate the entries below in the first column:

Subtract 2 times row 1 from row 2:

\[
R_2 \leftarrow R_2 - 2 R_1
\]
\[
(2 - 21) = 0, \quad (15 - 21) = 13, \quad (5 - 24) = 5 - 8 = -3, \quad (0 - 20) = 0

\]

Subtract 1 times row 1 from row 3:

\[
R_3 \leftarrow R_3 - R_1
\]
\[
(1 - 1) = 0, \quad (3 - 1) = 2, \quad (2 - 4) = -2, \quad (0 - 0) = 0
\]

Subtract 1 times row 1 from row 4:

\[
R_4 \leftarrow R_4 - R_1
\]
\[
(1 - 1) = 0, \quad (2 - 1) = 1, \quad (2 - 4) = -2, \quad (0 - 0) = 0
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 1 & 4 & 0 \\
0 & 13 & -3 & 0 \\
0 & 2 & -2 & 0 \\
0 & 1 & -2 & 0
\end{bmatrix}
\]

Step 3: Create leading 1 in row 2

Divide row 2 by 13:

\[
R_2 \leftarrow \frac{1}{13} R_2
\]
\[
(0, 1, -\frac{3}{13}, 0)
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 1 & 4 & 0 \\

0 & 1 & -\frac{3}{13} & 0 \\
0 & 2 & -2 & 0 \\
0 & 1 & -2 & 0
\end{bmatrix}
\]

Step 4: Eliminate entries above and below the leading 1 in column 2

- For row 3: subtract 2 times row 2:

\[
R_3 \leftarrow R_3 - 2 R_2
\]
\[
(0, 2 - 21) = 0, \quad (-2 - 2(-\frac{3}{13})) = -2 + \frac{6}{13} = -\frac{26}{13} + \frac{6}{13} = -\frac{20}{13}
\]

- For row 4: subtract 1 times row 2:

\[
R_4 \leftarrow R_4 - R_2
\]
\[
(0, 1 - 1, -2 - (-\frac{3}{13})) = (0, 0, -2 + \frac{3}{13}) = (0, 0, -\frac{26}{13} + \frac{3}{13}) = (0, 0, -\frac{23}{13})
\]

- For row 1: subtract 1 times row 2:

\[
R_1 \leftarrow R_1 - R_2
\]
\[
(1, 1 - 1, 4 - (-\frac{3}{13})) = (1, 0, 4 + \frac{3}{13}) = (1, 0, \frac{52}{13} + \frac{3}{13}) = (1, 0, \frac{55}{13})
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 0 & \frac{55}{13} & 0 \\
0 & 1 & -\frac{3}{13} & 0 \\
0 & 0 & -\frac{20}{13} & 0 \\
0 & 0 & -\frac{23}{13} & 0

\end{bmatrix}
\]

Step 5: Create leading 1 in row 3

Divide row 3 by \(-\frac{20}{13}\):

\[
R_3 \leftarrow R_3 \times \left(-\frac{13}{20}\right)
\]

\[
(0, 0, 1, 0)
\]

Updated matrix:

\[
\begin{bmatrix}
1 & 0 & \frac{55}{13} & 0 \\
0 & 1 & -\frac{3}{13} & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -\frac{23}{13} & 0
\end{bmatrix}
\]

Step 6: Eliminate entries above row 3's pivot

- For row 1:

\[
R_1 \leftarrow R_1 - \left(\frac{55}{13}\right) R_3
\]
\[
\frac{55}{13} - \frac{55}{13} \times 1 = 0
\]
Similarly, the third element:

\[
\frac{55}{13} - \frac{55}{13} \times 1 = 0
\]

- For row 2:

```
\[
R_2 \leftarrow R_2 + \frac{3}{13} R_3
\]
\[
-\frac{3}{13} + \frac{3}{13} \times 1 = 0
\]

- For row 4:

\[
R_4 \leftarrow R_4 + \frac{23}{13} R_3
\]
\[
-\frac{23
```

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