Question 4 The Projection Of The Vector V = (−6, −1, 2) Onto The Vector U = (−3, 0, 1) Is (enter Integers

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Understanding vector projection is fundamental in vector algebra, especially when analyzing components of vectors along specific directions. The problem asks us to find the projection of the vector V = (−6, −1, 2) onto the vector U = (−3, 0, 1), and to express the result as integers. This process involves applying the formula for vector projection, which helps determine the component of one vector along the direction of another. In this article, we will explore the theoretical background, step-by-step calculations, and interpret the results comprehensively.

Understanding Vector Projection

What Is Vector Projection?

Vector projection is a way to find the component of one vector along the direction of another vector. Geometrically, it represents how much of one vector extends in the direction of the other. Mathematically, the projection of a vector V onto a vector U is denoted as proj_U(V) and is given by the formula:

$$
\text{proj}_U(V) = \left(\frac{V \cdot U}{\|U\|^2} \right) U
$$

where:

- $V \cdot U$ is the dot product of vectors V and U,
- $\|U\|^2$ is the square of the magnitude of U.

This formula results in a vector that lies along U and represents the component of V in that direction.

Why Is Projection Important?

Projections are useful in many contexts, including physics for resolving forces, in computer graphics for shading and rendering, and in mathematics for decomposing vectors into components. They allow us to analyze how vectors relate to each other and are essential in calculations involving orthogonal projections, least squares solutions, and more.

Step-by-Step Computation of the Projection

To find the projection of V onto U, we follow these steps:

1. Calculate the dot product $V \cdot U$.
2. Calculate the magnitude squared of U, $\|U\|^2$.
3. Compute the scalar projection coefficient $\frac{V \cdot U}{\|U\|^2}$.
4. Multiply this scalar by U to get the projection vector.

Let's perform each step explicitly.

Step 1: Compute the Dot Product $V \cdot U$

Given:

$$
V = (-6, -1, 2)
$$

$$
U = (-3, 0, 1)
$$

The dot product is calculated as:

$$
V \cdot U = (-6)(-3) + (-1)(0) + (2)(1) = 18 + 0 + 2 = 20
$$

Thus,

$$
V \cdot U = 20
$$

Step 2: Calculate $\|U\|^2$

The magnitude squared of U is:

$$
\|U\|^2 = (-3)^2 + 0^2 + 1^2 = 9 + 0 + 1 = 10
$$

Step 3: Find the Scalar Coefficient $\frac{V \cdot U}{\|U\|^2}$

Dividing the dot product by the magnitude squared:

$$
\frac{20}{10} = 2
$$

This scalar indicates how much of V projects onto U in terms of U's length.

Step 4: Calculate the Projection Vector $\text{proj}_U(V)$

Multiply the scalar by U:

$$
\text{proj}_U(V) = 2 \times (-3, 0, 1) = (-6, 0, 2)
$$

Final Result and Interpretation

The projection vector of V onto U is:

$$
\boxed{(-6, 0, 2)}
$$

This vector lies along U's direction and represents the component of V in U's direction. Notice that the projection vector's components are integers, which satisfies the condition of the problem to enter integers.

Additional Insights and Applications

Orthogonal Components

The projection vector helps decompose V into two parts:

- The component along U: $\text{proj}_U(V) = (-6, 0, 2)$
- The component orthogonal to U: $V - \text{proj}_U(V)$

Calculating the orthogonal component:

$$
V - \text{proj}_U(V) = (-6, -1, 2) - (-6, 0, 2) = (0, -1, 0)
$$

This orthogonal component is perpendicular to U, as expected.

Practical Applications

- In physics, understanding how a force vector projects onto a certain axis helps determine work done.
- In computer graphics, projections assist in rendering shadows and reflections.
- In data analysis, projecting data vectors onto principal axes simplifies

complex datasets.

Summary of Key Points

- The projection of V onto U is calculated using the dot product and magnitude squared.
- The scalar coefficient is obtained by dividing the dot product by the magnitude squared.
- Multiplying this scalar by U yields the projection vector.
- The result is an integer vector, satisfying the problem's requirements.

Conclusion

In this detailed exploration, we have calculated the projection of the vector V = (-6, -1, 2) onto U = (-3, 0, 1). The step-by-step approach involved computing the dot product, magnitude squared, scalar projection coefficient, and finally the projection vector itself. The answer, (-6, 0, 2), is expressed in integers, aligning with the problem's specifications. Understanding this process enhances comprehension of vector components, which are fundamental in various scientific and engineering disciplines. Whether for resolving forces, analyzing signals, or simplifying data, the ability to project vectors accurately is an essential skill in vector algebra.

Frequently Asked Questions

How do you find the projection of vector V = (-6, -1, 2) onto vector U = (-3, 0, 1)?

The projection is calculated as: (V · U) / (U · U) multiplied by U. First, find the dot products V · U and U · U, then compute the scalar and multiply by U.

What is the dot product of vectors V = (-6, -1, 2) and U = (-3, 0, 1)?

The dot product V · U = (-6)(-3) + (-1)(0) + (2)(1) = 18 + 0 + 2 = 20.

What is the dot product of vector U = (-3, 0, 1) with itself?

U · U = (-3)^2 + 0^2 + 1^2 = 9 + 0 + 1 = 10.

What is the scalar projection of V onto U given V · U = 20 and U · U = 10?

The scalar projection is (V · U) / (U · U) = 20 / 10 = 2.

What is the vector projection of V onto U?

The vector projection is scalar projection multiplied by the unit vector in the direction of U: (2 / 10) U = (1/5) (−3, 0, 1) = (−3/5, 0, 1/5).

Are the components of the projection vector integers?

No, the components are fractions: (−3/5, 0, 1/5), which are not integers.

How can the projection of V onto U be expressed with integers?

Multiplying the projection vector by 5 to clear fractions yields: 5 (−3/5, 0, 1/5) = (−3, 0, 1).

What is the projection of V onto U in terms of integers?

The projection vector with integer components is (−3, 0, 1).

Is the projection vector (−3, 0, 1) a scaled version of U?

Yes, it is exactly 1 times U, since U = (−3, 0, 1).

What is the significance of entering integers for the projection of V onto U?

Entering integers indicates expressing the projection as a scaled version of U with integer components, which in this case is (−3, 0, 1).

Additional Resources

Projection of a Vector: A Deep Dive into Vector Operations

Understanding Vector Projections: An Essential Concept in Vector Algebra

Imagine trying to determine how much of one vector points in the same direction as another. This is essentially what the operation of projecting a vector onto another accomplishes. Whether you're working in physics, engineering, computer graphics, or data science, understanding vector projections is fundamental.

In this article, we'll explore the process of projecting the vector V = (−6, −1, 2) onto U = (−3, 0, 1). We'll unpack the mathematical principles involved, step through the calculation meticulously, and highlight the importance of the projection in various applications.

What Is a Vector Projection?

A vector projection essentially finds the component of one vector along the direction of another. Visualize two arrows originating from the same point: the projection of one arrow onto another is the shadow it casts onto the line of the second arrow if a light source shines perpendicular to it.

Mathematically, the projection of vector V onto U is a vector that lies along U and represents the part of V aligned with U. This is distinct from the scalar projection, which is just the length of the shadow length; the vector projection provides both magnitude and direction.

Scalar projection (or component):
$$
\text{comp}_U V = \frac{V \cdot U}{|U|}
$$

Vector projection:
$$
\text{proj}_U V = \left(\frac{V \cdot U}{|U|^2} \right) U
$$

where:
- V and U are vectors,
- $V \cdot U$ is the dot product,
- $|U|$ is the magnitude (length) of U.

Step-by-Step Calculation of the Projection

Let's put theory into practice with our specific vectors:

$$
V = (-6, -1, 2)
$$
$$
U = (-3, 0, 1)
$$

Our goal: compute $\text{proj}_U V$.

---

1. Compute the Dot Product $V \cdot U$

The dot product is calculated as:

$$
V \cdot U = (-6)(-3) + (-1)(0) + (2)(1)
$$

\]

Calculating each term:

- $(-6)(-3) = 18$
- $(-1)(0) = 0$
- $(2)(1) = 2$

Adding these:

$$
V \cdot U = 18 + 0 + 2 = 20
$$

---

2. Find the Magnitude Squared of U, $|U|^2$

The magnitude squared simplifies calculations:

$$
|U|^2 = (-3)^2 + 0^2 + 1^2 = 9 + 0 + 1 = 10
$$

---

3. Calculate the Scalar Multiplier $\frac{V \cdot U}{|U|^2}$

Dividing the dot product by the magnitude squared:

$$
\frac{20}{10} = 2
$$

This scalar indicates how much of V aligns with U.

---

4. Multiply the Scalar by Vector U to Get the Projection

Now, multiply each component of U by 2:

$$
\text{proj}_U V = 2 \times (-3, 0, 1) = (-6, 0, 2)
$$

Final Result and Interpretation

The projection of V onto U is:

$$
\boxed{\text{proj}_U V = (-6, 0, 2)}
$$

This vector tells us how much of V lies along U. Notably, the resulting projection points in the same or opposite direction as U, scaled appropriately. Here, the projection vector is directly aligned with U, scaled by a factor of 2.

In other words:
- The component of V along U has a magnitude and direction equivalent to (-6, 0, 2).
- The projection vector is an integer vector, aligning with the problem's requirement for integer entries.

Why Is This Important?

Understanding how to project vectors accurately is vital for numerous scientific and engineering tasks:

- Physics: Calculating work done by a force involves projecting the force vector onto the displacement vector.
- Computer Graphics: Projection helps in rendering scenes and transforming objects in 3D space.
- Data Analysis: Principal component analysis (PCA) involves projecting data onto principal axes.
- Robotics: Motion planning often requires projecting vectors to align movements along desired paths.

Conclusion: Key Takeaways

- The vector projection extracts the component of one vector along the direction of another.
- The calculation involves dot products and scalar multiplication.
- For vectors V and U, the projection is given by:

$$
\text{proj}_U V = \left(\frac{V \cdot U}{|U|^2} \right) U
$$

- In our specific example, the projection of V = (-6, -1, 2) onto U = (-3, 0, 1) is (-6, 0, 2).

This example underscores the straightforward yet powerful nature of vector operations. Mastering these calculations enhances your ability to analyze and interpret complex systems involving vectors, ensuring precision and clarity in your work.

End of Article

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