Question 6 An Open Rectangular Box Is To Be Made With A Square Base, And Its Capacity Is To Be 4000 Cm^3.

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Designing a practical and efficient open rectangular box with a square base involves understanding the relationship between dimensions, volume, and surface area. In this article, we will explore the mathematical concepts behind creating such a box with a fixed capacity of 4000 cubic centimeters (cm^3). Whether you're a student preparing for exams or a craftsman interested in packaging design, understanding how to determine the optimal dimensions for this box is essential. Let's delve into the problem systematically, exploring the key steps and principles involved.

Understanding the Problem: Basic Concepts

Before we proceed with calculations, it's important to clarify the problem's parameters and what is asked. The main goal is to design an open rectangular box with a square base that can hold a volume of 4000 cm^3. The constraints and variables include:

- The base of the box is square, meaning its length and width are equal.
- The height of the box is to be determined.
- Since it is an open box, the top surface is open and does not contribute to the surface area.
- The capacity or volume of the box is fixed at 4000 cm^3.

The problem involves optimization—finding the dimensions that satisfy the volume constraint and possibly minimize the surface area or meet other design criteria.

Setting Up the Variables and Equations

To analyze the problem mathematically, assign variables to the dimensions of the box:

Defining Variables

- Let **x** be the length of a side of the square base (in centimeters).
- Let **h** be the height of the box (in centimeters).

Based on these variables, the volume (V) of the box can be expressed as:

$$V = x^2 \; h$$

Given that the volume is fixed at 4000 cm^3, the primary constraint is:

$$x^2 \; h = 4000$$

From this, we can express the height in terms of x:

$$h = 4000 / x^2$$

This relationship will be central to our analysis, as it links the base dimension to the height.

Calculating Surface Area and Optimization

While the problem states the capacity is fixed, often the goal is to minimize the material used or surface area for cost efficiency. For an open box, the surface area (A) comprises just the base and the four vertical sides.

Surface Area Formula

The surface area of the open box can be written as:

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A = area of base + area of four sides
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The area of the square base:

$$\text{Base area} = x^2$$

The area of the four vertical sides:

$$4 \; (x \; h)$$

Hence, the total surface area is:

$$A(x) = x^2 + 4x \quad h$$

Substituting h from earlier, we get:

$$A(x) = x^2 + 4x \quad (4000 / x^2) = x^2 + (16000 / x)$$

Our goal could be to find the value of x that minimizes A(x), which corresponds to using the least material for construction.

Finding the Optimal Dimensions

To minimize the surface area, differentiate A(x) with respect to x and find critical points.

Calculating the Derivative

$$A(x) = x^2 + (16000 / x)$$

Differentiate term-by-term:

$$A'(x) = 2x - (16000 / x^2)$$

Set the derivative equal to zero to find critical points:

$$2x - (16000 / x^2) = 0$$

Rearranged:

$$2x = 16000 / x^2$$

Multiply both sides by x^2:

$$2x^3 = 16000$$

Solve for x:

$$x^3 = 8000$$

Taking cube root:

$$x = \sqrt[3]{8000} = 20 \text{ cm}$$

This suggests that the square base should have sides of approximately 20 cm to minimize surface area.

Calculating the Corresponding Height

Recall the relation:

$$h = 4000 / x^2$$

Substitute x = 20 cm:

$$h = 4000 / (20)^2 = 4000 / 400 = 10 \text{ cm}$$

Thus, the optimal dimensions for the box are:

- Base side length: 20 cm
- Height: 10 cm

These dimensions satisfy the volume requirement and minimize the material used for constructing the open box.

Verifying the Results and Practical Considerations

It's important to verify that the critical point corresponds to a minimum. One way is to check the second derivative or analyze the behavior of A(x).

Second Derivative Test

Calculate the second derivative:

$$A''(x) = 2 + (2 \quad 16000) / x^3 = 2 + (32000 / x^3)$$

Since $x > 0$, the second derivative is positive, confirming a minimum at x = 20 cm.

Practical Implications

- The dimensions provide an efficient design, minimizing material without sacrificing capacity.
- The open-top design is suitable for packaging or storage where accessibility from above is needed.
- The dimensions are manageable and easy to manufacture, with a 20 cm square base and 10 cm height.

Summary and Key Takeaways

- The problem involves designing an open rectangular box with a square base and fixed volume.
- By defining variables and setting up equations, we can calculate the optimal dimensions for minimal surface area.
- The key steps include expressing height in terms of base side length, deriving the surface area function, and finding its minimum via calculus.
- The optimal design, in this case, involves a 20 cm by 20 cm square base and a height of 10 cm, which satisfies the volume constraint of 4000 cm^3.

Conclusion: Applying Mathematical Principles to Real-World Design

Understanding how to optimize the dimensions of a box with specific constraints is a fundamental aspect of mathematical modeling and engineering design. This process, combining algebra, calculus, and practical considerations, allows designers to create cost-effective, functional, and efficient containers. Whether for packaging, storage, or artistic projects, mastering these techniques empowers you to solve complex design problems with confidence.

Remember, the key to such problems lies in properly defining variables, setting up the correct equations, and applying calculus to find optimal solutions. With practice, you'll be able to approach similar problems with ease and precision, ensuring your designs are both practical and mathematically sound.

Frequently Asked Questions

What are the dimensions of the open rectangular box with a square base and a capacity of 4000 cm^3?

Let the side of the square base be x cm and the height be h cm. Since the volume is 4000 cm^3, we have $x^2h = 4000$. The dimensions depend on the chosen value of x, with $h = 4000 / x^2$.

How can we determine the optimal dimensions of the box to

minimize the material used?

To minimize the surface area, set up the surface area function in terms of x and h, then differentiate with respect to x, substitute $h = 4000 / x^2$, and find the value of x that minimizes the surface area.

What is the formula relating the side length of the base and height of the box?

The formula is $h = 4000 / x^2$, where x is the side length of the square base.

If the side of the square base is 20 cm, what is the height of the box?

Using $h = 4000 / x^2$, substituting x = 20 cm, $h = 4000 / (20)^2 = 4000 / 400 = 10$ cm.

How does changing the base side length affect the height of the box for a fixed volume?

As the base side length x increases, the height h decreases proportionally to $1 / x^2$, maintaining the fixed volume of 4000 cm^3.

What are the practical applications of designing such a box with specific volume and minimal material?

This design is useful in packaging, shipping, and storage industries where maximizing volume while minimizing material costs and weight is essential.

Can the dimensions be adjusted to make the box more stable or easier to manufacture?

Yes, adjusting dimensions for stability involves considering factors like the height-to-base ratio, manufacturing constraints, and material strength, which may lead to choosing dimensions that slightly deviate from the optimal for minimal material.

What is the significance of the square base in the design of this open box?

A square base simplifies calculations, provides uniform support, and often results in more efficient use of material compared to rectangular bases, making it a common choice in box design.

Additional Resources

Question 6: An open rectangular box is to be made with a square base, and its capacity is to be 4000 cm^3.

Designing an open rectangular box with specific volume requirements is a common problem in geometry and optimization, often encountered in manufacturing, packaging, and design. Whether you're a student preparing for exams or a professional involved in product design, understanding how to approach such problems methodically is essential. In this guide, we will explore the step-by-step process of solving this problem, including setting up equations, optimizing the dimensions, and understanding the underlying principles.

Understanding the Problem

Before jumping into calculations, it’s crucial to understand what the problem asks:

- Shape: An open rectangular box with a square base.
- Volume (Capacity): Exactly 4000 cm^3.
- Objective: To analyze or determine the possible dimensions (length, width, height) of such a box.

The problem does not specify whether you need to maximize volume, minimize surface area, or find dimensions for a given volume. Here, the main focus is on designing a box with a fixed volume, which often involves optimizing for other parameters such as minimizing material used or maximizing height.

Visualizing the Problem

Imagine a box with the following features:

- The base is square: both length and width are equal, say x cm.
- The height of the box is h cm.

Since the box is open at the top, the surface area considerations exclude the top surface.

Key dimensions:

- Base dimensions: x cm by x cm.
- Height: h cm.

Volume formula:

$$\text{Volume} = x^2 \times h$$

Given:

$$x^2 \times h = 4000 \quad \text{cm}^3$$

Setting Up the Mathematical Model

Step 1: Express the volume constraint

From the problem:

$$ x^2 h = 4000 $$

This can be rearranged to express h in terms of x:

$$ h = \frac{4000}{x^2} $$

This relation allows us to analyze how changing x affects h.

Step 2: Understand what you want to optimize

Depending on the goal, the problem could involve:

- Minimizing the surface area (to reduce material costs),
- Maximizing height (for storage capacity),
- Finding dimensions that satisfy certain constraints.

Assuming the typical approach, let’s consider minimizing the surface area of the open box, which is a common extension in such problems.

Calculating Surface Area

Since the box is open at the top, the surface area (SA) includes:

- The square base: x^2
- The four vertical sides: $4 \times x \times h$

Total surface area:

$$ SA = x^2 + 4xh $$

Substitute h from earlier:

$$ SA = x^2 + 4x \times \frac{4000}{x^2} = x^2 + \frac{16000}{x} $$

Optimization: Minimizing Surface Area

To find the dimensions that minimize material use, differentiate SA with respect to x and set the derivative to zero.

Step 1: Derive the function

$$ SA(x) = x^2 + \frac{16000}{x} $$

Step 2: Find the critical points

Differentiate:

$$ \frac{d}{dx} SA(x) = 2x - \frac{16000}{x^2} $$

Set the derivative equal to zero:

$$ 2x - \frac{16000}{x^2} = 0 $$

Rearranged:

$$ 2x = \frac{16000}{x^2} $$

Multiply both sides by x^2:

$$ 2x^3 = 16000 $$

Solve for x:

$$ x^3 = \frac{16000}{2} = 8000 $$

$$ x = \sqrt[3]{8000} $$

Calculate:

$$ x \approx \sqrt[3]{8000} $$

Since $20^3 = 8000$:

$$ x = 20 \, \text{cm} $$

Step 3: Find h

Recall:

$$ h = \frac{4000}{x^2} $$

Calculate:

$$ h = \frac{4000}{(20)^2} = \frac{4000}{400} = 10 \, \text{cm} $$

Final Dimensions and Interpretation

The dimensions that minimize the surface area for the open box with volume 4000 cm^3 are approximately:

- Base side length (x): 20 cm
- Height (h): 10 cm

Verification:

- Volume check:

$$ 20^2 \times 10 = 400 \times 10 = 4000 \text{ cm}^3 $$

- Surface area:

$$ SA = 20^2 + 4 \times 20 \times 10 = 400 + 800 = 1200 \text{ cm}^2 $$

This configuration provides an efficient design with minimal material usage.

Additional Considerations

1. Alternative Objectives

- If the goal is to maximize height for a fixed volume, then h increases as x decreases (since $h = \frac{4000}{x^2}$), but very small x may be impractical.
- If material cost depends on surface area, the above optimization is relevant.
- For maximizing volume given surface area constraints, different approaches are needed.

2. Practical Implications

- Material efficiency: The dimensions derived minimize material use while satisfying volume constraints.
- Manufacturing considerations: Slight adjustments may be necessary for manufacturing tolerances.

Summary of Key Steps

- Set up the volume equation: $x^2 h = 4000$.
- Express one variable in terms of the other: $h = \frac{4000}{x^2}$.
- Define the surface area function: $SA = x^2 + 4x h$.
- Substitute h into the surface area: $SA = x^2 + \frac{16000}{x}$.
- Differentiate SA with respect to x, find critical points.
- Solve for x to minimize the surface area.
- Calculate h using the volume constraint.

Final Thoughts

Designing an open rectangular box with a square base and a fixed capacity involves a harmonious balance between geometric constraints and optimization principles. By translating the problem into algebraic expressions and employing calculus for optimization, you can determine the most efficient dimensions for your specific needs. This approach is not only applicable to this particular problem but also forms a foundation for solving a broad class of real-world engineering and design challenges involving volume, surface area, and material optimization.

In essence, understanding the relationship between the base size, height, and volume allows for precise control over the design parameters, ensuring both functional and economical solutions in practical applications.

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