R(t)=(ln(t2+1))i+(tan1t)j+(t2+1)k Is The Position Vector Of A Particle In Space At Time T. Find The Angle

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Understanding the motion of particles in space is a fundamental aspect of vector calculus and physics. When given a position vector function like R(t) = (ln(t^2+1))i + (tan t)j + (t^2+1)k, it describes the position of a particle in three-dimensional space at any given time t. To analyze the particle's movement comprehensively, one of the common questions posed is: What is the angle between the particle's position vector at a specific time and another vector, such as the velocity vector or a reference axis?

This article delves into how to find the angle associated with such a position vector, covering the necessary derivatives, calculations, and interpretations involved in understanding the particle's motion.

Understanding the Position Vector R(t)

The position vector R(t) in space is given by:

\[
R(t) = \left(\ln(t^2 + 1) \right) \mathbf{i} + \left(\tan t \right) \mathbf{j} + \left(t^2 + 1 \right) \mathbf{k}
\]

Here, each component corresponds to the x, y, and z coordinates respectively:

- \(x(t) = \ln(t^2 + 1) \)
- \(y(t) = \tan t \)
- \(z(t) = t^2 + 1 \)

This vector function describes the trajectory of a particle moving in three-dimensional space. As t varies, the particle's position changes according to these functions.

Calculating the Derivative of R(t): Velocity Vector

To understand the movement and find angles involving the position vector, the first step is to determine the velocity vector \(R'(t) \). The velocity vector indicates the direction and rate of change of the particle's position at any given time.

The derivatives of each component are:

- $x'(t) = \frac{d}{dt} \ln(t^2 + 1) = \frac{2t}{t^2 + 1}$
- $y'(t) = \frac{d}{dt} \tan t = \sec^2 t$
- $z'(t) = \frac{d}{dt} (t^2 + 1) = 2t$

Thus, the velocity vector is:

$$
R'(t) = \left(\frac{2t}{t^2 + 1} \right) \mathbf{i} + \sec^2 t \, \mathbf{j} + 2t \, \mathbf{k}
$$

Understanding both the position and velocity vectors is essential when computing angles, as these vectors often form the basis of the angle calculations.

What Does the Angle Represent?

In the context of particle motion, the angle between the position vector $R(t)$ and the velocity vector $R'(t)$ at a particular time t can reveal whether the particle is moving directly away from or towards the origin, or whether its path is changing direction.

Another common scenario involves finding the angle between the position vector and a fixed reference vector, such as the x-axis, y-axis, z-axis, or a specific vector related to the problem's context.

For this article, we'll focus on how to find the angle between the position vector $R(t)$ and the velocity vector $R'(t)$, which provides insights into the particle's motion.

Formula for the Angle Between Two Vectors

The angle θ between two vectors $\mathbf{A}$ and $\mathbf{B}$ in space is given by the dot product formula:

$$
\cos \theta = \frac{ \mathbf{A} \cdot \mathbf{B} }{ |\mathbf{A}| |\mathbf{B}| }
$$

where:

- $\mathbf{A} \cdot \mathbf{B}$ is the dot product of the vectors.
- $|\mathbf{A}|$ and $|\mathbf{B}|$ are the magnitudes (lengths) of the vectors.

Applying this to $R(t)$ and $R'(t)$:

$$
\cos \theta = \frac{ R(t) \cdot R'(t) }{ | R(t) | \times | R'(t) | }
$$

Once $\cos \theta$ is computed, the angle θ can be found using the inverse cosine function:

$$
\theta = \arccos \left(\frac{ R(t) \cdot R'(t) }{ | R(t) | \times | R'(t) | } \right)
$$

Calculating the Dot Product $R(t) \cdot R'(t)$

The dot product between $R(t)$ and $R'(t)$ is:

$$
R(t) \cdot R'(t) = x(t) x'(t) + y(t) y'(t) + z(t) z'(t)
$$

Substituting the functions:

$$
\begin{aligned}
x(t) &= \ln(t^2 + 1) \\
x'(t) &= \frac{2t}{t^2 + 1} \\
y(t) &= \tan t \\
y'(t) &= \sec^2 t \\
z(t) &= t^2 + 1 \\
z'(t) &= 2t
\end{aligned}
$$

Calculating each term:

$$
x(t) x'(t) = \ln(t^2 + 1) \times \frac{2t}{t^2 + 1}
$$

$$
y(t) y'(t) = \tan t \times \sec^2 t
$$

$$
z(t) z'(t) = (t^2 + 1) \times 2t
$$

Therefore, the dot product:

$$
R(t) \cdot R'(t) = \left(\ln(t^2 + 1) \times \frac{2t}{t^2 + 1} \right) +
$$

$$
\left(\tan t \times \sec^2 t \right) + \left((t^2 + 1) \times 2t \right)
$$

Calculating the Magnitudes $| R(t) |$ and $| R'(t) |$

The magnitude of the position vector:

$$
| R(t) | = \sqrt{ x(t)^2 + y(t)^2 + z(t)^2 }
$$

which is:

$$
| R(t) | = \sqrt{ \left(\ln(t^2 + 1) \right)^2 + (\tan t)^2 + (t^2 + 1)^2 }
$$

Similarly, the magnitude of the velocity vector:

$$
| R'(t) | = \sqrt{ \left(\frac{2t}{t^2 + 1} \right)^2 + (\sec^2 t)^2 + (2t)^2 }
$$

Calculating these explicitly:

$$
| R'(t) | = \sqrt{ \frac{4t^2}{(t^2 + 1)^2} + \sec^4 t + 4t^2 }
$$

Calculating the Angle θ

Putting it all together, the angle θ at a given time t is:

$$
\boxed{
\theta(t) = \arccos \left(\frac{ R(t) \cdot R'(t) }{ | R(t) | \times | R'(t) | } \right)
}
$$

This expression can be evaluated numerically for specific values of t or analyzed qualitatively to understand the particle's behavior over time.

Special Cases and Interpretation

Understanding the behavior of the angle at specific points in time can reveal important insights:

- At \(t = 0 \):

- \(x(0) = \ln(0^2 + 1) = 0 \)
- \(y(0) = \tan 0 = 0 \)
- \(z(0) = 0^2 + 1 = 1 \)

- \(x'(0) = 0 \)
- \(y'(0) = \sec^2 0 = 1 \)
- \(z'(0) = 0 \)

- Dot product:

\[
R(0) \cdot R'(0) = 0 \times 0 + 0 \times 1 + 1 \times 0 = 0
\]

- Magnitudes:

\[
| R(0) | = \sqrt{ 0^2 + 0^2 + 1^2 } = 1
\]

\[
| R'(0) | = \sqrt{ 0^2 +

Frequently Asked Questions

What does the vector function R(t) = (ln(t^2 + 1))i + (tan t)j + (t^2 + 1)k represent in the context of particle motion?

It represents the position vector of a particle moving in space at time t, with each component indicating the particle's position along the x, y, and z axes respectively.

How do you interpret the components of R(t) in terms of particle movement?

The components show how the particle's x-position varies as ln(t^2 + 1), y-position as tan t, and z-position as t^2 + 1 over time.

How can we find the angle between the position vector R(t) and a given axis at time t?

The angle can be found using the dot product formula: cos θ = (R(t) · A) / (|R(t)| |A|), where A is the axis vector. For example, to find the angle with the x-axis, use A = i.

What is the formula to compute the angle between the position vector R(t) and the x-axis?

The angle θ between R(t) and the x-axis is given by $\cos \theta = (\ln(t^2 + 1)) / |R(t)|$, where $|R(t)|$ is the magnitude of the position vector.

How do you calculate the magnitude of R(t) for a given time t?

The magnitude $|R(t)|$ is calculated as $\sqrt{(\ln(t^2 + 1))^2 + (\tan t)^2 + (t^2 + 1)^2}$.

What steps are involved in finding the angle between R(t) and the z-axis at a specific time t?

First, compute the dot product $R(t) \cdot k = t^2 + 1$. Then, find $|R(t)|$ as described, and use $\cos \theta = (t^2 + 1) / |R(t)|$ to find the angle θ with the z-axis.

Additional Resources

$R(t) = (\ln(t^2 + 1))i + (\tan t)j + (t^2 + 1)k$ is the position vector of a particle in space at time t. Find the angle

Understanding the motion of particles through space involves analyzing their position vectors, which encode their location at any given moment. In this context, we are presented with a position vector:

$$R(t) = (\ln(t^2 + 1)) \mathbf{i} + (\tan t) \mathbf{j} + (t^2 + 1) \mathbf{k}$$

and tasked with finding the angle between the position vector $R(t)$ and another vector, most likely its velocity vector $R'(t)$. This problem not only involves calculus but also a deep understanding of vector operations, derivatives, and trigonometric functions. Let's explore this problem step-by-step, starting from fundamentals, moving to derivatives, and ultimately calculating the angle.

Understanding the Position Vector

The position vector $R(t)$ describes the location of a particle in three-dimensional space at an arbitrary time t. Its components are:

- x-component: $x(t) = \ln(t^2 + 1)$
- y-component: $y(t) = \tan t$
- z-component: $z(t) = t^2 + 1$

This vector encapsulates the particle's position in space at any time t. The components involve a mix of elementary functions such as logarithms, tangent, and quadratic functions, indicating complex particle motion.

Calculating the Derivative $R'(t)$

To analyze the motion further and eventually find the angle between $R(t)$ and another vector, we need the derivative $R'(t)$, which represents the velocity vector of the particle.

Step 1: Derive each component

- $x(t) = \ln(t^2 + 1)$

$$
x'(t) = \frac{1}{t^2 + 1} \times 2t = \frac{2t}{t^2 + 1}
$$

- $y(t) = \tan t$

$$
y'(t) = \sec^2 t
$$

- $z(t) = t^2 + 1$

$$
z'(t) = 2t
$$

Step 2: Write the velocity vector

$$
R'(t) = \left(\frac{2t}{t^2 + 1} \right) \mathbf{i} + \sec^2 t \ \mathbf{j} + 2t \ \mathbf{k}
$$

This vector indicates how the position of the particle changes with respect to time.

The Angle Between Vectors: Concept and Formula

The main goal is to determine the angle between the position vector $R(t)$ and its derivative $R'(t)$. The angle θ between two vectors $\mathbf{A}$ and $\mathbf{B}$ is given by the dot product formula:

$$
\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}
$$

where

- $\mathbf{A} \cdot \mathbf{B}$ is the dot product,
- $\|\mathbf{A}\|$ and $\|\mathbf{B}\|$ are the magnitudes (norms) of the vectors.

In our case:

- $\mathbf{A} = R(t)$
- $\mathbf{B} = R'(t)$

The angle $\theta(t)$ can be expressed as:

$$
\theta(t) = \arccos \left(\frac{ R(t) \cdot R'(t) }{ \| R(t) \| \| R'(t) \| } \right)
$$

Calculating this requires explicit computation of the dot product and the norms.

Step-by-Step Calculation of the Dot Product

1. Dot product $R(t) \cdot R'(t)$:

$$
R(t) \cdot R'(t) = x(t) \times x'(t) + y(t) \times y'(t) + z(t) \times z'(t)
$$

Substituting the known functions:

$$
\begin{aligned}
& \ln(t^2 + 1) \times \frac{2t}{t^2 + 1} + \tan t \times \sec^2 t + (t^2 + 1) \times 2t \\
=\, & \frac{2t \ln(t^2 + 1)}{t^2 + 1} + \tan t \sec^2 t + 2t (t^2 + 1)
\end{aligned}
$$

This expression combines algebraic, logarithmic, and trigonometric components.

Step-by-Step Calculation of the Magnitudes

2. Magnitude of $R(t)$:

$$
\| R(t) \| = \sqrt{ x(t)^2 + y(t)^2 + z(t)^2 } = \sqrt{ (\ln(t^2 + 1))^2 + \tan^2 t + (t^2 + 1)^2 }
$$

3. Magnitude of $R'(t)$:

$$
\| R'(t) \| = \sqrt{ \left(\frac{2t}{t^2 + 1} \right)^2 + (\sec^2 t)^2 + (2t)^2 }
$$

$$
= \sqrt{ \frac{4t^2}{(t^2 + 1)^2} + \sec^4 t + 4t^2 }
$$

Practical Approach to Computing the Angle

In most cases, computing the exact value of $\theta(t)$ analytically is complex due to the involved expressions. Instead, the typical approach involves:

- Choosing a specific value of t,
- Computing the components numerically,
- Plugging into the dot product and magnitude formulas,
- Calculating $\cos \theta$, and then,
- Using the arccos function to find $\theta(t)$.

This process provides an approximate value of the angle at specific moments in time, offering insights into the particle's motion direction relative to its position.

Significance and Applications of the Calculation

Understanding the angle between position and velocity vectors is crucial in physics and engineering, especially in fields involving motion analysis, such as:

- Kinematics: Determining whether a particle moves directly away or towards its initial position.
- Navigation and Robotics: Ensuring paths are optimized and understanding directional changes.
- Astrodynamics: Calculating angles between position vectors and velocity vectors to analyze orbital characteristics.

In our context, knowing how the position vector relates to the velocity at different times can reveal whether the particle is moving radially outward, inward, or tangentially, which is vital for understanding its trajectory.

Final Remarks

The problem of finding the angle between the position vector $R(t)$ and its derivative involves a blend of calculus, algebra, and trigonometry. While an exact symbolic expression might be cumbersome, numerical methods and computational tools can facilitate accurate evaluations at specific time points.

This exploration underscores the importance of mathematical tools for analyzing particle motion in space, with applications spanning physics, engineering, and even computer graphics. Recognizing the interplay between position, velocity, and the angles between them enhances our understanding of dynamic systems and their behaviors.

In conclusion, by differentiating the given position vector, calculating the dot product, and applying the formula for the angle between vectors, we gain comprehensive insights into the particle's motion. Whether for academic purposes or practical applications, mastering these techniques is fundamental in the field of vector calculus and motion analysis.

R T Ln T2 1 I Tan1t J T2 1 K Is The Position Vector Of A Particle In Space At Time T Find The Angle

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