Rational Function H Is Continuous, With A Horizontal Asymptote At Y = 1. Which Function Could Be Function

Rational Function H Is Continuous, With A Horizontal Asymptote At Y = 1. Which Function Could Be Function?

Understanding the nature of rational functions and their asymptotic behavior is fundamental in algebra and calculus. When given a rational function H that is continuous everywhere within its domain and features a horizontal asymptote at y = 1, it prompts us to explore what form such a function might take. In this article, we will delve into the characteristics that define such functions, analyze possible forms, and provide detailed examples demonstrating how to construct rational functions with the specified properties.

Fundamentals of Rational Functions and Asymptotes

What Is a Rational Function?

A rational function is any function that can be expressed as the quotient of two polynomials:

$$ H(x) = \frac{P(x)}{Q(x)} $$

where $P(x)$ and $Q(x)$ are polynomials, and $Q(x) \neq 0$.

Key features of rational functions include:

- Domain restrictions: Points where $Q(x) = 0$ are excluded from the domain, often leading to vertical asymptotes.
- Behavior at infinity: As $x \to \pm \infty$, the function may approach a finite limit (horizontal asymptote), or diverge.

Horizontal Asymptotes and Their Significance

A horizontal asymptote is a horizontal line $y = L$ that the graph of a function approaches as $x \to \pm \infty$.

For rational functions:

- If the degree of numerator $P(x)$ is less than the degree of denominator $Q(x)$, then the horizontal asymptote is at $y = 0$.
- If the degrees are equal, the asymptote is at $y =$ ratio of leading coefficients.
- If the degree of numerator exceeds that of denominator, no horizontal asymptote exists (but there may be an oblique/slant asymptote).

Given that the asymptote is at $y = 1$, we are most interested in functions where the degrees of numerator and denominator are equal, or functions that can be adjusted to satisfy this property.

Constructing Rational Functions with a Horizontal Asymptote at Y = 1

Conditions for Horizontal Asymptote at Y = 1

To have a horizontal asymptote at $y = 1$:

- The degrees of numerator $P(x)$ and denominator $Q(x)$ should be equal.
- The ratio of the leading coefficients should be 1, i.e.,

$$\lim_{x \to \pm \infty} H(x) = \frac{\text{Leading coefficient of } P}{\text{Leading coefficient of } Q} = 1$$

- The function should be continuous over its domain, which implies that any discontinuities (like vertical asymptotes) are properly accounted for, and the function is defined everywhere except at points where the denominator is zero.

General Form of Such Rational Functions

Based on the above, a general form is:

$$H(x) = \frac{a_n x^n + \dots + a_0}{a_n x^n + \dots + d_0}$$

where:

- $a_n \neq 0$, $d_0 \neq 0$,
- The leading coefficients a_n and d_n satisfy $\frac{a_n}{d_n} = 1$.

Examples of Rational Functions with Horizontal Asymptote $y = 1$

Simple Examples with Equal Degree Polynomials

1. Function with numerator and denominator of degree 1:

$$H(x) = \frac{x + c}{x + d}$$

The horizontal asymptote as $x \to \pm \infty$ is:

$$\lim_{x \to \pm \infty} \frac{x + c}{x + d} = 1$$

since the leading coefficients are both 1.

- For example: $H(x) = \frac{x + 2}{x + 3}$

This function is continuous for all $x \neq -3$, where it has a vertical asymptote, but is otherwise continuous.

2. Function with quadratic numerator and denominator:

$$H(x) = \frac{x^2 + a x + b}{x^2 + c x + d}$$

If the leading coefficients are both 1, the asymptote at $y = 1$ is achieved.

- For example: $H(x) = \frac{x^2 + 2x + 1}{x^2 + x + 5}$

As $x \to \pm \infty$, $H(x) \to 1$.

Key Point:

Such functions will tend toward the asymptote at $y=1$, except possibly at points where the denominator is zero.

Adjusting the Function to Achieve Continuity

While rational functions are often discontinuous at points where the denominator is zero, certain modifications or domain restrictions can ensure the function is continuous over its domain.

- Removing discontinuities: If a factor cancels in numerator and denominator, the resulting function can be continuous at the canceled point.

- Domain considerations: For the function to be continuous over its entire domain (excluding points where the denominator is zero), the domain must exclude the poles.

- Ensuring continuity: If the problem considers the function over its domain, then the function is continuous everywhere except at the vertical asymptotes.

Constructing Specific Rational Functions with the Given Properties

Method 1: Direct Construction Based on Leading Coefficients

To explicitly construct such a function:

1. Choose numerator $P(x)$ and denominator $Q(x)$ polynomials with same degree.
2. Set their leading coefficients to be equal; for simplicity, both 1.
3. Include lower-degree terms as desired to shape the function.

Example:

$$ H(x) = \frac{x^2 + 3x + 2}{x^2 + x + 1} $$

- As $x \to \pm \infty$, $H(x) \to 1$.

- The function is continuous everywhere except at points where the denominator is zero.

Method 2: Incorporating a Constant Term to Shift the Asymptote

Alternatively, functions can be designed to exactly approach $y=1$, even if the degrees differ:

$$ H(x) = 1 + \frac{k}{Q(x)} $$

where $Q(x)$ is a polynomial that grows large as $x \to \pm \infty$, ensuring $H(x) \to 1$.

- For example:

$$ H(x) = 1 + \frac{2}{x^2 + 1} $$

- As $x \to \pm \infty$, $H(x) \to 1$.

- The function is continuous everywhere, with no vertical asymptotes (since denominator is never zero).

- If vertical asymptotes are desired, choose $Q(x)$ such that it equals zero at some point.

Summary of Key Points

- To have a horizontal asymptote at $y=1$, the degrees of numerator and denominator polynomials should be equal, with matching leading coefficients.

- The general form of such functions is:

$$H(x) = \frac{a_n x^n + \dots + a_0}{a_n x^n + \dots + d_0}$$

with $a_n/d_n = 1$.

- The function's continuity over its domain depends on the domain excluding points where the denominator is zero.

- Adjusting lower-degree terms can shape the function's behavior elsewhere, but the asymptote at $y=1$ remains due to the dominant degree and leading coefficients.

- Alternatively, functions of the form $H(x) = 1 + \frac{\text{bounded term}}{\text{polynomial}}$ approach the asymptote at infinity and can be continuous everywhere if the denominator never zeroes.

Conclusion

In conclusion, the class of rational functions that are continuous (except at vertical asymptotes) and have a horizontal asymptote at $y = 1$ can be characterized by their degrees and leading coefficients. The simplest and most representative functions are ratios of polynomials with equal degrees and matching leading coefficients, such as $\frac{x + c}{x + d}$ or quadratic forms like $\frac{x^2 + a x + b}{x^2 + c x + d}$. By adjusting coefficients and carefully selecting domain restrictions, one can construct an array of functions fitting these criteria. The key is ensuring the asymptotic behavior aligns with the horizontal asymptote at $y=1$, which hinges on the polynomial degrees and ratios of their leading coefficients.

Frequently Asked Questions

What characteristics does a rational function need to have to be continuous with a horizontal asymptote at y

= 1?

The function must be continuous for all values in its domain and approach y = 1 as x approaches infinity or negative infinity, typically meaning the degrees of numerator and denominator are equal with their leading coefficients ratio being 1.

How can I determine if a given rational function has a horizontal asymptote at y = 1?

Compare the degrees of numerator and denominator; if they are equal, the horizontal asymptote is at y = (leading coefficient of numerator) / (leading coefficient of denominator). For y = 1, the leading coefficients must be equal.

Can you give an example of a rational function that is continuous and has a horizontal asymptote at y = 1?

Yes, for example, f(x) = (2x + 3) / (2x + 5). As x approaches infinity, f(x) approaches 1, with the degrees equal and leading coefficients both 2, ensuring the horizontal asymptote at y = 1.

Is it possible for a rational function to be continuous everywhere and have a horizontal asymptote at y = 1?

Yes, if the rational function is defined for all real x (no vertical asymptotes) and its end behavior approaches y = 1, then it can be continuous everywhere with that horizontal asymptote.

What restrictions are there on the numerator and denominator for the function to have a horizontal asymptote at y = 1?

The degrees of numerator and denominator must be equal, and the ratio of their leading coefficients must be 1. Additionally, the function must be defined for all x to ensure continuity.

How do vertical asymptotes affect the continuity of the rational function with a horizontal asymptote at y = 1?

Vertical asymptotes occur where the denominator is zero and the numerator is non-zero, causing discontinuities. To be continuous everywhere, the function must have no vertical asymptotes, meaning its denominator cannot be zero anywhere.

What is the general form of a rational function that has a horizontal asymptote at y = 1 and is continuous?

A general form can be f(x) = (ax + b) / (ax + c), where the degrees are equal, leading coefficients are equal (a), and the function is defined for all x (no zeros in the denominator).

Additional Resources

Rational Function H Is Continuous, With A Horizontal Asymptote At Y = 1. Which Function Could Be?

In the realm of mathematical functions, rational functions serve as fundamental building blocks with a broad spectrum of applications spanning engineering, physics, economics, and beyond. They exhibit intriguing properties, especially concerning their asymptotic behavior and continuity. Among these properties, the presence of a horizontal asymptote at $y=1$ and the continuity of the function are particularly noteworthy, prompting an in-depth investigation into the possible forms such a function might take.

This article aims to dissect the characteristics of rational functions with a horizontal asymptote at $y=1$, analyze the conditions under which the function remains continuous across its domain, and ultimately identify the potential candidates that fit these criteria. Through rigorous exploration, we aim to answer the question: Which rational functions could be such that H is continuous, with a horizontal asymptote at $y=1$?

Understanding Rational Functions and Their Behavior

Before delving into specific functions, it's essential to establish what constitutes a rational function and how their properties relate to asymptotes and continuity.

Definition of Rational Functions

A rational function $R(x)$ is any function expressible as the ratio of two polynomials:

$$
R(x) = \frac{P(x)}{Q(x)}
$$

where:

- $P(x)$ and $Q(x)$ are polynomials.
- $Q(x) \neq 0$ for all x in the domain.

The domain of $R(x)$ is thus all real numbers except where $Q(x) = 0$.

Asymptotic Behavior of Rational Functions

Rational functions often exhibit asymptotic behavior, which is characterized by the behavior of the function as $x \to \pm \infty$ or near points where the function is undefined.

- Horizontal asymptotes describe the behavior of the graph as $x \to \pm \infty$.
- Oblique or slant asymptotes occur when degrees of numerator and denominator differ appropriately.

Understanding the asymptote at $y=1$ requires examining the end behavior of the rational function as $x \to \pm \infty$.

Conditions for a Horizontal Asymptote at $y=1$

A rational function $R(x) = \frac{P(x)}{Q(x)}$ has a horizontal asymptote at $y = L$ if:

$$
\lim_{x \to \pm \infty} R(x) = L
$$

For $L=1$, this becomes:

$$
\lim_{x \to \pm \infty} \frac{P(x)}{Q(x)} = 1
$$

This limit exists and equals 1 if and only if the degrees of $P(x)$ and $Q(x)$ are equal, and the leading coefficients satisfy:

$$
\frac{\text{leading coefficient of } P(x)}{\text{leading coefficient of } Q(x)} = 1
$$

Key implications:

- The degrees of numerator and denominator polynomials must be equal.
- The ratio of their leading coefficients must be 1.

Ensuring Continuity of the Function

Continuity of the rational function across its domain hinges on the absence of discontinuities. Typically, rational functions are discontinuous at points where $Q(x) = 0$, unless the numerator also zeroes out at the same points, allowing for simplification.

Removable Discontinuities (Holes)

Suppose $R(x) = \frac{P(x)}{Q(x)}$ has a common factor $(x - a)$ in numerator and denominator:

$$
R(x) = \frac{(x - a) \cdot P_1(x)}{(x - a) \cdot Q_1(x)}
$$

which simplifies to:

$$
R(x) = \frac{P_1(x)}{Q_1(x)}
$$

The discontinuity at $x = a$ is removable (a "hole") if the limit exists there, which it does if $P_1(a)$ and $Q_1(a)$ are finite and $Q_1(a) \neq 0$.

To have $R(x)$ be continuous everywhere in its domain (excluding points where $Q(x)=0$), the function must be defined at those points (by defining the value explicitly or removing the discontinuity).

Implication for the Function with a Horizontal Asymptote at $y=1$

Since the horizontal asymptote is at $y=1$, the function’s end behavior approaches 1, and the function can be made continuous everywhere except possibly at points where $Q(x)=0$. To ensure overall continuity, the function should either:

- Not have discontinuities at these points (i.e., the zeros of $Q(x)$ are removable by factoring), or
- Exclude such points from the domain (i.e., discontinuities are vertical asymptotes).

Constructing Candidate Functions

Based on the above conditions, the general form of rational functions with a horizontal asymptote at $y=1$, which can be continuous throughout their domain (excluding points where $Q(x)=0$), can be characterized as follows:

$$
H(x) = \frac{P(x)}{Q(x)} \quad \text{where} \quad \deg P = \deg Q, \quad \text{and}
$$

$$
\quad \lim_{x \to \pm \infty} H(x) = 1
$$

Specifically:

$$
H(x) = \frac{a_n x^n + \dots + a_0}{a_n x^n + \dots + a_0}
$$

with leading coefficients satisfying:

$$
\frac{a_n}{a_n} = 1
$$

and the numerator and denominator polynomials can be constructed to ensure the function is continuous everywhere except at points where $Q(x) = 0$, which can be handled as removable discontinuities or ignored depending on the context.

Examples of Such Functions

1. Simple Polynomial Ratio:

$$
H(x) = \frac{x + 2}{x + 3}
$$

- Degree numerator = degree denominator = 1
- Leading coefficients = 1 in numerator and denominator
- $\lim_{x \to \pm \infty} H(x) = 1$
- Discontinuity at $x = -3$, which is removable if defined appropriately.

2. Quadratic Example:

$$
H(x) = \frac{2x^2 + 3x + 1}{2x^2 + x + 4}
$$

- Leading coefficients both 2
- Degree = 2
- Limit as $x \to \pm \infty$:

$$
\lim_{x \to \pm \infty} H(x) = \frac{2x^2}{2x^2} = 1
$$

- Discontinuities at zeros of $2x^2 + x + 4$, which do not occur for real x, so the function is continuous everywhere in $\mathbb{R}$.

Addressing Special Cases and Constraints

While the above examples suggest a broad class of functions fitting the criteria, several nuances and constraints need to be addressed:

Polynomials Dividing Each Other

- If numerator and denominator are proportional polynomials, the function simplifies to a constant c, where $c=1$ for the asymptote condition.
- For instance:

$$
H(x) = \frac{a \cdot P(x)}{a \cdot Q(x)} = \frac{P(x)}{Q(x)}
$$

where the ratio of leading coefficients is 1, as before.

Handling Discontinuities and Domain Considerations

- To ensure overall continuity, rational functions should either:

- Have zeros in the denominator canceled out by numerator factors, creating holes that can be filled with explicit definitions, or
- Be defined only on open subsets excluding points where $Q(x)=0$, with vertical asymptotes.

- The choice depends on the context—whether discontinuities are permissible or if the function must be globally continuous.

Conclusion: Which Function Could Be?

Based on the rigorous analysis, the class of rational functions that are continuous (except possibly at removable discontinuities) and have a horizontal asymptote at $y=1$ are characterized by:

- Having numerator and denominator polynomials of equal degree.
- The leading coefficients of numerator and denominator polynomials are equal, ensuring

the limit at infinity is 1.
- The function can be modified to be continuous across its domain by canceling common factors, thereby removing potential discontinuities at zeros of the denominator.

In essence, a suitable example of such a function is:

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