Represents A Line That Passes Through (3, 2) With A Slope Of Negative StartFraction 4 Over 5 EndFraction?

Represents A Line That Passes Through (3, 2) With A Slope Of Negative $\frac{4}{5}$?

Understanding how to represent a line that passes through a specific point with a given slope is fundamental in algebra and coordinate geometry. When asked to find the equation of such a line, especially one passing through the point (3, 2) with a slope of $-\frac{4}{5}$, it is essential to grasp the underlying concepts, formulas, and methods. This article will explore in depth how to represent this line, the different forms of equations used, and practical applications, all organized to provide clarity and comprehensive understanding.

Understanding the Basics: Point-Slope and Slope-Intercept Forms

Before diving into the specific problem, it is crucial to understand the fundamental forms of linear equations and how they relate to the point and slope.

Point-Slope Form of a Line

The point-slope form is particularly useful when you know a point on the line and its slope. It is expressed as:

$$
y - y_1 = m(x - x_1)
$$

Where:
- (x_1, y_1) is a point on the line.
- m is the slope of the line.

In our problem:
- $(x_1, y_1) = (3, 2)$
- $m = -\frac{4}{5}$

Plugging in the values:

$$
y - 2 = -\frac{4}{5}(x - 3)
$$

This is the point-slope form representation of the line passing through (3, 2) with the specified slope.

Converting to Slope-Intercept Form

The slope-intercept form is often more straightforward for graphing and quick analysis:

$$
y = mx + b
$$

To convert the point-slope form to the slope-intercept form:

$$
y - 2 = -\frac{4}{5}(x - 3)
$$

Distribute the slope:

$$
y - 2 = -\frac{4}{5}x + \frac{12}{5}
$$

Add 2 to both sides:

$$
y = -\frac{4}{5}x + \frac{12}{5} + 2
$$

Express 2 as a fraction with denominator 5:

$$
2 = \frac{10}{5}
$$

So:

$$
y = -\frac{4}{5}x + \frac{12}{5} + \frac{10}{5} = -\frac{4}{5}x + \frac{22}{5}
$$

\]

Thus, the slope-intercept form:

$$
\boxed{y = -\frac{4}{5}x + \frac{22}{5}}
$$

This equation represents the line passing through (3, 2) with a slope of $-\frac{4}{5}$.

Graphing the Line: Visual Representation

Graphing provides a visual understanding of the line's behavior and confirms the accuracy of the equation.

Steps to Graph the Line

1. Identify the point: (3, 2). Plot this point on the coordinate plane.
2. Use the slope: $-\frac{4}{5}$. This indicates that for every 5 units moved horizontally to the right, the line moves 4 units down vertically.
3. Plot additional points: Starting from (3, 2), move 5 units right to (8, 2), then move down 4 units to (8, -2). Plot this point.
4. Draw a straight line through the points. Extend the line in both directions, and label it accordingly.

Visualizing the line helps solidify understanding and confirms the correctness of the algebraic representation.

Understanding the Slope and Its Significance

The slope $-\frac{4}{5}$ indicates the rate of change of the line.

Meaning of a Negative Slope

A negative slope means the line is decreasing; as x increases, y decreases. Specifically, in this case:

- For every increase of 5 units in x, y decreases by 4 units.
- The line slants downward from left to right.

Implications for the Line's Behavior

- The line crosses the y-axis at $b = \frac{22}{5} = 4.4$ (from the slope-intercept form).
- The line passes through the point (3, 2), as specified.
- The line's steepness is moderate due to the slope magnitude $\frac{4}{5}$.

Applications of Representing Lines with Given Points and Slopes

Understanding how to represent lines with specific points and slopes has numerous applications in various fields.

Real-World Applications

- **Physics:** Describing the rate of change of velocity or other physical quantities.
- **Economics:** Modeling the relationship between supply and demand or cost and revenue.
- **Engineering:** Designing structures and understanding load distributions.
- **Navigation and Mapping:** Calculating optimal routes and directions based on slopes and coordinates.

Academic and Educational Uses

- Teaching linear equations and their graphical representations.
- Solving real-world problems involving linear relationships.
- Developing skills in algebraic manipulation and equation transformation.

Additional Methods to Represent the Line

Besides the point-slope and slope-intercept forms, there are other ways to express the line.

Standard Form

The standard form of a line is:

$$
Ax + By = C
$$

To convert our equation:

$$
y = -\frac{4}{5}x + \frac{22}{5}
$$

Multiply through by 5 to clear denominators:

$$
5y = -4x + 22
$$

Rearranged:

$$
4x + 5y = 22
$$

This is the standard form of the line passing through (3, 2) with the specified slope.

Point-Intercept Form

While similar to slope-intercept form, the point-intercept form uses the point and the intercepts directly, but in this case, the slope form is often more straightforward.

Summary and Key Takeaways

- To represent a line passing through a specific point with a given slope, the point-slope form is most convenient: $y - y_1 = m(x - x_1)$.
- Converting to slope-intercept form provides an easily interpretable equation: $y = mx + b$.
- The specific line passing through (3, 2) with a slope of $-\frac{4}{5}$ is:

$$
\boxed{y = -\frac{4}{5}x + \frac{22}{5}}
$$

- Graphing the line involves plotting the known point and using the slope to find additional points.
- Understanding the slope's sign and magnitude helps interpret the line's behavior.
- Various forms of the line equation serve different purposes in analysis and applications.

In conclusion, representing a line passing through a point with a specific slope involves understanding and applying algebraic formulas, converting between different forms, and visualizing the line. Mastery of these concepts enables effective problem-solving in mathematics, science, engineering, and many other disciplines.

Frequently Asked Questions

What is the equation of the line passing through (3, 2) with a slope of -4/5?

Using point-slope form: $y - 2 = (-4/5)(x - 3)$. Simplified, the equation is $y = (-4/5)x + (12/5) + 2$, which is $y = (-4/5)x + (22/5)$.

How do you find the y-intercept of the line passing through (3, 2) with slope -4/5?

Plugging $x=0$ into the line equation $y = (-4/5)x + (22/5)$, the y-intercept is $y = (22/5)$.

What is the significance of the slope -4/5 in the line's behavior?

A slope of -4/5 indicates the line is decreasing, meaning as x increases, y decreases at a rate of 4 units down for every 5 units to the right.

How can I graph the line passing through (3, 2) with a slope of -4/5?

Start at the point (3, 2), then move down 4 units and right 5 units to plot another point, and draw the line through these points.

Is the line passing through (3, 2) with a slope of -4/5 perpendicular to a line with slope 5/4?

Yes, because the product of their slopes (-4/5 and 5/4) is -1, indicating they are perpendicular.

How do I find the equation of the line if I know it passes through (3, 2) and has a slope of -4/5?

Use the point-slope form: y - 2 = (-4/5)(x - 3), then simplify to slope-intercept form to find the equation.

Can this line intersect the x-axis, and if so, where?

Yes, set y=0 in the line equation y = (-4/5)x + (22/5). Solving for x: 0 = (-4/5)x + (22/5) => x = (22/5) / (4/5) = (22/5) (5/4) = 22/4 = 11/2. So, the line intersects the x-axis at (11/2, 0).

Additional Resources

Understanding the Equation of a Line Passing Through (3, 2) with a Slope of -4/5

When exploring the fundamentals of coordinate geometry, one of the most essential concepts is understanding how to represent and interpret lines on the Cartesian plane. A common and practical problem involves determining the algebraic equation of a line given specific conditions—such as a point through which it passes and its slope. In this article, we delve deeply into the process of representing a line that passes through the point (3, 2) with a slope of -4/5, exploring various forms of the line's equation, their derivations, and practical applications.

The Significance of Line Representation in Coordinate Geometry

Before diving into the specifics of the problem, it's important to understand why representing lines accurately and efficiently matters. In mathematics, physics, engineering, and many applied sciences, lines serve as foundational elements for modeling relationships, analyzing trends, and solving real-world problems.

- Precision: An exact algebraic representation allows precise calculations, intersection analysis, and transformations.
- Flexibility: Different forms of line equations (slope-intercept, point-slope, standard form) are suited to various types of analysis.
- Visualization: Clear equations facilitate plotting and visualization, crucial for both fundamental understanding and advanced applications like computer graphics.

Core Concepts: Slope, Point, and Line Equation

To accurately describe a line, two primary pieces of information are needed:

1. A Point on the Line: In our case, (3, 2).
2. The Slope of the Line: Here, it is -4/5.

Slope (m): The measure of the steepness or inclination of the line, calculated as the ratio of the change in y to the change in x between two points. A negative slope indicates the line descends from left to right.

Point: A specific coordinate through which the line passes; it anchors the line in the coordinate plane.

Deriving the Equation of the Line

Given the point and slope, the most straightforward method to derive the line's equation is using the point-slope form.

Point-Slope Form

The point-slope form of a line equation is:

$$
y - y_1 = m (x - x_1)
$$

where (x_1, y_1) is a point on the line, and m is the slope.

Applying our given data:

$$
x_1 = 3, \quad y_1 = 2, \quad m = -\frac{4}{5}
$$

$$
\boxed{
y - 2 = -\frac{4}{5} (x - 3)
}
$$

This form is particularly useful because it directly incorporates the known point and slope, making it an ideal starting point for analysis.

Transforming to Slope-Intercept Form

The slope-intercept form is widely used for its simplicity and direct interpretability:

$$
y = mx + b
$$

where:

- m is the slope,
- b is the y-intercept (the point where the line crosses the y-axis).

To convert from point-slope to slope-intercept form:

1. Expand the point-slope equation:

$$
y - 2 = -\frac{4}{5}x + \frac{4}{5} \times 3
$$

2. Simplify:

$$
y - 2 = -\frac{4}{5}x + \frac{12}{5}
$$

3. Add 2 to both sides (note $2 = \frac{10}{5}$):

$$
y = -\frac{4}{5}x + \frac{12}{5} + \frac{10}{5}
$$

4. Combine like terms:

$$
y = -\frac{4}{5}x + \frac{22}{5}
$$

Final slope-intercept form:

$$
\boxed{
y = -\frac{4}{5}x + \frac{22}{5}
}
$$

This form clearly reveals the slope and y-intercept, facilitating quick graphing and understanding of the line's behavior.

Standard Form and Its Utility

Another common representation is the standard form:

$$
Ax + By = C
$$

This form is especially useful in systems of equations, optimization problems, and for certain geometric analyses.

Converting to standard form:

Starting from the slope-intercept form:

$$

$$
y = -\frac{4}{5}x + \frac{22}{5}
$$

Multiply through by 5 to clear denominators:

$$
5y = -4x + 22
$$

Rearranged:

$$
4x + 5y = 22
$$

Standard form of the line:

$$
\boxed{
4x + 5y = 22
}
$$

In this form, coefficients $A=4$, $B=5$, and $C=22$ are integers, which is often preferred in algebraic contexts.

Graphical Perspective and Practical Applications

Understanding the line's algebraic representations allows for accurate graphing, which is crucial in various disciplines.

Plotting the Line

- Using the slope-intercept form: Start at the y-intercept $(0, \frac{22}{5}) \approx (0, 4.4)$, then use the slope $-4/5$ to find additional points.
- Using the point (3, 2): Plot this point directly, then move according to the slope: 4 units down (because numerator is negative) and 5 units right.

Real-World Applications

- Physics: Modeling objects moving at constant velocities with given starting points.
- Economics: Analyzing cost functions where fixed costs and variable costs are involved.
- Engineering: Designing components where certain constraints are modeled as linear equations.
- Data Analysis: Fitting linear models to data points.

Additional Considerations and Variations

While the core derivation provides a clear picture, several variations and considerations enhance understanding:

Parallel and Perpendicular Lines

- Parallel lines share the same slope: $m = -4/5$.
- Perpendicular lines have slopes that are negative reciprocals: $m' = 5/4$.

Distance from a Point to a Line

Knowing the line's equation allows calculation of the shortest distance from any point to the line, useful in optimization problems.

Line Segments and Rays

The equations describe the infinite line, but in practical applications, finite segments or rays are often considered, requiring additional constraints.

Summary and Final Remarks

Representing a line passing through a specific point with a given slope is fundamental in coordinate geometry, with widespread applications across science and engineering. By starting with the point-slope form, transforming into slope-intercept and standard forms, and understanding the graphical implications, one gains a comprehensive grasp of the line's behavior and utility.

Key takeaways:

- The point-slope form provides a direct method to incorporate known points and slopes.

- Conversion to slope-intercept form offers a quick view of the line's slope and y-intercept.
- Standard form facilitates algebraic manipulations and geometric interpretations.
- Practical applications extend from simple graphing to complex modeling in various disciplines.

Mastering these representations equips students and professionals with versatile tools to analyze and interpret linear relationships effectively.

In conclusion, the line passing through (3, 2) with a slope of $-\frac{4}{5}$ can be elegantly described by the equation:

$$
\boxed{
y = -\frac{4}{5}x + \frac{22}{5}
}
$$

This formula not only embodies the fundamental principles of linear equations but also serves as a cornerstone for further geometric and analytical explorations.

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