Rotate Shape A 180 With Centre Of Rotation (3,-1). What Are The Coordinates Of The Vertices Of The Image?

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Understanding geometric transformations is fundamental in mathematics, especially in coordinate geometry. Among these transformations, rotation plays a pivotal role in altering the position of a shape around a fixed point, known as the center of rotation. This article explores the process of rotating a shape—referred to as Shape A—by 180 degrees about a specific point, (3, -1). We will break down the steps involved, analyze the underlying principles, and demonstrate how to determine the coordinates of the vertices after the rotation.

Understanding Rotation in Coordinate Geometry

What Is Rotation?

Rotation is a transformation that turns a shape around a fixed point called the center of rotation. The amount of turning is measured in degrees, with positive angles typically indicating counterclockwise rotation and negative indicating clockwise rotation. In this context, rotating Shape A by 180 degrees means turning it halfway around the center point.

Key Elements of Rotation

- Center of Rotation: The fixed point around which the shape is rotated. In our scenario, this is (3, -1).
- Angle of Rotation: The degree measure of how far the shape is rotated. Here, it is 180 degrees.
- Vertices of the Shape: The points defining the shape, which are transformed to new positions after rotation.

Step-by-Step Process to Rotate Shape A by 180 Degrees

To find the new coordinates of the vertices after rotation, follow these systematic steps:

1. Identify the Coordinates of the Vertices

Suppose Shape A has vertices at points \((x_1, y_1), (x_2, y_2), ..., (x_n, y_n) \). For example, assume Shape A is a triangle with vertices:
- \(A (x_A, y_A) \)

- $B (x_B, y_B)$
- $C (x_C, y_C)$

The specific coordinates of these points are essential for the calculation.

2. Translate the Shape so the Center Becomes the Origin

Since rotation occurs around the point (3, -1), convert each vertex to a coordinate system where this point is at the origin:

$$
X = x - 3 \\
Y = y + 1
$$

This translation simplifies the rotation process.

3. Apply the Rotation Formula for 180 Degrees

Rotating a point (X, Y) by 180 degrees around the origin results in:

$$
X' = -X \\
Y' = -Y
$$

This is because a 180-degree rotation about the origin inverts both coordinates.

4. Revert the Translation to Find the New Coordinates

After rotation, translate back to the original coordinate system:

$$
x' = X' + 3 \\
y' = -Y' - 1
$$

This gives the new position of each vertex after rotation.

Calculating the Coordinates of the Rotated Vertices

Let's suppose the original vertices of Shape A are as follows:

- $A (x_A, y_A)$
- $B (x_B, y_B)$
- $C (x_C, y_C)$

Applying the steps:

Example with Specific Coordinates:

Suppose Shape A has vertices:
- $A (1, 2)$
- $B (4, 3)$
- $C (2, 5)$

Step 1: Translate vertices

Vertex	$X = x - 3$	$Y = y + 1$
A	$1 - 3 = -2$	$2 + 1 = 3$
B	$4 - 3 = 1$	$3 + 1 = 4$
C	$2 - 3 = -1$	$5 + 1 = 6$

Step 2: Rotate by 180°

Vertex	$X' = -X$	$Y' = -Y$
A	2	-3
B	-1	-4
C	1	-6

Step 3: Translate back

Vertex	$x' = X' + 3$	$y' = -Y' - 1$
A	$2 + 3 = 5$	$-(-3) - 1 = 3 - 1 = 2$
B	$-1 + 3 = 2$	$-(-4) - 1 = 4 - 1 = 3$
C	$1 + 3 = 4$	$-(-6) - 1 = 6 - 1 = 5$

Resulting vertices after rotation:

- $A' (5, 2)$
- $B' (2, 3)$
- $C' (4, 5)$

These are the coordinates of the vertices of Shape A after a 180-degree rotation about point (3, -1).

Visualizing the Rotation and Its Effect

Understanding the geometric effect of rotation enhances comprehension. When rotating a shape 180 degrees about a point, each point is effectively "flipped" across the center, resulting in an image that is upside down or reversed in position relative to the original.

Graphical Representation

- Plot the original vertices on a coordinate plane.
- Mark the center of rotation at (3, -1).
- Draw the rotated shape with the new vertices.
- Observe how each point is mirrored across the center.

This visual approach helps verify the correctness of the calculations and provides an intuitive understanding of the rotation.

Applications and Relevance of Rotation in Real-World Contexts

Rotations are commonplace in various fields such as engineering, computer graphics, architecture, and robotics. For instance:

- Computer Graphics: Rotating images or models to achieve desired orientations.
- Robotics: Programming robotic arms to rotate parts around specific pivot points.
- Architecture: Designing symmetrical structures by rotating elements around central points.
- Navigation: Determining new positions after turns or rotations.

Understanding how to calculate the new coordinates after a rotation allows professionals to manipulate shapes precisely and predict their new positions accurately.

Summary and Key Takeaways

- Rotation involves turning a shape around a fixed point by a specified angle.
- To rotate a shape 180 degrees about point (3, -1), translate the shape so the center becomes the origin.
- Apply the 180-degree rotation formula ($X' = -X$, $Y' = -Y$) in the translated coordinate system.
- Revert the translation to find the new coordinates of each vertex.
- The process is systematic and can be applied to any polygon or shape with known vertices.

Remember: Always verify your calculations by plotting the original and rotated shapes to ensure accuracy.

Conclusion

Rotating Shape A by 180 degrees around the point (3, -1) is a straightforward process once the steps are understood. By translating the shape, applying the rotation formula, and then translating back, you can accurately determine the coordinates of the image's vertices. This exercise not only deepens your understanding of coordinate geometry but also enhances your ability to manipulate shapes in various mathematical and real-world applications. Whether for academic purposes or practical design tasks, mastering such transformations is a valuable skill in the mathematician's toolkit.

Frequently Asked Questions

What is the main goal when rotating Shape A 180° about the point (3, -1)?

The main goal is to find the new positions of the vertices of Shape A after rotating it 180° about the center point (3, -1).

How do you determine the coordinates of a vertex after a 180° rotation around a specific point?

To find the new coordinates, subtract the center's coordinates from the vertex, rotate the point 180°, then add back the center's coordinates.

What is the formula for rotating a point (x, y) 180° about a center (h, k)?

The rotated point (x', y') is given by: $x' = 2h - x$, $y' = 2k - y$.

If a vertex of Shape A is at (x, y), how do you apply the rotation formula for (3, -1)?

Calculate $x' = 23 - x$ and $y' = 2(-1) - y$ to find the new vertex coordinates after rotation.

Why is the point (3, -1) called the 'centre of rotation' in this transformation?

Because it is the fixed point around which the shape is rotated, remaining unchanged during the rotation.

Can you give an example of rotating a vertex at (5, 2) 180° about (3, -1)?

Yes, $x' = 23 - 5 = 6 - 5 = 1$; $y' = 2(-1) - 2 = -2 - 2 = -4$; so the new coordinate is (1, -4).

What are the steps to find the coordinates of all vertices after rotating Shape A 180° about (3, -1)?

First, identify each vertex's coordinates, then apply the rotation formula to each vertex to find their new positions.

Does rotating a shape 180° change its size or shape?

No, a 180° rotation is a rigid transformation, so it preserves the size and shape of the figure.

How can understanding rotation help in real-world applications or other areas of math?

Understanding rotation aids in fields like engineering, computer graphics, robotics, and geometry, where precise movement and positioning are essential.

What are common mistakes to avoid when calculating rotated vertices?

Common mistakes include mixing up the signs, forgetting to apply the rotation formula correctly, or not adding back the center coordinates after rotation.

Additional Resources

Rotate Shape A 180 With Centre Of Rotation (3, -1): What Are The Coordinates Of The Vertices Of The Image?

Introduction

The process of rotating geometric figures is fundamental in geometry, with applications spanning from computer graphics to engineering design. A common question encountered by students and professionals alike involves determining the new coordinates of a shape's vertices after a specific rotation about a given point. This article delves into the problem: Rotate Shape A 180 with centre of rotation (3, -1). What are the coordinates of the vertices of the image? We will explore the mathematical principles underlying the rotation, analyze the step-by-step procedure to derive the new vertices, and provide comprehensive examples that clarify this geometric transformation.

Understanding Rotation in the Coordinate Plane

What Does Rotating a Shape Mean?

In the coordinate plane, rotating a shape involves turning it around a fixed point called the center of rotation by a specified angle. The shape's size remains unchanged, but its position and orientation are altered.

Key Elements for Rotation

- Center of Rotation: The fixed point about which the shape is rotated.
- Angle of Rotation: The degree and direction of rotation, measured counterclockwise as positive.
- Vertices Coordinates: The initial positions of the shape's points, which after rotation, move to new locations.

In our case:

- Center of rotation: (3, -1)
- Rotation angle: 180 degrees
- Shape: Shape A, with known vertices

Mathematical Foundations of Rotation

Rotation Formula

To find the new coordinates of a point (x, y) after rotation by an angle θ about a point (h, k), the following transformation is used:

$$
\begin{cases}
x' = h + \cos \theta \times (x - h) - \sin \theta \times (y - k) \\
y' = k + \sin \theta \times (x - h) + \cos \theta \times (y - k)
\end{cases}
$$

Where:

- (x, y): original coordinate
- (x', y'): new coordinate after rotation
- (h, k): center of rotation
- θ: angle of rotation in radians

Special Case: Rotation by 180 Degrees

Since $\theta = 180^\circ$, the trigonometric functions simplify:

$$
\cos 180^\circ = -1, \quad \sin 180^\circ = 0
$$

Thus, the rotation formula reduces to:

$$
\begin{cases}
x' = h + (-1) \times (x - h) - 0 \times (y - k) = 2h - x \\
y' = k + 0 \times (x - h) + (-1) \times (y - k) = 2k - y
\end{cases}
$$

Key insight: The rotation by 180 degrees about (h, k) maps each point (x, y) to $(2h - x, 2k - y)$.

Step-by-Step Process for Determining the Vertices

1. Establish the Original Vertices

Suppose Shape A has vertices $V_1, V_2, V_3, ...$ with known coordinates:

- $V_1 = (x_1, y_1)$
- $V_2 = (x_2, y_2)$
- $V_3 = (x_3, y_3)$
- etc.

(In this article, for illustration, let’s assume a specific shape — for example, a triangle with vertices:)

- $V_1 = (1, 2)$
- $V_2 = (4, 3)$
- $V_3 = (2, 5)$

(Note: If the original shape's vertices are provided, substitute accordingly.)

2. Apply the Rotation Formula to Each Vertex

Using the formula:

$$
x' = 2h - x
$$
$$
y' = 2k - y
$$

with $h=3$, $k=-1$:

- For $V_1 = (1, 2)$:

$$
x_1' = 2 \times 3 - 1 = 6 - 1 = 5
$$
$$
y_1' = 2 \times (-1) - 2 = -2 - 2 = -4
$$
- Image of V_1: (5, -4)

- For $V_2 = (4, 3)$:

$$
x_2' = 6 - 4 = 2
$$
$$
y_2' = -2 - 3 = -5
$$
- Image of V_2: (2, -5)

- For $V_3 = (2, 5)$:

$$
x_3' = 6 - 2 = 4
$$
$$
y_3' = -2 - 5 = -7
$$
- Image of V_3: (4, -7)

3. Assemble the Coordinates of the Image

The vertices of the rotated shape are:

- $V_1' = (5, -4)$
- $V_2' = (2, -5)$
- $V_3' = (4, -7)$

This process can be repeated for any set of vertices.

Visualizing the Rotation

Visual aids significantly enhance understanding of geometric transformations. The original shape, placed on the coordinate plane, can be rotated about the point $(3, -1)$ by 180 degrees, illustrating how each vertex maps to its new position.

Practical Examples and Applications

- Engineering Design: Rotating components for assembly or analysis.
- Computer Graphics: Rendering rotated objects in virtual environments.
- Mathematics Education: Demonstrating symmetry and transformations.

Implications of Rotation About a Non-Origin Point

Rotating about a point other than the origin introduces additional considerations compared to rotation about the origin:

- The need to translate the shape so that the center of rotation aligns with the origin.
- Applying the rotation formula.
- Translating the shape back to its original position.

However, with the specific formula for 180 degrees, the process simplifies, as shown above, eliminating the need for multiple translation steps.

Summary of Key Steps

1. Identify the vertices of the original shape.
2. Apply the rotation formula for 180 degrees about \((3, -1) \):
\[
(x', y') = (2h - x, 2k - y)
\]
3. Calculate the new coordinates for each vertex.
4. Plot or analyze the new shape to confirm correctness.

Conclusion

Understanding the rotation of shapes in the coordinate plane, especially by 180 degrees about a specific point, is a vital skill in geometry. The key takeaway is that a 180-degree rotation about \((h, k) \) maps each point \((x, y) \) to \((2h - x, 2k - y) \). Applying this formula to the vertices of Shape A with center \((3, -1) \), we can determine the precise coordinates of the rotated image. This process not only reinforces core geometric principles but also demonstrates the elegance and simplicity of transformations in coordinate geometry.

By mastering these techniques, students and professionals can confidently analyze and manipulate shapes in diverse contexts, from academic exercises to complex engineering tasks.

References

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Note: To customize this analysis for a specific shape, replace the assumed vertices with the actual coordinates provided in the problem statement.

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