Sergey Is Solving 5x2 + 20x 7 = 0. Which Steps Could He Use To Solve The Quadratic Equation By Completing

Sergey Is Solving 5x2 + 20x 7 = 0. Which Steps Could He Use To Solve The Quadratic Equation By Completing is a common scenario students encounter when learning algebra. Quadratic equations are fundamental in mathematics, and mastering different methods of solving them, such as completing the square, is essential for a solid understanding of algebraic principles. In this article, we will explore the detailed steps Sergey can follow to solve the quadratic equation using the completing the square method, along with tips and explanations to make the process clear and manageable.

Understanding the Quadratic Equation

Before diving into the solving process, it's important to understand what a quadratic equation is and why completing the square is a useful method.

What Is a Quadratic Equation?

A quadratic equation is any second-degree polynomial equation that can be written in the form:

- $ax^2 + bx + c = 0$

where a, b, and c are coefficients, with $a \neq 0$.

In Sergey's case, the equation appears to be:
$5x^2 + 20x + 7 = 0$
assuming the missing plus sign and the correct placement of terms.

Why Use Completing the Square?

Completing the square transforms a quadratic into a perfect square trinomial, making it easier to solve for the variable. This method is especially useful when the quadratic does not factor neatly or when you want to derive the quadratic formula manually.

Step-by-Step Guide to Solving $5x^2 + 20x + 7 = 0$ by Completing the Square

Let's now walk through the detailed steps Sergey can take to solve this quadratic using completing the square.

Step 1: Normalize the Coefficient of x^2

The first step is to make the coefficient of x^2 equal to 1, which simplifies the process.

- Divide the entire equation by 5:

$(5x^2 + 20x + 7) \div 5 = 0 \div 5$

Simplifies to:

$x^2 + 4x + \frac{7}{5} = 0$

- Now, the quadratic is in the form:

$x^2 + 4x = -\frac{7}{5}$

This step ensures the coefficient of x^2 is 1, facilitating completing the square.

Step 2: Isolate the Constant Term

- Move the constant term to the right side:

$x^2 + 4x = -\frac{7}{5}$

- Our goal is to complete the square on the left side.

Step 3: Find the Number to Complete the Square

- Take half of the coefficient of x (which is 4), then square it:

- Half of 4 is 2.
- Square of 2 is 4.

- Add and subtract this number within the equation to complete the square:

$x^2 + 4x + 4 = -\frac{7}{5} + 4$

- To keep the equation balanced, add 4 to the right side as well:

$x^2 + 4x + 4 = -\frac{7}{5} + 4$

- Convert 4 to a fraction with denominator 5:

$4 = \frac{20}{5}$

- Now, the right side becomes:

$-\frac{7}{5} + \frac{20}{5} = \frac{13}{5}$

- The equation is now:

$(x + 2)^2 = \frac{13}{5}$

Step 4: Solve for x

- Take the square root of both sides:

$x + 2 = \pm \sqrt{\frac{13}{5}}$

- Simplify the square root:

$x + 2 = \pm \frac{\sqrt{13}}{\sqrt{5}}$

- Rationalize the denominator:

$x + 2 = \pm \frac{\sqrt{13} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \pm \frac{\sqrt{65}}{5}$

- Finally, solve for x:

$x = -2 \pm \frac{\sqrt{65}}{5}$

This gives the two solutions for the quadratic equation.

Summary of the Completing the Square Method

To recap, Sergey can follow these summarized steps:

1. Divide the entire quadratic by the coefficient of x^2 to normalize it.
2. Move the constant term to the right side of the equation.
3. Calculate half of the coefficient of x and square it.
4. Add and subtract this square inside the equation to complete the square.
5. Rearrange the perfect square trinomial into a squared binomial.
6. Take the square root of both sides, considering both positive and negative roots.
7. Solve for x, simplifying the radical expression as needed.

Additional Tips for Solving Quadratics by Completing the Square

- Always ensure the quadratic is in the proper form before starting.
- When dividing to normalize, remember to do so to every term.
- Be cautious when dealing with fractions; convert all numbers to common denominators for easier calculations.
- Rationalize denominators after taking square roots to keep expressions simplified.
- Remember that completing the square is a versatile method that can be used to derive the quadratic formula as well.

When to Use Completing the Square

Completing the square is particularly useful when:
- The quadratic does not factor easily.
- You want to derive the quadratic formula.
- The coefficients are convenient for completing the square.
- You are learning about the properties of quadratics and their graphs.

Conclusion

Sergey’s process of solving the quadratic equation $5x^2 + 20x + 7 = 0$ by completing the square involves systematic steps that, when followed carefully, lead to the solutions efficiently. By normalizing the coefficient, completing the square, and simplifying radicals, Sergey can confidently find the roots of the quadratic. Mastering this method enhances problem-solving skills and deepens understanding of quadratic functions, making it an essential technique in algebra.

Whether for exams, homework, or conceptual understanding, practicing completing the square with various equations will help Sergey and students alike become proficient in solving quadratics with confidence and precision.

Frequently Asked Questions

What is the first step Sergey should take to solve the quadratic equation $5x^2 + 20x + 7 = 0$ by completing the square?

He should start by dividing the entire equation by 5 to simplify the coefficients, resulting in $x^2 + 4x + 7/5 = 0$.

How does Sergey find the value to complete the square

in the quadratic equation?

He takes half of the coefficient of x (which is 4), divides it by 2 to get 2, then squares it to get 4, which will be used to complete the square.

What is the next step after calculating the value to complete the square?

He adds and subtracts this value (4) inside the equation to rewrite it as a perfect square trinomial.

How does Sergey rewrite the quadratic equation after completing the square?

He writes it as $(x + 2)^2 + (\text{constant term}) = 0$, adjusting the constants accordingly to maintain equality.

What is the purpose of completing the square in solving quadratic equations?

Completing the square transforms the quadratic into a perfect square trinomial, making it easier to solve for x by taking square roots.

What are the final steps Sergey should follow to find the solutions after completing the square?

He isolates the squared term, takes the square root of both sides (considering $\pm$), and then solves for x accordingly.

Can completing the square be used for all quadratic equations, and what are its limitations?

Yes, it can be used for all quadratic equations, but it is most straightforward when the quadratic is easily turned into a perfect square; otherwise, it may be more complex than using the quadratic formula.

Additional Resources

Sergey Is Solving $5x^2 + 20x + 7 = 0$. Which Steps Could He Use To Solve The Quadratic Equation By Completing

In the world of algebra, quadratic equations are fundamental, often serving as the gateway to understanding more complex mathematical concepts. When faced with an equation such as $5x^2 + 20x + 7 = 0$, students and mathematicians alike seek efficient strategies to find the solutions. Among the various methods, completing the square stands out for its elegance and conceptual clarity. This article explores the step-by-step process Sergey can follow to solve the quadratic equation by completing the square, offering a detailed and reader-friendly guide through the method's nuances.

Understanding the Quadratic Equation and Completing the Square

Before diving into the specific steps, it’s essential to grasp the nature of quadratic equations and the rationale behind completing the square.

A quadratic equation is any second-degree polynomial of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, and $a \neq 0$. The solutions to the quadratic equation—the roots—are the values of x that satisfy the equation.

Completing the square is a method that transforms a quadratic into a perfect square trinomial, making it easier to solve for x. The core idea involves rewriting the quadratic in the form $(x + d)^2 = e$, from which the solutions can be directly obtained by taking square roots.

This method not only provides solutions but also offers insights into the parabola's vertex form, making it a valuable tool for visualization and understanding.

Step-by-Step Process for Completing the Square in Sergey’s Equation

The specific quadratic equation Sergey is working with is:

$5x^2 + 20x + 7 = 0$

Let's explore how he can methodically complete the square to find the solutions.

Step 1: Normalize the Coefficient of x^2

The first step involves ensuring that the coefficient of x^2 is 1, simplifying subsequent calculations.

- Divide the entire equation by 5:

$(5x^2 + 20x + 7) \div 5 = 0 \div 5$

Which simplifies to:

$x^2 + 4x + (7/5) = 0$

This step makes it easier to focus on completing the square for the x terms.

Step 2: Isolate the Constant Term

Next, move the constant term to the other side to prepare for completing the square:

- Subtract (7/5) from both sides:

$x^2 + 4x = - (7/5)$

Now, the quadratic part is isolated on the left.

Step 3: Complete the Square on the Left Side

To complete the square:

- Identify the coefficient of x: 4

- Divide it by 2: 4 ÷ 2 = 2

- Square this result: $2^2 = 4$

- Add this square to both sides:

$x^2 + 4x + 4 = - (7/5) + 4$

This creates a perfect square trinomial on the left, which factors neatly.

- Express the left side as a squared binomial:

$(x + 2)^2 = - (7/5) + 4$

Step 4: Simplify the Right Side

Now, combine the right side:

- Convert 4 to a fraction with denominator 5:

4 = 20/5

- Add the fractions:

- (7/5) + (20/5) = (20/5) - (7/5) = (13/5)

- Note the sign:

Since the original was minus, the right side becomes:

$(x + 2)^2 = (20/5) - (7/5) = (13/5)$

So, the equation simplifies to:

$(x + 2)^2 = 13/5$

Step 5: Solve for x by Taking Square Roots

The next step involves taking the square root of both sides:

- Apply square root to both sides:

$x + 2 = \pm \sqrt{(13/5)}$

- Simplify the square root:

$\sqrt{(13/5)} = \sqrt{13} / \sqrt{5}$

- Rationalize the denominator:

$\sqrt{13} / \sqrt{5} (\sqrt{5} / \sqrt{5}) = (\sqrt{13} \sqrt{5}) / 5 = \sqrt{(13 5)} / 5 = \sqrt{65} / 5$

Thus, the solutions are:

$x + 2 = \pm (\sqrt{65}) / 5$

Final step: Isolate x:

- Subtract 2 from both sides:

$x = -2 \pm (\sqrt{65}) / 5$

Expressing the Solutions and Interpretation

The solutions Sergey obtains are:

$x = -2 + (\sqrt{65}) / 5$

and

$x = -2 - (\sqrt{65}) / 5$

These solutions are irrational (involving square roots), indicating that the quadratic has no rational roots but two real roots expressed in simplified radical form. The process of completing the square not only finds these roots but also reveals the structure of the quadratic's solutions.

Advantages of Completing the Square

Using the completing the square method offers several benefits:

- Conceptual Clarity: It provides a clear understanding of how quadratic equations relate to perfect squares.

- Versatility: It works regardless of whether roots are real or complex, especially when combined with the quadratic formula.

- Graphical Insight: Completing the square rewrites the quadratic in vertex form, revealing the parabola’s vertex and symmetry.

- Foundation for Derivatives and Integrals: The method underpins more advanced calculus techniques involving quadratic functions.

Additional Considerations for Sergey

While completing the square is systematic, there are some key points Sergey should keep in mind:

- Handling fractions: When the constant term is a fraction, ensure accurate addition and subtraction.

- Dealing with irrational roots: Rationalizing denominators simplifies radical expressions.

- Alternative methods: If the quadratic equation becomes more complicated, the quadratic formula might be more straightforward. However, completing the square remains a valuable skill for understanding the structure of solutions.

- Checking solutions: Substituting the solutions back into the original equation verifies their correctness, especially when dealing with irrational roots.

Conclusion: Mastering Completing the Square for Quadratic Equations

Sergey’s journey through solving $5x^2 + 20x + 7 = 0$ exemplifies the power and elegance of completing the square. By systematically normalizing the coefficients, isolating the quadratic terms, completing the square, and then solving for x, he uncovers the roots that satisfy the original equation. This method underscores a fundamental principle in algebra: transforming complex equations into manageable forms to reveal their solutions.

Whether used for solving specific equations or gaining deeper insights into quadratic functions' behavior, completing the square remains an essential tool in the mathematician's toolkit. For Sergey, mastering this technique not only solves the current problem but also lays a foundation for tackling more advanced mathematical challenges with confidence and clarity.

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