Show That The Result Of Applying A Cnot Gate Followed By A Hadamard Gate (on The First Qubit) To 2 Qubits

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Quantum computing is a rapidly advancing field that leverages the principles of quantum mechanics to perform computations far beyond the capabilities of classical computers. Central to this domain are quantum gates—fundamental operations that manipulate qubits, the basic units of quantum information. Understanding how different quantum gates interact and transform qubit states is crucial for designing quantum algorithms and understanding quantum circuit behavior.

In this article, we will explore a specific sequence of operations: applying a controlled-NOT (CNOT) gate followed by a Hadamard (H) gate on the first qubit, acting on a two-qubit system. Our goal is to analyze and demonstrate the resulting state after these operations, providing detailed derivations and insights into the process.

Overview of Quantum Gates Involved

Before delving into the calculations, it’s essential to understand the individual gates used in this sequence.

Controlled-NOT (CNOT) Gate

- The CNOT gate is a two-qubit gate that flips the state of the target qubit if the control qubit is in the state $|1\rangle$.
- Its matrix representation in the computational basis ($|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$) is:

$$
\text{CNOT} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

- It acts as follows:
- $|00\rangle \rightarrow |00\rangle$
- $|01\rangle \rightarrow |01\rangle$
- $|10\rangle \rightarrow |11\rangle$
- $|11\rangle \rightarrow |10\rangle$

Hadamard (H) Gate

- The Hadamard gate creates superpositions and is defined by the matrix:

$$
H = \frac{1}{\sqrt{2}}
\begin{bmatrix}
1 & 1 \\
1 & -1 \\
\end{bmatrix}
$$

- When applied to a single qubit, it transforms basis states as:
- $|0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
- $|1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

Initial State of the Two-Qubit System

The analysis depends on the initial state of the two qubits. For completeness, we consider a general initial state:

$$
|\psi_{initial}\rangle = \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle
$$

where $\alpha, \beta, \gamma, \delta$ are complex coefficients satisfying normalization:

$$
|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1
$$

To illustrate the process concretely, we often analyze the case where the initial state is a basis state, such as $|00\rangle$, but the derivation applies generally.

Step-by-Step Derivation of the Resulting State

The sequence of operations is:

1. Apply the CNOT gate to the initial two-qubit state.
2. Apply the Hadamard gate to the first qubit of the resulting state.

Let's analyze each step carefully.

Step 1: Applying the CNOT Gate

Given the initial state:

$$
|\psi_{initial}\rangle = \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle
$$

The CNOT acts on this state as:

$$
|\psi_{after\, CNOT}\rangle = (\text{CNOT}) |\psi_{initial}\rangle
$$

Applying CNOT to each basis state:

- $|00\rangle \rightarrow |00\rangle$
- $|01\rangle \rightarrow |01\rangle$
- $|10\rangle \rightarrow |11\rangle$
- $|11\rangle \rightarrow |10\rangle$

Therefore:

$$
|\psi_{after\, CNOT}\rangle = \alpha |00\rangle + \beta |01\rangle + \gamma |11\rangle + \delta |10\rangle
$$

Rearranged as:

$$
|\psi_{after\, CNOT}\rangle = \alpha |00\rangle + \beta |01\rangle + \delta |10\rangle + \gamma |11\rangle
$$

Step 2: Applying the Hadamard Gate to the First Qubit

Since the Hadamard acts only on the first qubit, we need to understand how it transforms each basis state:

- $|0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
- $|1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

Applying to the two-qubit states:

$$

|0\rangle \otimes |b\rangle \to \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |b\rangle
\]

\[
|1\rangle \otimes |b\rangle \to \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes |b\rangle
\]

Applying this to each component:

\[
|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[\left(|0\rangle + |1\rangle \right) \left(\alpha |0\rangle + \beta |1\rangle \right) \right] + \frac{1}{\sqrt{2}} \left[\left(|0\rangle - |1\rangle \right) \left(\delta |0\rangle + \gamma |1\rangle \right) \right]
\]

Expressed explicitly:

\[
|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle \left(\alpha |0\rangle + \beta |1\rangle \right) + |1\rangle \left(\alpha |0\rangle + \beta |1\rangle \right) \right] + \frac{1}{\sqrt{2}} \left[|0\rangle \left(\delta |0\rangle + \gamma |1\rangle \right) - |1\rangle \left(\delta |0\rangle + \gamma |1\rangle \right) \right]
\]

Now, distribute and combine terms:

\[
|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle (\alpha |0\rangle + \beta |1\rangle) + |1\rangle (\alpha |0\rangle + \beta |1\rangle) \right] + \frac{1}{\sqrt{2}} \left[|0\rangle (\delta |0\rangle + \gamma |1\rangle) - |1\rangle (\delta |0\rangle + \gamma |1\rangle) \right]
\]

Grouping similar terms:

\[
|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle (\alpha |0\rangle + \beta |1\rangle + \delta |0\rangle + \gamma |1\rangle) \right] + \frac{1}{\sqrt{2}} \left[|1\rangle (\alpha |0\rangle + \beta |1\rangle - \delta |0\rangle - \gamma |1\rangle) \right]
\]

Factor within the brackets:

\[
|\psi_{final}\rangle = \frac{1}{\sqrt{2}} |0\rangle \left[(\alpha + \delta) |0\rangle + (\beta + \gamma) |1\rangle \right] + \frac{1}{\sqrt{2}} |1\rangle \left[(\alpha - \delta) |0\rangle + (\beta - \gamma) |1\rangle \right]
\]

Expressed as a superposition over the basis states:

\[

|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[(\alpha + \delta) |0 0\rangle + (\beta + \gamma) |0 1\rangle + (\alpha - \delta) |1 0\rangle + (\beta - \gamma) |1 1\rangle \right]
\]

Special Cases and Examples

To make the derivation more concrete, consider specific initial states:

Example 1: Initial State $|00\rangle$

Set \(\alpha=1, \beta

Frequently Asked Questions

What is the effect of applying a CNOT gate followed by a Hadamard gate on the first qubit in a two-qubit system?

Applying a CNOT gate followed by a Hadamard gate on the first qubit transforms the initial state into a new entangled or superposed state depending on the initial state, effectively performing a basis change and entanglement operation that is fundamental in quantum algorithms.

How does the combination of a CNOT and a Hadamard gate affect the entanglement between two qubits?

The sequence can create or modify entanglement; specifically, applying a CNOT followed by a Hadamard on the control qubit can generate Bell states from certain initial states, thus establishing maximal entanglement between the qubits.

Can you show the mathematical transformation of a two-qubit state after applying a CNOT followed by a Hadamard on the first qubit?

Yes. For example, starting with $|00\rangle$, after the CNOT, the state remains $|00\rangle$; then applying Hadamard on the first qubit transforms it into $(|0\rangle + |1\rangle)/\sqrt{2} \otimes |0\rangle$, resulting in an entangled or superposed state, depending on the initial input.

What is the significance of applying a Hadamard gate after a

CNOT in quantum algorithms like quantum teleportation or superdense coding?

This combination is essential for creating superpositions and entanglement that enable protocols like teleportation and superdense coding, as it allows the manipulation of qubit bases and correlation structures needed for these algorithms.

How would you verify the final state after applying a CNOT followed by a Hadamard on the first qubit experimentally?

You would perform quantum state tomography to reconstruct the density matrix of the two-qubit system after the operations, then analyze the resulting state for expected entanglement and superposition properties predicted by the theoretical transformation.

Additional Resources

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Quantum computing is revolutionizing the way we think about information processing, promising unprecedented capabilities in fields ranging from cryptography to complex simulations. Central to this quantum revolution are quantum gates—fundamental operations that manipulate qubits, the quantum counterparts of classical bits. Among these, the CNOT (Controlled-NOT) gate and the Hadamard gate are two of the most pivotal. Understanding how these gates interact, especially when applied sequentially to multi-qubit systems, is fundamental for designing quantum algorithms and circuits.

In this article, we explore a specific quantum operation: applying a CNOT gate followed by a Hadamard gate on the first qubit of a two-qubit system. Our goal is to analyze this sequence thoroughly, demonstrating the resulting quantum state transformations step-by-step. This examination not only deepens our grasp of quantum gate interactions but also illustrates the elegant mathematics underpinning quantum computing.

Understanding the Building Blocks: Quantum Gates and Their Functions

Before diving into the combined operation, it’s essential to understand the individual gates involved: the CNOT and the Hadamard.

The CNOT Gate: Controlled-X Operation

The CNOT gate is a two-qubit gate that flips the state of the target qubit if and only if the control qubit is in the $|1\rangle$ state. Its matrix representation in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ is:

$$
\text{CNOT} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

This gate is fundamental for entangling qubits and implementing quantum algorithms like quantum teleportation and error correction.

Operational Effect:

- If the first qubit (control) is $|0\rangle$, the second (target) remains unchanged.
- If the first qubit is $|1\rangle$, the second qubit flips: $|0\rangle \rightarrow |1\rangle$, $|1\rangle \rightarrow |0\rangle$.

The Hadamard Gate: Creating Superposition

The Hadamard gate transforms a single qubit into a superposition of states. Its matrix form is:

$$
H = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1 \\
\end{bmatrix}
$$

Applying H to $|0\rangle$ yields:

$$
H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
$$

Similarly, applying H to $|1\rangle$ yields:

$$
H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)
$$

The Hadamard is instrumental in quantum algorithms, enabling qubits to exist in multiple states simultaneously—a key quantum advantage.

Initial State and Sequence of Operations

Let's consider an arbitrary initial two-qubit state:

$$
|\psi_{initial}\rangle = |\phi\rangle \otimes |\chi\rangle
$$

where:

$$
|\phi\rangle = a|0\rangle + b|1\rangle, \quad |\chi\rangle = c|0\rangle + d|1\rangle
$$

with complex coefficients satisfying normalization:

$$
|a|^2 + |b|^2 = 1, \quad |c|^2 + |d|^2 = 1
$$

Our goal is to analyze the effect of applying:

1. The CNOT gate, with the first qubit as control and the second as target.
2. The Hadamard gate on only the first qubit.

Sequence:

$$
|\psi_{final}\rangle = (H \otimes I) \cdot \text{CNOT} \; |\psi_{initial}\rangle
$$

where I denotes the identity operation on the second qubit.

Applying the CNOT Gate

Step 1: Express the initial state explicitly

$$
|\psi_{initial}\rangle = (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)
$$

Expanding:

\[
|\psi_{initial}\rangle = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle
\]

Step 2: Apply the CNOT gate

Recall the CNOT action:

- $|00\rangle \to |00\rangle$
- $|01\rangle \to |01\rangle$
- $|10\rangle \to |11\rangle$ (since control is $|1\rangle$, target flips)
- $|11\rangle \to |10\rangle$

Applying CNOT:

\[
\begin{aligned}
\text{CNOT} |\psi_{initial}\rangle &= ac|00\rangle + ad|01\rangle + bc|11\rangle + bd|10\rangle
\end{aligned}
\]

Rearranged:

\[
|\psi_{after\,CNOT}\rangle = ac|00\rangle + ad|01\rangle + bd|10\rangle + bc|11\rangle
\]

Applying the Hadamard Gate on the First Qubit

Step 3: Express the post-CNOT state

The state after CNOT (before Hadamard):

\[
|\psi_{after\,CNOT}\rangle = ac|00\rangle + ad|01\rangle + bd|10\rangle + bc|11\rangle
\]

Step 4: Apply \(H \otimes I\)

The Hadamard acts only on the first qubit, transforming:

\[
|0\rangle \to \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
\]
\[
|1\rangle \to \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)
\]

Apply this to each term:

- For $|0\rangle$ in the first qubit:

\[
|0\rangle \to \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
\]

- For $|1\rangle$ in the first qubit:

\[
|1\rangle \to \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)
\]

Now, rewrite each component:

1. \(ac|00\rangle = ac \cdot |0\rangle \otimes |0\rangle\)

\[
\Rightarrow ac \cdot \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes |0\rangle
\]

2. \(ad|01\rangle = ad \cdot |0\rangle \otimes |1\rangle\)

\[
\Rightarrow ad \cdot \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes |1\rangle
\]

3. \(bd|10\rangle = bd \cdot |1\rangle \otimes |0\rangle\)

\[
\Rightarrow bd \cdot \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes |0\rangle
\]

4. \(bc|11\rangle = bc \cdot |1\rangle \otimes |1\rangle\)

\[
\Rightarrow bc \cdot \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes |1\rangle
\]

Step 5: Combine all terms

\[
|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[ac (|0\rangle + |1\rangle) \otimes |0\rangle + ad (|0\rangle + |1\rangle) \otimes |1\rangle + bd (|0\rangle - |1\rangle) \otimes |0\rangle + bc (|0\rangle - |1\rangle) \otimes |1\rangle \right]
\]

Distribute and group similar terms:

\[

|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \left[(ac + bd)|0\rangle \otimes |0\rangle + (ac - bd)|1\rangle \otimes |0\rangle + (ad + bc)|0\rangle \otimes |1\rangle + (ad - bc)|1\rangle \otimes |1\rangle \right]
\]

Final Expression and Interpretation

The resulting state after applying the CNOT followed by the Hadamard on the first qubit is:

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